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Teacher Pension Enhancements and Staffing in an Urban School District

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Abstract

Many states enhanced benefits in teacher retirement plans during the 1990s. This paper examines the school staffing effects of one such enhancement in a major urban school district with mostly high poverty schools. Pension rule changes in 1999 for St. Louis public school teachers resulted in very large increases in pension wealth for active teachers, as well as a powerful increase in "push" incentives for earlier retirement. Simple descriptive statistics on retirement patterns before and after the enhancements suggest much earlier retirement resulted. Shorter teaching spells imply a steady state with more teaching vacancies and a larger share of novice teachers in classrooms. To better understand the long run effects of these changes and alternatives policies, the authors estimate a structural model of teacher retirement. Simulations of retirement behavior for a representative senior teacher point to shorter completed teaching spells and earlier retirement age as a result of the enhancements. By contrast, moving from the post-1999 to a DC- type plan would extend the teaching career of a representative senior teacher by roughly three years. Simulations of voluntary DC conversation plans suggest that many senior teachers would enroll, thereby reducing workforce turnover, and overall pension costs.

1 Introduction

Most public school teachers are covered by traditional, or more precisely, final average salary defined-benefit (FAS-DB) pension plans that base a teacher's retirement annuity on the average of the highest (typically final) several years of earnings. These types of plans are commonplace for state and local government employees, but DB plans of any sort have largely disappeared in the private sector, in favor of defined contribution (DC) plans such as 401k, IRA or 403b plans (Butrica, Iams, Smith, and Toder (2009)). The costs of teacher pension plans have risen sharply over time and account for a growing share of per pupil expenditures. In current dollars, employer teacher pension costs (excluding Social Security and teacher contributions) amounted to $520 per student or 4.8 percent of current per student expenditures in March, 2004. By September 2017, these costs had risen to $1284 per student or 10.5 percent of per student expenditures[1].

The fiscal stress for schools and districts resulting from these rising pension costs has been widely noted (e.g., Moody and Randazzo (2020)). There are a variety of factors that have contributed to these rising costs, most notably failure of state or local governments to make actuarially necessary contributions and less than forecast returns on plan investments. However, one often overlooked factor is the cost of pension enhancements that occurred during the 1990s. In response to the stock market boom during the 1990's, many pension plans enhanced benefits, and educator pensions were among the most actively enhanced. Between 1999 and 2001 alone, for example, the National Conference of State Legislators reported that educator pensions were enhanced in more than half the states (Koedel, Ni, and Podgursky, 2014).

Our focus in this paper is not on the rising costs of teacher plans, per se, but on the effect of pension rule enhancements on teacher labor supply and school staffing in St. Louis, an urban district that has difficulty attracting and retaining experienced teachers. In 1999,

[1]http://www.uaedreform.org/downloads/2017/12/employer-contributions-per-pupil-12-20-17.pdf

the pension formula for St. Louis teachers was changed substantially, with a large increase in the replacement rate accompanied by a cap on total benefits as percentage of final salary. These major changes in the pension rules provide an opportunity to study how pension rules influence teacher retirement decisions.

In general, pension enhancements, by lowering the price of leisure (retirement relative to work, tend to encourage more of the former. Studies of senior teachers consistently show a high degree of responsiveness in retirement timing to pension system incentives (Brown, 2013; Costrell and McGee, 2010; Furgeson, Strauss, and Vogt, 2006; Knapp, Brown, Hosek, Mattock, and Asch, 2016; Ni and Podgursky, 2016. Given popular concern with "teacher shortages", especially in high need schools, the consequences of expensive HR decisions that have the effect of reducing teacher labor supply deserve careful scrutiny.

In this paper we highlight the powerful "push" and "pull" incentives in the Saint Louis plan and show descriptively how changes in pension wealth accrual are associated with changes in retirement behavior in short panels. To explore the longer term effects and alternative policies, we estimate a structural model of retirement behavior, with parameters describing underlying teacher preferences that are invariant to pension rules. There are important advantages of estimating a structural model. First, it allows us to isolate and unbundle the effects of more complex changes plan rules. Second, it allows us to predict the effect of the rule changes in retirement beyond the sample period, since it takes many years for the full effects of pension rule changes on teacher retirement decisions to materialize. Finally, and most importantly, structural estimates allow us to examine the effect of alternative plans. In the context of St. Louis, it allows us to explore the labor supply and fiscal effects of partial or total conversion from the current FAS-DB to defined contribution (DC alternatives.

In the next section we describe the operation of the St. Louis pension plan and the manner in which pension wealth accrues over a teacher's work life, with the ensuing "pull" and "push" incentives. We show how the 1999 enhancements dramatically increased the "push" incentives for retirement and report descriptive data for retirement age and experience of teachers before and after the enhancements. We then specify and estimate our structural retirement model. The model is found to exhibit good fit within and out-of-sample. Using the structural model estimates we then simulate the long run effect of the enhancements on a representative senior teacher. These simulations imply a significant reduction in the length of teaching careers as a result of the enhancements. We then consider some DC alternatives, including voluntary DC conversions.

2 Pension Wealth Accrual Under Plan Changes

Teachers in Missouri are in three separate FAS-DB systems. Teachers in the St. Louis and Kansas City districts are covered by Social Security and are in two separate district plans. The rest of the public school teachers in the state are in the Public School Retirement System (PSRS), and are not covered by the Social Security system. There is no reciprocity between the St. Louis and other plans, meaning that the years of service in the formula below apply only to service in the current plan and not to any prior teaching employment.

Table 1 summarizes the St. Louis plan rules before and after 1999. Benefits at retirement are determined by a standard FAS-DB formula:

$$Annual\ Benefit = rf \times S \times FAS$$

where rf stands for replacement factor. S denotes years of teaching experience in the system. FAS is the average of the highest consecutive three years of salary, which are typically the three years prior to retirement. There is also a provision for early retirement at age 60 with a reduced annuity for teachers who do not qualify for regular retirement. After 1999, St. Louis teachers became eligible for a regular pension when they reached age 65 with at least five years of service or when the sum of their years of service and age equals at least 85 (known as "Rule of 85").

During the 1990's there were some small pension rule changes. However, the big change occurred in 1999. Prior to 1999, the replacement factor was 1.25 percent. After 1999, the replacement factor increased to 2 percent retroactively. For example, before 1999, a retired teacher with 30 years teaching experience and a final average salary of \$50,000 received an \$18,750 annuity ($.0125 \times 30 \times \$50,000$). After 1999, this same teacher would receive an annual annuity of \$30,000 ($.02 \times 30 \times \$50,0000$). This is a very large enhancement awarded to all active teachers, including those who were about to retire.

The new benefit was capped at 60 percent of final average salary. Thus a teacher with 31 or more years teaching experience can still only receive 60 percent of her final average salary, in spite of the fact that she contributes five percent of salary to the plan. Table 1 reports the pension rules before and after enhancement.[2]

($Table$ 1)

One way to assess the retirement incentives arising from these pension plan rules is to compute pension wealth at different points in a teacher's work life. Pension wealth is calculated as the expected present value of pension benefits at retirement age r as follows:

$$PW(r) = \sum_{s \geq r} (1+r)^{r-s} \pi(\, s|r)\, P(\, s|r\,),$$

where $\pi(\, s|r)$ is the conditional probability of survival given by age-dependent mortality rates, and $P(\, s|r\,)$ is the pension benefit if the teacher retires at age r. Pension wealth

[2]There were some subsequent rule changes regarding pension COLAs. Before 2000, "COLA adjustments were equal to any increase of 1% or greater in the CPI, subject to an annual maximum of 3% and **a cumulative maximum increase of 10%**". Effective July 1, 2000, retired members received a catch-up COLA at least 65% of CPI. After 2002, COLAs are paid to retired members when such COLAs are approved by both the Board of Education and the Board of Trustees.

captures the market value of obtaining this stream of annuity payments over time.

Following Costrell and Podgursky (2009), pension wealth accumulation can be calculated as follows:

$$pw(r) = \frac{PW(r) - (1 + inf) * PW(r - 1)}{Salary(r)},$$

where $PW(r)$ is pension wealth if retiring at age r, inf is inflation rate, $Salary(r)$ is salary at age r.

Figure 1 compares the pension wealth before and after pension enhancement for a representative teacher in St. Louis who enters the system at age 25 and works continuously until separation. Figure 2 compares the pension wealth accumulation under the pension rules before and after pension enhancement.

($Figure$ 1-2)

Figures 1 and 2 are consistent with the prior literature analyzing the "pull" and "push" incentives implied by different pension rules, e.g., Costrell and Podgursky (2009). The steeply rising accrual up to peak value tends to lock in teachers, while the decline in pension wealth past the peak tends to push teachers in to retirement. What is notable in both figures is how much more powerful the "push" incentives became after 1999. This is a direct result of the 60 percent of FAS salary cap. Pension wealth accrued faster because of the sharp increase in the formula factor from 1.25 percent to 2.0 percent, and then declines sharply because of the 60 percent cap. The result is much more powerful incentives pushing our representative teacher into retirement at age 55.

3 Retirement Behavior Before and After Pension Enhancements

In this section we compare the empirical retirement behavior before and after the 1999 pension enhancements. The data used are administrative data for St. Louis public school teachers. We examine two cohorts of teachers aged 50-62 at the beginning of the period and track them forward for five years. The pre-enhancement cohort begins in Fall, 1992 and is followed to Fall, 1996, well before enactment or discussion of the 1999 enhancement. The post-enhancement cohort begins in Fall, 2008 and is tracked to Fall 2012. Again, this is well beyond the initial implementation of the enhancement. We have a cohort of 696 female and 184 male teachers in the 1993 cohort. By the end of the panel 186 females and 69 males had retired. The 2009 cohort had 692 female teachers and 198 male teachers of whom 396 female teachers and 116 male teachers had retired by the end of the panel. Table 2 reports basic descriptive statistics for the two cohorts.

(*Table* 2)

Table 2 shows that teacher retirement behavior differs sharply under pre- and post-enhancements rules in two important ways. The first difference is that in the post enhancement period teachers retire earlier than in the pre-enhancement period. This is seen clearly in Figure 3 which reports the employment survival rates over the five years. The post enhancement survival curve is well below the earlier cohorts for both female and male teachers.

Figures 4 and 5 report age and experience distributions for the two cohorts. Here it is important to note we only observe completed age and experience spells for teachers who have retired. Nonetheless, among the retirees, the cumulative age distribution is clearly displaced left for females. The results for males are somewhat more ambiguous. For both groups, however, there is a pronounced left displacement of the experience distribution.

($Figure$ 3-5)

It is noteworthy that the average experience of the senior teachers at the beginning of the post-enhancement subsample period is considerably lower than at the beginning of the pre-enhancement sample despite the fact that the average age is nearly identical. In 2009, at the beginning of the post-enhancement sample the female teachers are about 2.8 years less experienced than their same age-group counterpart in 1993, and male teachers in 2009 are about 4.8 years less experienced. This substantial difference in experience suggests in the post-enhancement sample teachers older than 50 tended to retire earlier (with less experience) due to the change in pension incentives. There are two implications of this observation. First, conditional on age, one would expect less experienced senior teachers retire later, hence the earlier retirement by the post-enhancement cohort shown in Figures 4 and 5 is even more striking. Second, some teachers of age 50-62 are retirement-eligible prior to the beginning of the sample period. Their decision to stay on the job reflects preferences unobserved to the researchers. The distributions of these preferences are likely differ at the beginning of the pre- and post-enhancement periods because the pension rules themselves have reshaped the pool of senior teachers. This illustrates why it is important to take account of this baseline sample selection in estimating a structural retirement model. The parameters of the structural model influence retirement behavior prior to the sample period (hence sample selection) as well as retirement behavior in the sample period (see Appendix A.2 for modelling details.)

Before turning to structural estimation, we round out our empirical discussion by presenting simple regression-adjusted retirement rates for the two cohorts. Table 3 reports the results of fitting a simple linear probability model that predicts probability of retirement as a function of baseline age, experience, and whether a teacher is in the post-enhancement cohort. The coefficient on the post-enhancement cohort dummy implies that the probability of retirement, averaged over all age and experience cells was 37 percentage points higher for females, and 30 percentage points higher for males.

$(Table\ 3)$

The evidence in Tables 2 and 3 and Figures 3-5 suggests that the 1999 enhancements significantly shorted teaching careers and that the "push" effects were very potent. However, to explore further the long-run effect of these changes on senior teachers, and consider how teachers would respond to alternate retirement plans, in the next section we estimate a structural model of retirement behavior.

4 Estimating a Dynamic Retirement Model

Stock and Wise (1990) developed a structural model of retirement which was extended by Ni and Podgursky (2016) to an analysis of public school teachers. We apply this model, with several innovations, to an analysis of St. Louis teachers.

Consider an employed teacher who has not retired at the beginning of year t. The present value of expected lifetime utility for the teacher if she retires in year r ($r \geq t$) is:

$$E_t V_t(r) = max_{c_1,c_2} E_t \{ \sum_{s=t}^{r-1} \beta^{s-t} \pi(s|t) U_w(Y_s, B_s(c_1)) + \sum_{s=r}^{T} \beta^{s-t} \pi(s|t) U_r(P_s(c_2), B_s(c_1)) \} \quad (4.1)$$

where

$$U_w(Y_s,\ B_s(c_1)) = (k_s((1-c)Y_s + I_s^1 B_s(c_1)))^{\gamma} + \omega_s, \quad (4.2)$$

$$U_r(P_s(c_2),\ B_s(c_1)) = (I_s^2 P_s(c_2) + I_s^1 B_s(c_1))^\gamma + \xi_s. \tag{4.3}$$

$$I_s^1 = \begin{cases} 1 & if\ s \geq c_1 \\ 0 & otherwise \end{cases} \tag{4.4}$$

$$I_s^2 = \begin{cases} 1 & if\ s \geq c_2 \\ 0 & otherwise \end{cases} \tag{4.5}$$

The value function V depends on future annual earnings Y_s before retirement, pension benefits $P_s(c_2)$ if the teacher starts collecting a pension benefit in year c_2, and Social Security benefits $B_s(c_1)$ if she starts collecting Social Security in year c_1. The collection years c_1 and c_2 are optimally chosen to maximize the expected utility given the retirement year r. The parameter c is the contribution rate for teachers before retirement, and k_s represents the teacher's preference for teaching versus retirement, which, in turn, depends on the teacher's age in year s. We assume $k_s = k(\frac{60}{age})^{k_1}$.

The expected gain from postponing retirement to year r is:

$$G_t(r) \ = \ E_t V_t(r) \ - E_t V_t(t). \tag{4.6}$$

Each year, an employed teacher has two choices: continue teaching or retire. The expected gain from postponing retirement can be seen as the "option value" of continuing to work, which is a key feature of this model. Retirement occurs if the value of continuing teaching is less than the value of retiring, i.e., the option value of continued teaching is negative.

The teacher's future salary, pension benefits, and Social Security benefits are assumed to be predictable[3]. We know the salary of teachers in the first period and then we predict the future salary with forecasts based on historical data.

The expected gain from retiring in year r (later than t) is

$$G_t(r) = E_t V_t(r) - E_t V_t(t) = g_t(r) + K_t(r)\nu_t, \tag{4.7}$$

where the first term, $g_t(r) = \sum_{s=t}^{r-1} \beta^{s-t}\pi(s|t)\,((k_s((1-c)Y_s + I_s^1 B_s(c_1)))^\gamma)+$

$\sum_{s=r}^{T} \beta^{s-t}\pi(s|t)\,((I_s^2 P_s(c_2) + I_s^1 B_s(c_1))^\gamma - [\sum_{s=t}^{T} \beta^{s-t}\pi(s|t)\,(I_s^2 P_s(c_2) + I_s^1 B_s(c_1))^\gamma]$, depends on pension rules. The second term, $K_t(r) = \sum_{s=t}^{r-1}(\beta\rho)^{s-t}\pi(s|t)$, depends on the mortality rates. Both terms depend on the parameters we need to estimate. The preference error $\nu_t = \omega_t - \xi_t$. We assume ν_t follows an AR(1) process: $\nu_t = \rho\nu_{t-1} + \epsilon_t$, where ϵ_t is iid $N(0, \sigma^2)$.

Suppose $r^\dagger$ solves $max_{r\in\{t+1,t+2,......T\}} E_t V_t(r)$. Thus, the teacher will continue working at t if $G_t(r^\dagger) = E_t V_t(r^\dagger) - E_t V_t(t) > 0$. The probability of retirement at time t is $\mathrm{P}[R = t] = \mathrm{P}[G_t(r) \leq 0, \forall r \geq t+1] = \mathrm{P}[G_t(r^\dagger) \leq 0]$, which can be represented as

$$\mathrm{P}[g_t(r_t^\dagger)/K_t(r_t^\dagger) \leqslant -\nu_t], \tag{4.8}$$

In this model, there are six unknown parameters to be estimated, which are listed in Table 4. Details concerning the construction and estimation of the sample likelihood function are given in the Appendix A.1.

($Table$ 4)

The data we use for estimation includes all teachers aged 50-62 at the 2009-10 school year, which is the cohort of teachers under the post-enhancement pension rules described in

[3]Teachers' salaries are largely determined by the district's salary schedule and can be predicted using the following formula: $Salary_{t+i} = Salary_t * exp(b_1 * i + b_2 * [(t+i)^2 - t^2] + b_3 * [(t+i)^3 - t^3])$, where b1=0.03569, b2=-0.00053652, b3=-0.00000372.

Table 2. We track this cohort of teachers for the next five years. Using Maximum Likelihood estimation (MLE) methods in Ni and Podgursky (2016), we obtain the estimates of six structural parameters reported in Table 5.

$(Table\ 5)$

All of these parameter estimates are statistically significant, plausible, and estimated with reasonable to high precision. Compared with the estimates in Ni and Podgursky (2016), these "deep" parameters are quite similar even though the St. Louis teachers are in a separate and substantially different FAS-DB pension system (e.g., St. Louis teachers are covered by Social Security, teachers in the state-wide plan are not).

The parameter β measures the teacher's discount rate. The parameter γ is significantly less than unity for both female and males, which implies risk-aversion. σ and ρ are two parameters related to unobserved retirement preferences. These unobserved preferences ("preference shocks") are substantial and persistent (i.e., ρ is positive and significant). Since the option value is the key variable in this model that predicts retirement behavior, the large standard deviation estimates of 4001.108 and 3770.432, for females and males respectively, are reasonable since they capture all other omitted variables that influence retirement decisions. The parameters k and k_1 together measure the disutility of teaching relative to retirement. These estimates imply that the attractiveness of retirement (leisure) relative to teaching rises with age (Ni and Podgursky, 2016).

The structural model provides a good fit to these data both in and out of sample. Figure 6 reports employment survival rates in-sample for the 2009 senior teacher cohort and out-of-sample for the 1993 cohort. Note that the 1993 out-of-sample cohort is a non-overlapping group of teachers who face a very different set of pension rules. Nonetheless, the fit is good for both groups. More extensive plots of age and experience are provided in the Appendix

B. Overall, the structural model exhibits good fit, both in and out-of-sample.

($Figure$ 6)

5 Simulating the Effects of Pension Enhancements

As noted in the introduction, the 1999 enhancement of the replacement factor was accompanied by a cap on pension benefits of 60% of the final average salary. Using the estimated structural model we conduct counter-factual simulations on the effect of each aspect of the pension rule changes.

In order to remove the influence of the composition of teachers in the post-enhancement sample and highlight the policy effect, we choose a representative female and male teacher who is 50 years old and with 25 years of experience. Such a teacher is entering the retirement window. We analyze the effects of the pension enhancements and three counterfactual pension alternatives over a 30 year time horizon, by which time the teacher will have retired.

Table 6 reports the effects of the different pension rules on the timing of retirement. We treat the after-enhancement pension rules as our baseline case (Column 2) and subtract enhancements. Column 5 reports the effect of eliminating all of the enhancements. The elimination of pension cap of 60%, a reduction of 0.75% in the replacement factor, and a COLA cap leads to a higher level of average retirement age and extends the teaching career by 2.6 years for females and 1.9 years for males. This shows that the less remunerative and uncapped pre-enhancement pensions exerted a weaker "push" effect, and hence yielded longer teaching careers.

($Table$ 6)

Table 6 also decomposes the effects of the pension enhancements into the components due to increasing the replacement factor and the effect of a benefit cap. Eliminating the cap on pension benefits increases the average career experience at retirement by 0.7 years for female teachers and 0.5 years for male teachers. Imposing the COLA cap increases retirement experience from 0.3-0.6 years. Reducing the replacement factor from 2% to 1.25% increases the average retirement experience by 1.3 years for female and 1.1 years for male teachers.

In short, the somewhat surprising finding from Table 6 is that while the 60% cap, which has a clear "push" effect, shorted teaching careers, the larger effect came from the increase in the replacement factor from 1.25% to 2%. The COLA cap also had a potent effect.

6 The Effect of DC plans

6.1 Mandatory Conversion for Senior Teachers

Over the last several decades many private sector employers have frozen or closed their DB plans and put workers into defined contribution (DC) plans (Butrica et al. (2009)). The final column of Table 6 simulates the effect of such a policy for St. Louis teachers. This plan would freeze accrual of pension wealth under the current DB plan and convert all future contributions toward a less expensive (but still generous by private sector standards) DC plan. Specifically, the teacher would receive the full value of her accrued pension wealth. We assume a 4% annual nominal return on this balance. Going forward, the teacher would contribute 5% of her salary to the DC plan (which matches her contribution rate to the DB plan) and the district would contribute 10%. The district contribution rate is less than the current cost of the DB plan.

The last column of Table 6 shows the behavior of our representative senior teacher under such a hypothetical DC plan. The mandatory conversion to a DC plan produces a substantial increase in additional teaching years and retirement age. As compared to the current plan, completed experience at retirement rises by 3.3 and 2.6 years for females and males, respectively. Average age at retirement rises from 56.4 to 59.7 for females and 56.5 to 59.1 for males. These substantial increases from a DC conversion are robust to modest changes in underlying assumptions regarding returns or contribution. This exercise highlights the potent effect of the "push" incentives built into both the current and pre-enhancement DB plans. A DC conversion neutralizes the "push" and thus substantially extends a teaching career.

Of course a mandatory conversion to DC may reduce the welfare of late-career teachers who are far from reaching the pension wealth peak. Thus in the next section we consider effects of a voluntary DC conversion.

6.2 Voluntary Conversion for Senior Teachers

While many private sector employers over the last several decades have closed their DB plans and moved employees to DC plans, state courts have generally found such policies illegal under state constitutions for state and local workers, including teachers (Monahan (2010)). However, given the large DC effects observed in Table 6, it is worth considering some voluntary policies that would potentially lower plan liabilities and extend teaching careers. This is particularly relevant given the large unfunded liabilities of many state teacher plans, and the current COVID19-associated fiscal problems. Since these are voluntary DC conversions, they would not run afoul of the courts. In this section we consider effects of one-time voluntary conversions to a DC plan designed to reduce employer costs and pension fund liabilities.

We simulate the outcome of the following experiment. We assume that senior teachers in the current DB plan are offered a one-time conversion option in the initial year t. Suppose in year t a teacher with (age, experience) (a,e) has DB pension wealth W(a,e). We consider an experiment in which the teacher is offered a one-time option to convert from the STL DB plan to a DC plan with an initial balance of 80% or 90% of W(a,e), i.e., the teacher gets a "haircut" of 10 to 20 percent of the balance of her DC plan.

Going forward the value in this account grows by the nominal interest rate of 4% (on the fund balance) and the teacher and district each contribute 10% of teacher current salary each year until retirement. Modelling details of the DC plan are given in Appendix C.

The teacher has three choices in the initial period of this offer: retire under the current DB plan, remain in the DB plan and continue teaching, or convert from the DB to DC plan and continue teaching. After the initial period, the remaining teachers choose either to retire or continue teaching but there is no further option to switch pension plans. The options are (1) $d_t = 1$: retire in t under DB with value $V_t(t)$, (2) $d_t = 2$: stay with DB and continue teaching, (2) $d_t = 3$: switch the current pension wealth to DC in t.[4]

We use retirement as the reference and write the expected gain from retirement in year r under DB over retirement in the current period t as: $G_t^{DB}(r) = \mathbb{E}_t V_t(r) - \mathbb{E}_t V_t(t)$ where $\mathbb{E}_t V_t(.)$ is the expected value function under the DB plan defined in (4.1). Let $r^{\dagger}$=$argmax$ $G_t^{DB}(r)$ for $r > t$. The highest net value for a SW teacher choosing option 2 is $G_t^{DB}(r^{\dagger})$.

Similarly, we write the expected gain from retirement in year r under DC over retiring in t as $G_t^{DC}(r) = \mathbb{E}_t V_t^{DC}(r) - \mathbb{E}_t V_t(t)$. Let $r^{\#}$=$argmax$ $G_t^{DC}(r)$ for $r > t$. The highest net value for a SW teacher choosing option 3 is $G_t^{DC}(r^{\#})$.

Let the best net value relative to retirement be $G_t = max\{0, G_t^{DB}(r^{\dagger}), G_t^{DC}(r^{\#})\}$.

The optimal choice given age, experience, and preference error is $d_t = 1$ if $G_t = 0$; $d_t = 2$ if $G_t = G_t^{DB}(r^{\dagger})$; $d_t = 3$ if $G_t = G_t^{DC}(r^{\#})$. Since the optimal choice depends on the preference

[4]Under the assumptions of the modelling exercise and the fact that the DC conversion returns less than 100 percent of pension wealth, a fourth option – retire immediately under DC – is dominated by immediate retirement under the existing DB plan.

error ν_t it may differ for teachers of the same age and experience.

($Table$ 7)

We report in Table 7 the percentage of teachers who would take a one-time offer to convert from the current DB plan to the DC plan under the terms described above under for several age-experience combinations. To make this exercise more policy relevant, we consider two scenarios regarding the conversion option. In the upper panel, teachers are offered the option of receiving 90% of their pension wealth with ongoing employer DC contributions. In the lower panel, the conversion rate is 80% of pension wealth. Thus both options reduce employer and pension fund costs, first, through less than full conversion of accumulated pension wealth, and second, through continued employment with lower pension contributions. In general, teachers with low pension wealth stay in the DB plan. Teachers with higher pension wealth but who prefer teaching tend to convert to the DC plan.

The DC plan extends the average teaching career. This is primarily because the DC conversion removes the penalty imposed by the 60% cap in the DB plan. For example, with an 90% conversion, the average extension of a teaching career for a teacher aged 56 with 24 years experience (averaged over takers and non-takers) is 1.3 years.

Turning to the fiscal impact of the voluntary DC conversion, pension costs under the DC plan for the employer include the of DB pension wealth in the initial period plus the 10% annual salary matching prior to retirement. With more years of teaching (and hence fewer years of retirement) under the DC plan, and less than full conversion, average pension costs fall for teachers who accept the offer. For a 56 year old teacher with 24 service years, average costs fall by $119,377. Although fewer teachers take an 80% conversion as compared to 90%, average savings per teacher generally increase. The pension cost saving is higher for the 56-24 age-experience group because they have accumulated a sizable pension wealth to start the DC cash balance (in contrast to the 54-22 group but they are still many years away from expected retirement (in contrast the 60-28 group. For the 56-24 teachers, the flexibility of the DC plan is particularly valuable and they are willing to commit to the DC plan at a lower expected pension wealth.

While these voluntary conversion simulations undoubtedly push the envelope for our structural equation model, which is estimated off of retirement decisions, not buyouts, they do at least suggest that many senior teachers might opt into voluntary DC conversions that would yield cost-savings for districts and pension plans.

7 Conclusion

Many states enhanced benefits in teacher retirement plans during the 1990s. This paper examines the effect of a major enhancement for teachers in St. Louis public schools in 1999. Pension rule changes in 1999 resulted in very large increases in pension wealth for active teachers, as well as a powerful increase in "push" incentives for earlier retirement. Simple descriptive statistics on retirement patterns before and after the enhancements suggest earlier retirement resulted. However, to better understand the long run effects of these changes, we estimate a structural retirement model using panel data for the St. Louis teachers. Simulations based on the structural estimates imply shorter completed teaching spells and earlier retirement ages for a representative senior teacher as a result of the enhancements. These shorter teaching spells imply a steady state with more teaching vacancies and a larger share of novice teachers in classrooms. In contrast, conversion to a DC type plan, by eliminating the "push" incentives into retirement, would substantially lengthen expected teaching careers. A mandatory conversion to a DC plan would substantially extend expected teaching career among senior teachers. Our simulations also suggest that many senior teachers would opt into a voluntary DC plan as well, which would extend teacher careers and reduce pension fund liabilities.

As a result of rising pension plan costs and a growth in unfunded liabilities, retirement benefits for new St. Louis teachers (hired in 2018 or later) were cut and contribution rates for new teachers increased sharply from 5 to 9 percent of salary. However, it will be many years before these newly hired teachers will be eligible for retirement. In the meantime, the pension incentives described in this paper will be shaping retirement incentives, and shortening teaching careers of incumbent teachers for many years to come. More generally, the elevated risk in stock market and sharp drop in state and local tax revenues associated with the COVID19 pandemic will undoubtedly exacerbate financing challenges for these public employee plans. The structural methods used in this paper can help policy makers better understand the short and long run staffing and fiscal consequences of changes in pension plan design.

Tables and Figures

Table 1: Summary of Pension Rules

	Before 1999	After 1999
Normal Pension	$Age + Experience \geq 85 (Rule\, of\, 85)$	$Age + Experience \geq 85 (Rule\, of\, 85)$
	or $Age \geq 65$ with $Experience \geq 5$	or $Age \geq 65$ with $Experience \geq 5$
Early Pension	$Age \geq 60$	$Age \geq 60$
Replacement Factor	1.25%	2%
Maximum Benefit	No	60%

Table 2: Sample Statistics: Before and After 1999 Enhancements

Female Teachers	# of Teachers		Age		Experience	
	93-97	09-13	93-97	09-13	93-97	09-13
Female Teachers in 1993(2009)	696	692	55.29	55.88	23.36	20.55
Retired in 1993(2009)	37	64	58.70	56.70	26.19	24.75
Retired in 1994(2010)	34	131	59.97	58.71	29.88	24.31
Retired in 1995(2011)	32	48	60.91	58.92	30.06	23.79
Retired in 1996(2012)	39	56	60.69	59.09	30.36	22.77
Retired in 1997(2013)	46	97	60.59	60.29	29.48	24.68
Not Retired	508	296	58.27	58.54	25.99	22.33
Male Teachers	# of Teachers		Age		Experience	
	93-97	09-13	93-97	09-13	93-97	09-13
Male Teachers in 1993(2009)	184	198	55.00	56.19	23.54	18.78
Retired in 1993(2009)	16	17	57.38	56.41	27.38	22.24
Retired in 1994(2010)	6	35	59.67	58.86	30.67	21.74
Retired in 1995(2011)	18	7	56.83	58.71	26.44	18.57
Retired in 1996(2012)	12	22	60.42	60.14	31.92	21.55
Retired in 1997(2013)	17	35	60.71	60.26	30.53	22.46
Not Retired	115	82	58.00	59.11	25.54	21.62

Note: The table compares two cohorts of teachers aged 50-62 in the 1992-93 and 2008-09 school years. We track the two cohorts forward for five years. The age and experience columns report the averages for teachers in the base year, those who retired in each subsequent year, and who were not retired at the end of the sample period.

Table 3: Regression Estimates: Linear Probability Models

	Dependent variable: Retirement	
	(1) Female	(2) Male
With Pension Enhancement	0.366***	0.300***
	(0.039)	(0.077)
Experience	0.011***	0.010***
	(0.001)	(0.003)
Age	0.048***	0.040***
	(0.003)	(0.007)
Salary($000)	−0.003**	−0.006*
	(0.002)	(0.003)
Constant	−2.524***	−1.872***
	(0.184)	(0.376)
Observations	1,388	382
R^2	0.264	0.161
Adjusted R^2	0.262	0.152
Residual Std. Error	0.424 (df = 1383)	0.461 (df = 377)
F Statistic	124.116*** (df = 4; 1383)	18.090*** (df = 4; 377)

Note: The table shows the results of linear probability model using the two cohorts of teachers in table 2 in the base year.
Model: $Retire_i = \beta_0 + \beta_1 Enhancement_i + \beta_2 Experience_i + \beta_3 Age_i + \beta_4 Salary_i + \epsilon_i$;
Dummy variable $Retire = 1$ if the teacher retired during the panel;
Dummy variable $With\,Pension\,Enhancement = 1$ if the teacher is in the 09-13 cohort;
*p<0.1; **p< 0.05; ***p<0.01. The standard errors are in parentheses.

Table 4: Structural Model Parameters

Parameters	Economic Interpretation
$\beta \in (0,1)$	discount factor
$k \in (0,1)\,\&\,k_1 > 0$	$k_s = k(\frac{60}{age})^{k_1}$: preference of teaching versus retiring
$\gamma \in (0,1]$	curvature in the utility function ($\gamma < 1$ indicates concavity)
$\sigma > 0$	magnitude of unobserved preference shocks
$\rho \in (-1,1)$	persistence in unobserved preference shocks

Table 5: Estimates of Structural Parameters

	Female	Male
β	0.964	0.963
	(0.014)	(0.026)
k	0.682	0.688
	(0.061)	(0.108)
k_1	0.900	0.751
	(0.100)	(0.189)
σ	4001.108	3770.432
	(388.878)	(853.972)
γ	0.684	0.693
	(0.015)	(0.027)
ρ	0.637	0.682
	(0.030)	(0.053)
Log-Likelihood	-1053.697	-314.897
Number of Teachers	692	198

Note: Parameters are estimated by post-enhancement (09-13) cohort.

The standard errors are in parentheses.

Table 6: Effects of Alternative Pension Rules on A Representative Senior Teacher

Representative Female Teacher		With Enhancements	Policy A	Policy B	No Enhancements	DC Policy
	Replacement Factor	2%	2%	2%	1.25%	n.a.
Pension Rules	Cap	60%	No	No	No	n.a.
	COLA Cumulative Max.	No Cap	No Cap	1.1	1.1	n.a.
Average Retired Age		56.4	57.1	57.7	59.1	59.7
Average Retired Experience		31.4	32.1	32.7	34.1	34.7
Additional Years of Teaching		6.3	7.0	7.7	9.0	9.7
Difference in Additional Years		-	0.7	1.3	2.6	3.3
Representative Male Teacher		With Enhancements	Policy A	Policy B	No Enhancements	DC Policy
	Replacement Factor	2%	2%	2%	1.25%	n.a.
Pension Rules	Cap	60%	No	No	No	n.a.
	COLA Cumulative Max.	No Cap	No Cap	1.1	1.1	n.a.
Average Retired Age		56.5	57.0	57.3	58.4	59.1
Average Retired Experience		31.5	32.0	32.3	33.4	34.1
Additional Years of Teaching		6.4	6.9	7.2	8.3	9.0
Difference in Additional Years		-	0.5	0.8	1.9	2.6

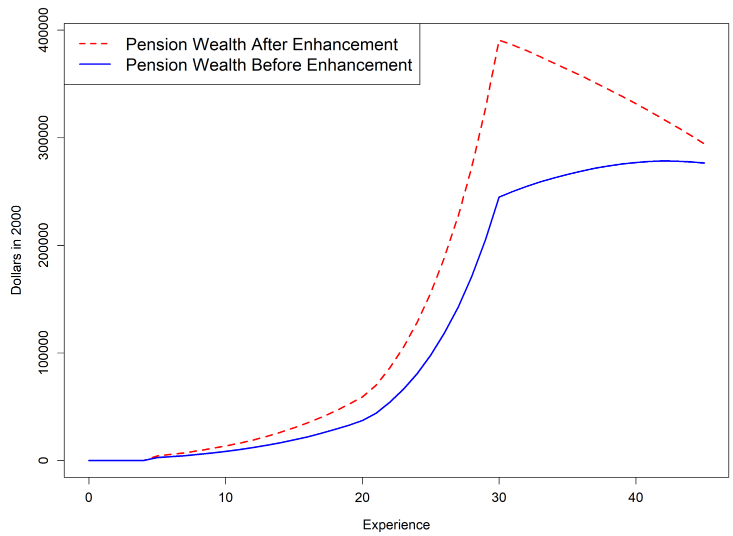
Note: The simulation tracks a representative female and male teacher aged 50 with 25 years experience for the next 30 years under four cases: a baseline with enhancements; Policy A eliminates the pension cap of 60%; Policy B further imposes the maximum increase of 10% on the retirement COLA adjustment; before-enhancement policy; Defined Contribution (DC) Policy with investment returns of 4%. Additional teaching years is the number of years the teacher is expected to teach before retirement. Difference in additional teaching years is the additional teaching years under an alternative policy minus that under the baseline DB policy (with the enhancements in place.) The DC plan assumes that teachers contribute 5% and the district contributes 10% of salary.

Table 7: Effects of a One-Time Voluntary DC Option on Representative Senior Teachers

Female Teachers	Age	Experience	Acceptance Rate	Difference in Additional Teaching Years	Pension Cost Saving
90% Conversion	54	22	0	0	$0
	56	24	0.803	1.327	$119,377
	58	26	0.698	0.739	$38,659
	60	28	0.787	0.662	$11,053
	62	30	0.719	0.303	$5,478
80% Conversion	54	22	0	0	$0
	56	24	0.769	1.398	$136,822
	58	26	0.671	0.789	$66,451
	60	28	0.767	0.633	$48,808
	62	30	0.700	0.256	$40,965
Male Teachers	**Age**	**Experience**	**Acceptance Rate**	**Difference in Additional Teaching Years**	**Pension Cost Saving**
90% Conversion	54	22	0.234	0.417	$37,580
	56	24	0.727	0.926	$96,147
	58	26	0.661	0.598	$29,711
	60	28	0.791	0.544	$2,880
	62	30	0.736	0.279	-$2,314
80% Conversion	54	22	0.141	0.307	$22,839
	56	24	0.703	0.997	$110,970
	58	26	0.639	0.646	$53,856
	60	28	0.775	0.532	$37,281
	62	30	0.721	0.247	$30,372

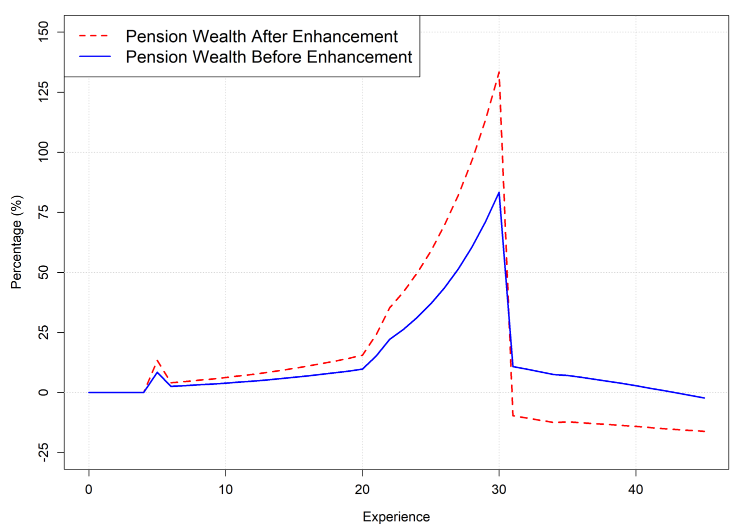
Note: The nominal return is assumed to be 4%, inflation rate is assumed to be 3%, and the employer and teacher contribution rate is assumed to be 10%, and 5%, respectively. “90% (80%) Conversion” means that the initial cash balance is 90% (80%) of current pension wealth if the teacher chooses to switch to the DC Plan. Acceptance rate is the probability of accepting the DC option. The difference in additional teaching years is the expected teaching years with the DC option minus that without the DC option. Pension cost saving is the expected pension cost reduction with the DC option from that without the DC option. In both cases the change is the average of overall all teachers of the given age/experience combination, not just the teachers who choose the DC option.

Figure 1: Pension Wealth Accrual for a Teacher Entering at Age 25



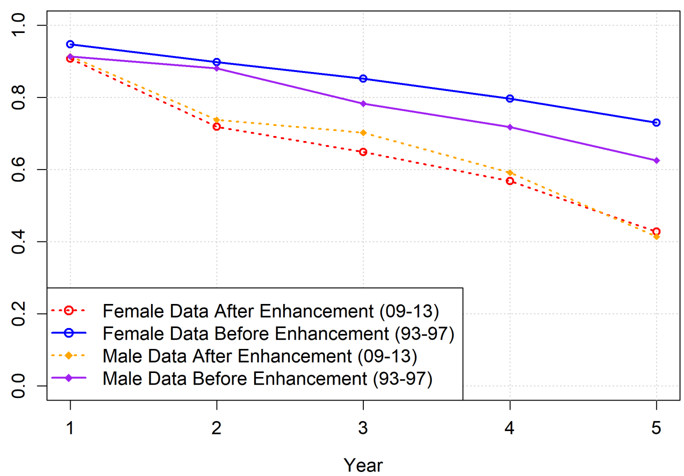
Note: The figure compares pension wealth accrual before and after the 1999 pension enhancements for a representative teacher in St. Louis who enters the system at age 25. For prediction of salary we use a cubic function of experience.

Figure 2: Annual Changes in Pension Wealth for a Teacher Entering at Age 25



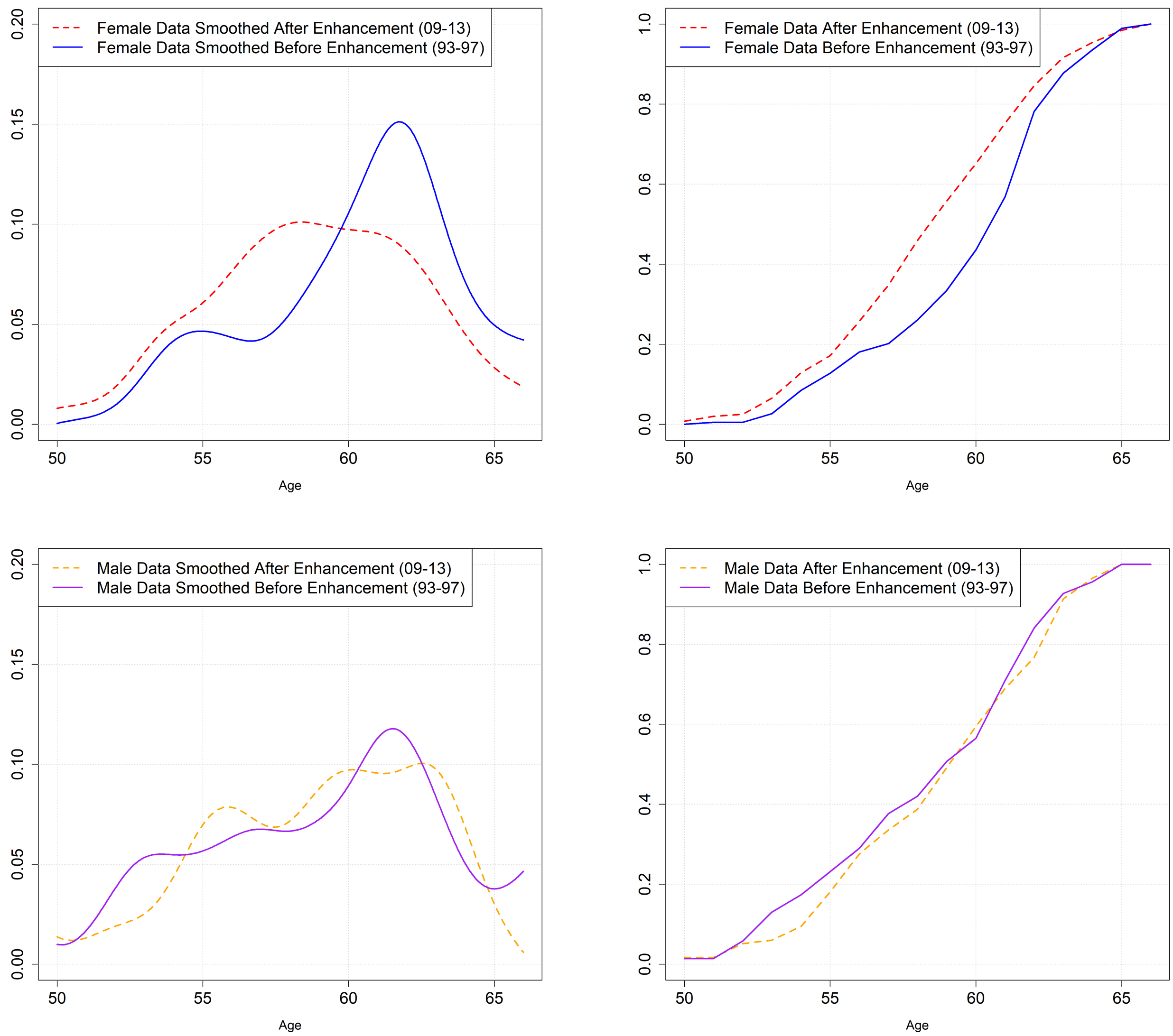
Note: The figure compares the pension wealth accumulation under the pension rules before and after the 1999 pension enhancements.

Figure 3: Employment Survival Rate Before and After Pension Enhancements



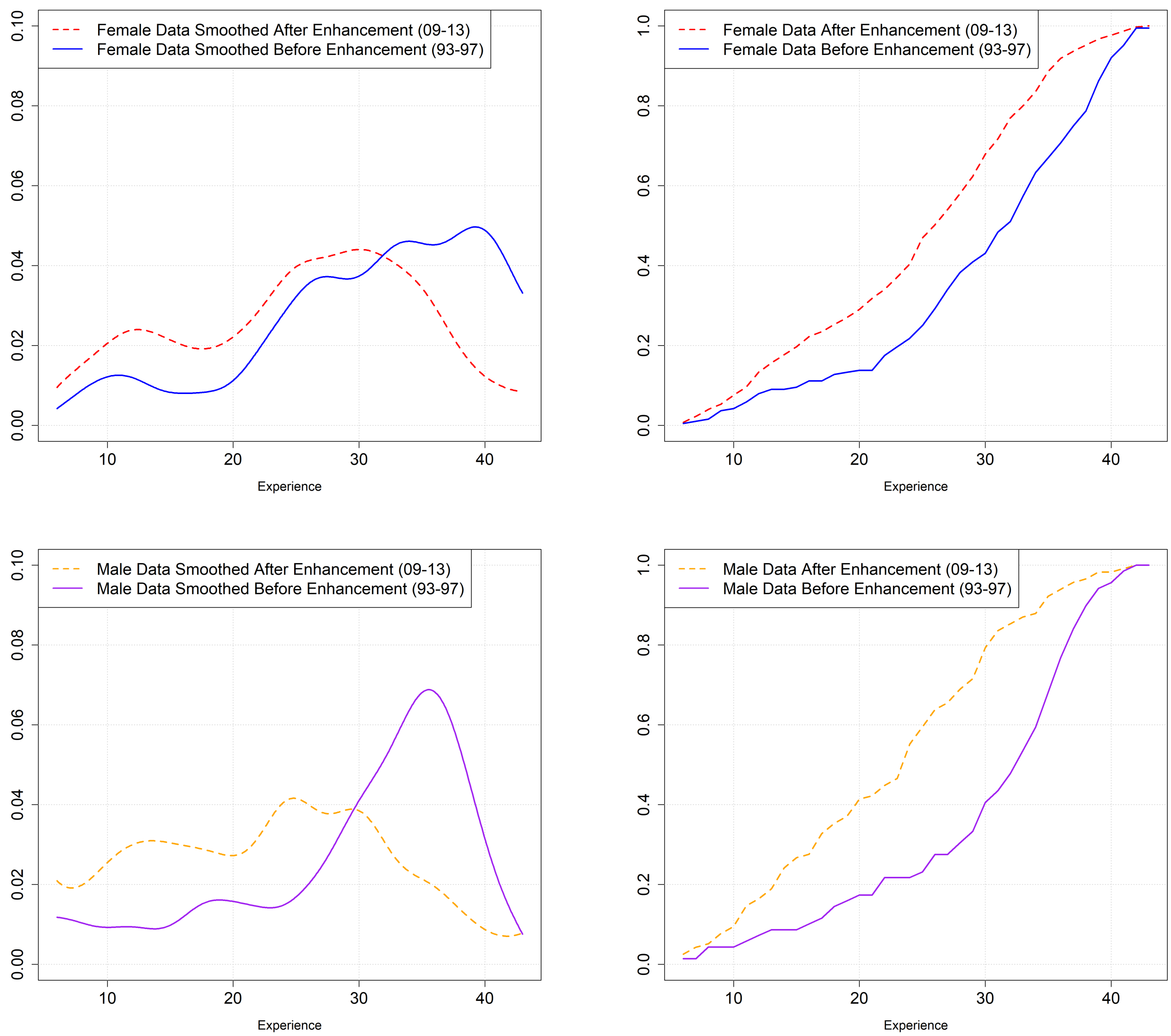
Note: The figure compares the employment survival rates for two cohorts of teachers (for female and male teachers) aged 50-62 at the base year and then tracks them for the next five years (93-97 and 09-13).

Figure 4: Age Distribution Before and After Pension Enhancements



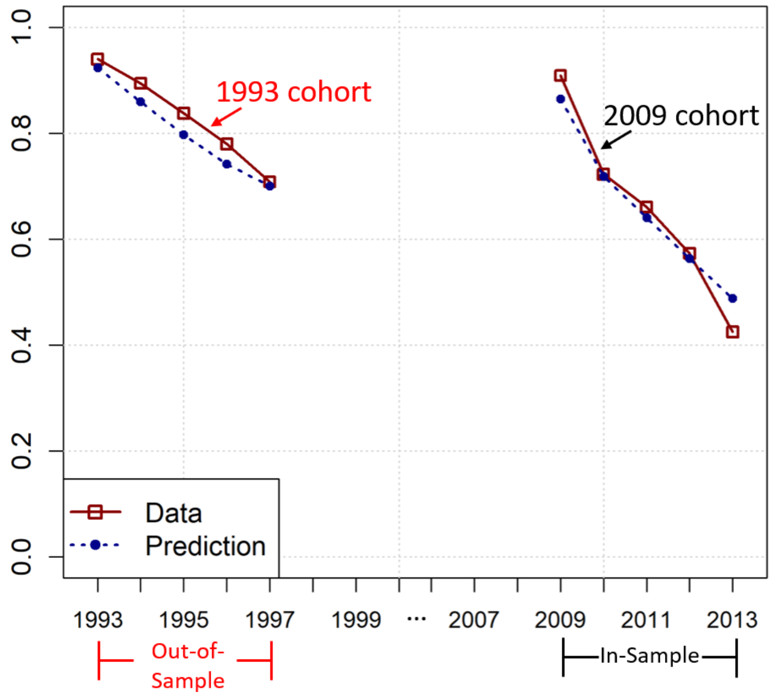
Note: The figure compares the age distributions for retired teachers (female and male teachers) before and after the 1999 enhancements. The teachers are aged 50-62 at the base year of two periods. The left graph compares the probability density, while the right graph compares the cumulative distribution. Please note that the figures are kernel-smoothed density estimates of data.

Figure 5: Experience Distribution Before and After Pension Enhancements



Note: The figure compares the experience distributions for retired teachers (female and male teachers) before and after the 1999 enhancements. The teachers are aged 50-62 in the base year of the two periods. The left graph compares the probability density, while the right graph compares the cumulative distribution. The figures are kernel-smoothed density estimates.

Figure 6: Observed and Predicted Survival Rate for All Teachers (In Sample and Out of Sample)



Note: The figure compares the observed and predicted survival rate of two cohorts of teachers: in-sample (2009 cohort) and out-of-sample (1993 cohort) teachers. Male and female teachers are pooled.

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Appendices

Appendix A Statistical Appendix

A.1 Numerical Computation of Retirement Probability

Continuing with the notation from Section 4, denote $f_t^+ = g_t(r^+)/K_t(r^+)$. Suppose year 1 is the first year of our sample period, and year N is the last year of the sample period. Denote $V_{t_1,t_2} = (\nu_{t_1}, ..., \nu_{t_2})$, $F^+_{t_1,t_2} = (f^+_{t_1}, ..., f^+_{t_2})$. The unconditional likelihood (unconditional probability) for that teacher i chooses to retire at year n ($n < N$) is

$$prob(V_{1,n} \in A_i) = prob[((V_{1,n-1} > -F^+_{1,n-1}) \cap (v_n < -f^+_n))] \tag{A.1}$$

The unconditional probability that teacher i chooses to stay until year N is

$$prob(V_{1,N} \in A_i) = prob[((V_{1,N} > -F^+_{1,N})] \tag{A.2}$$

These probabilities depend on pension rules, teachers' age and experience, and structural parameters $(\gamma, k, k_1, \beta, \sigma, \rho)$. Under the normality assumption of preference errors, computing the probabilities for the retirement decision made by teacher i involves a high dimensional numerical integration of the correlated preference errors V over the region A_i, for which we use the Geweke–Hajivassiliou–Keane (GHK) algorithm. The GHK algorithm transforms the problem of high dimensional integration to a problem of sequential one-dimensional integrations. For more applications of the GHK algorithm see for example Börsch-Supan and Hajivassiliou (1993).

A.2 Censoring in the Baseline Sample

The sample excludes those teachers whose age and experience would have put them in our sample but who chose to retire before our sample period. This results in sample selection bias. Sample selection bias of this sort will lead to overprediction of retirement for relatively young teachers (Ni and Podgursky, 2016). We correct for sample selection bias in our baseline sample by using conditional probability to construct the likelihood function. The probability of teachers retiring in our sample period is based on the condition that teachers are not retired at the beginning of sample period.

Suppose in the first year of our sample period, teacher i was eligible for retirement J years ago. The probability of teacher i retiring at time n<N conditional on being in the sample is:

$$prob(V_{1,n} \in A_i \mid appearing\,in\,sample)$$

$$= Prob\,(retiring\ at\ n \mid appearing\,in\,sample)$$

$$= \frac{Prob\,(retiring\ at\ n\ \&\,appearing\,in\,sample)}{Prob\,(appearing\,in\,sample)}$$

$$= Prob[((V_{1,n-1} > -F^{+}_{1,n-1}) \cap (v_n < -f^{+}_{n})) \mid\ V_{-J,0} > -F^{+}_{-J,0}]$$

$$= \frac{Prob[(V_{-J,n-1} > -F^{+}_{-J,n-1}) \cap (v_n < -f^{+}_{n})\,]}{Prob(V_{-J,0} > -F^{+}_{-J,0})}$$

The conditional probability of teacher i chooses to stay until the end of sample period N is:

$$prob(V_{1,N} \in A_i \mid appearing\,in\,sample)$$

$$= Prob\,(staying\ at\ N \mid appearing\,in\,sample)$$

$$= \frac{Prob\,(staying\ at\ N\ \&\,appearing\,in\,sample)}{Prob\,(appearing\,in\,sample)}$$

$$= Prob[((V_{1,N} > -F^{+}_{1,N})) \mid\ V_{-J,0} > -F^{+}_{-J,0}]$$

$$= \frac{Prob[(V_{-J,N} > -F^{+}_{-J,N}))\,]}{Prob(V_{-J,0} > -F^{+}_{-J,0})}$$

Given the observations of teachers $i = 1,,,I$ who retired during the sample period (at $n_i < N$) or not retired until the end of the sample period ($n_i = N$), the conditional likelihood for the whole sample is

$$L(\gamma,\ k,\ k_1, \beta, \sigma, \rho) = \prod_{i=1}^{I} prob(V_{1,n_i} \in A_i \mid appearing\, in\, sample). \tag{A.3}$$

We use the likelihood in equation (A.3) to estimate our structural model.

Appendix B Goodness of Fit

B.1 In-Sample Goodness of Fit

Figure B1 reports the observed and predicted distributions of age, experience, and the sum of age and experience for retired teachers. There is a sharp spike at 85 and 86 reflecting the "Rule of 85" (i.e., age + experience = 85) option for regular retirement. While the observed age and experience distributions are somewhat choppy because of the relatively small sample sizes (particularly for male teachers,) overall they are in line with the predicted distributions. All the above figures show that the model nicely mimics the distribution of both age and experience, in particular the "Rule of 85".

$(Figure\ B1)$

B.2 Out-of-Sample Goodness of Fit

The out-of-sample simulation provides a robust check for the validity of our model. The data we use for out-of-sample simulation includes all teachers aged 50-62 in the 1993-97 school year. The basic statistics are shown in Table 2. The table shows that under different rules the patterns of retirement rates differ before and after the enhancement as well as the distributions of teacher experience at the time of retirement. Using the estimated parameters in Table 5, we simulate the teachers retirement behaviors in 1993-97 school years. During this period, the teachers are under pre-enhancement pension rules. The out-of-sample simulation also take into account of the fact that teachers in 1993 are significantly different from those in 2009 in age and experience. Figure B2 reports the observed and predicted distribution of age, experience, and the sum of age and experience for retired teachers for the out-of-sample simulation.

$(Figure\ B2)$

B.3 Overall Goodness of Fit

Figure B3 provides both the in-sample and out-of-sample goodness of fit of the survival rate. Figure B4 plots the observed and predicted in- and out-of-sample cumulative distributions of experience at the time of retirement. Overall, the model with the same parameters in teacher preference can capture the difference in retirement patterns for teachers under different pension rules.

$(Figure\ B3\text{-}B4)$

Figure B1: Observed and Predicted Retired Age, Experience and Age+Experience Distribution of Female and Male Teachers (In Sample)

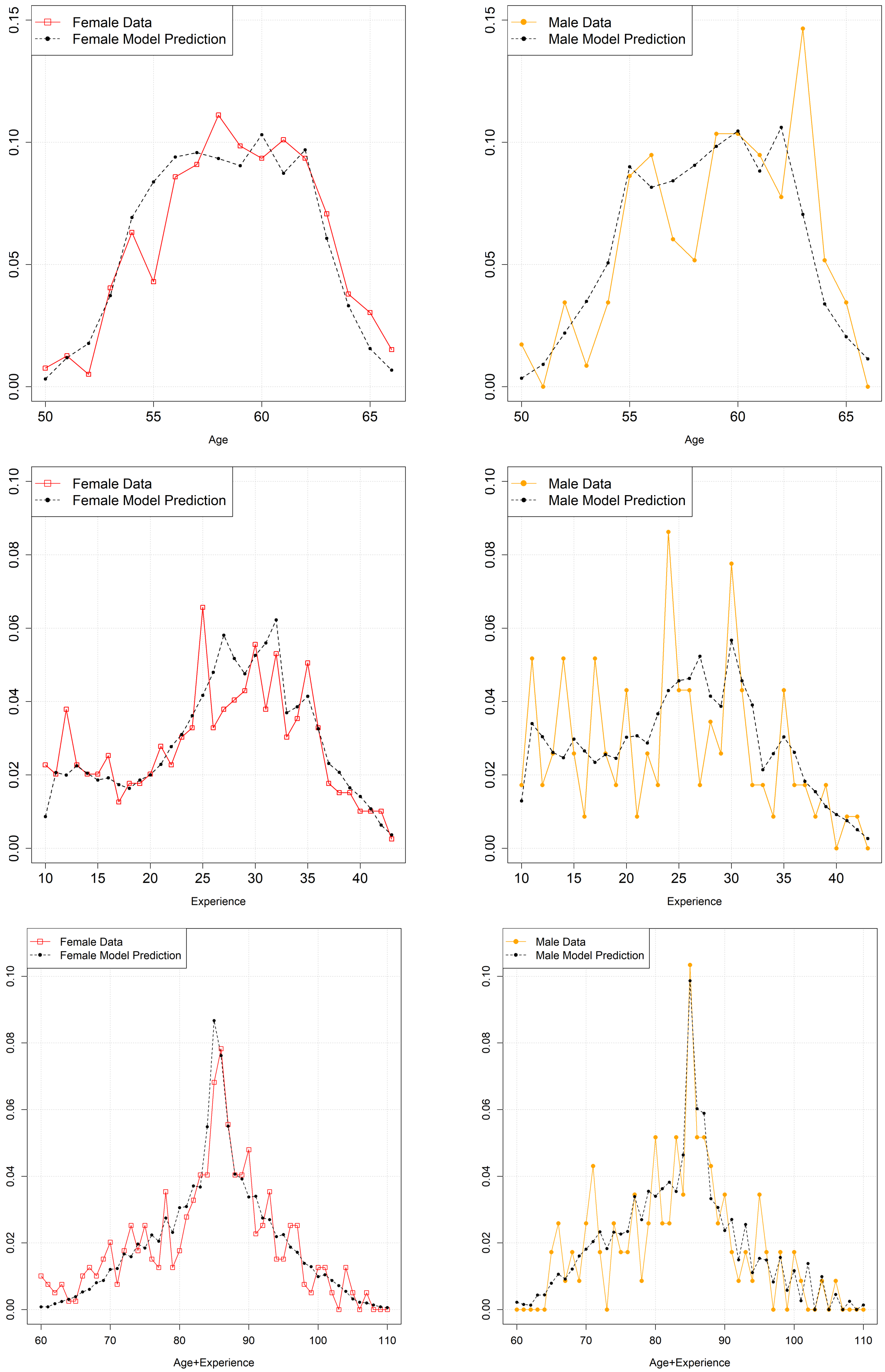


Figure B2: Observed and Predicted Retired Age, Experience and Age+Experience Distribution of Female and Male Teachers (Out of Sample)

Figure B3: Observed and Predicted Survival Rate (In Sample and Out of Sample)

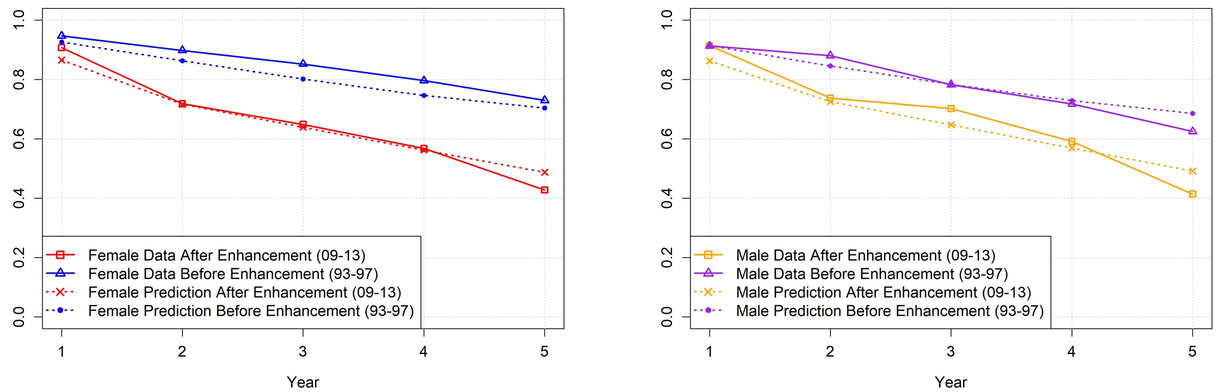
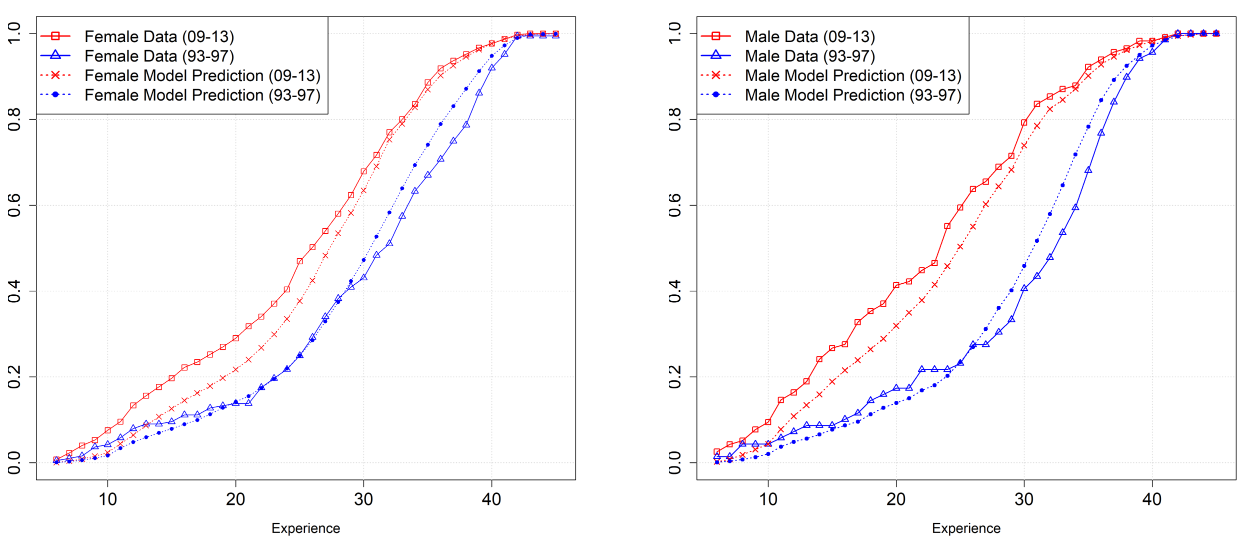


Figure B4: Observed and Predicted Retired Experience Cumulative Distribution of Female and Male Teachers (In Sample and Out of Sample)



Appendix C Defined Contribution Analytics

If in year t a teacher with age-experience (a, e) plans to retire in year $r^\dagger \geq t$, her expected utility under the DB plan is

$$\mathbb{E}_t V_t^{DB}(r^\dagger) = \mathbb{E}_t\{\sum_{s=t}^{r^\dagger-1} \beta^{s-t}[(k_s Y_s)^\gamma + \nu_s] + \sum_{s=r^\dagger}^{T} \beta^{s-t}[B_{s,r^\dagger}(DB)]^\gamma\} = E_t\{\bar{V}^{DB}(r^\dagger; a, e) + \sum_{s=t}^{r^\dagger-1} \pi(s|t)\beta^{s-t}\nu_s\},$$

the sum of the present value of expected flow of salary and benefit

$$\bar{V}^{DB}(r^\dagger; a, e) = \sum_{s=t}^{r^\dagger-1} \pi(s|t)\beta^{s-t}(k_s Y_s)^\gamma + \sum_{s=r^\dagger}^{T} \pi(s|t)\beta^{s-t}[B_{s,r^\dagger}(DB)]^\gamma$$

and that of preference errors.

We consider the following scenario. A teacher with (age,experience) (a, e) in the DB plan in a base year have cash balance $W(a, e)$. Going forward the value in this account grows by the nominal interest rate (on the fund balance) and further annual contributions from teachers and districts of c fraction of salary for teachers and districts, for a combined total of $2c$ times salary. A teacher's account accumulates with annual contributions and nominal investment returns of $R - 1$ on the fund balance. The inflation rate is assumed to be i.

With the initial value in the DC plan $W_t = W(a, e)$, the teacher considers whether to retire or continue to work as in the SW model: her expected utility in period t is a function of expected retirement in year r (with $r = t, \cdots, T$ and $T = 101$ is an upper bound on age).

For a teacher with a DC account value $W_t = W(a, e)$, the account nominal value in year $r > t$ is the value of accumulation of contributions plus the compound return of the wealth in period t: $W_r = W_t R^{r-t} + \sum_{k=t+1}^{r} 2cY_k R^{r-k}$, Y_k is salary in year k.

When a teacher retires, the contribution to the account stops and an insurance company provides an actuarially fair annuity (B) equal to the cash value in the teacher's account.

For a teacher with a DC account value W_r, age $a+(r-t)$ at the time of retirement (the teacher's age is a in year t), let the expected nominal flow of the annuity be B_{r+n} in the $n-$th year of retirement. The retiree survives to $r+n$ with conditional probability $\pi(r+n|r)$. The expected account value and the expected payment evolve from the collection r as

$$W_{r+n} = W_{r+n-1}R - B_{r+n}, \quad B_{r+n} = \pi(r+n|r)(1+i)^n B_r.$$

We set $W_T = 0$. It follows that given age $a+(r-t)$ at the time of retirement has DC benefit

$$B_r = \frac{W_r(1+i)^{t-r}}{\sum_{n=1}^{T-a-(r-t)} \pi(r+n|r)(\frac{1+i}{R})^n.} \tag{C.1}$$

The benefit in time t dollar is $B_r(DC) = B_r(1+i)^{r-t}$.

In period t, the expected utility of retiring in period r is the discounted sum of pre- and post retirement expected utility

$$\mathbb{E}_t V_t^{DC}(r) = \mathbb{E}_t\{\sum_{s=t}^{r-1} \beta^{s-t}[(k_s(1-c)Y_s)^\gamma + \nu_s] + \sum_{s=r}^{T} \beta^{s-t}[B_r(DC)]^\gamma\},$$

where salary Y and and benefit B are in time t dollar; and the unobserved errors are the same AR(1) as in the DB model: $\nu_s = \rho\nu_{s-1} + \epsilon_s$. Let the optimal expected retirement year under the DC plan be $r^\#$. Then we can write

$$\mathbb{E}_t V_t^{DC}(r^\#) = \bar{V}^{DC}(r^\#; a, e) + \sum_{s=t}^{r^\#-1} \pi(s|t)\beta^{s-t}\rho^{s-t}\nu_t,$$

where $\bar{V}^{DC}(r^\#; a, e) = \sum_{s=t}^{r^\#-1} \pi(s|t)\beta^{s-t}(k_s(1-c)Y_s)^\gamma + \sum_{s=r^\#}^{T} \pi(s|t)\beta^{s-t}[B_{r^\#}(DC)]^\gamma$ is the expected utility under the DC plan of expected flow of salary and benefit and that of preference errors.