EXAMINIING MATHEATICAL MODELNG OF FIFTH GRADERS: USE OF INTERACTIVE COMPUTER SIMULATIONS

Azita Manouchehri
The Ohio State University
Manouchehri.1@osu.edu

Azin Sanjari
The Ohio State University
sanjaripirmahaleh.1@osu.edu

We analyzed 5th grade students' interactions with one computer-based modeling simulation to examine how they defined and prioritized variables in a dynamically simulated environment. Results revealed that exposure to the simulation environment helped students visualize continuous motions, interpret the different quantities present in the model, and connect how variables related to each other. Additionally, students became more precise and systematic in adjusting the variables to explore the problem, test hypotheses, and achieve desired outcomes.

Keywords: Modeling, Technology, Elementary School Education

Due to the need to establish connections between mathematics and real-world problems, studies in mathematics education have focused on mathematical modeling in recent years (Lesh et al., 2007). Calls for inclusion of modeling experiences in K-12 curriculum have been reflected in various Standards documents (CCSAM, NCTM). Despite this there is evidence that students are typically not provided with enough modelling opportunities in elementary and middle schools (Suh et al., 2016; Stohlmann & Albarracin, 2016). There has also been strong scholarly support for inclusion of technology as a "tool" for introducing learners to modeling activities Greefrath (2011), assisting them in visualizing real-world problems, and exploring their properties (e.g., Ferri, 2007). While the use of digital tools in modeling problems is studied in more detail among high school and undergraduate students, there has been limited research on their impact and the support they provide in encouraging mathematical modeling skills among elementary school learners (Greefrath et al., 2018; Geiger, 2011). In this work we aimed to address this gap by examining how interactions with one computer-based simulation influenced 5th graders' mathematical modeling process. In particular, we investigated: (1) the learners' perceptions and interpretations of a situation model concerning the impact of rate of change on distance travelled in time (2) ways that these interpretations and perceptions changed as the result of exposure to an interactive simulation depicting the same scenario.

Background Literature

Blum and Ferri (2009) define mathematical modeling as "the process of translating between the real world and mathematics in both directions" (p. 45), where real-world encompasses situations that lie outside the world of mathematics. The modeling process involves observing a real-world situation, conjecturing about it, conducting mathematical analysis, obtaining results, and evaluating the model by comparing its result with the real-world situation (Lingefjärd, 2004).

Mathematical modeling cycle describe the modeling process (Blum & Leiß, 2007; Borromeo Ferri, 2007), allow a focus on cognition, and provide a means for understanding how to trace individual thinking (Borromeo Ferri, 2007; Czocher, 2017). Blum's modeling cycle encompasses aspects of the modeling process described by Lingefjärd (2004) and serves as our theoretical framework for studying student's thought process. When encountering a real-world problem, the modeler initially produces a situation model. The situation model is then simplified to a mathematical model by adding structure and considering conditions and variables and restricted parameters. This formal mathematical model is analyzed, outputting mathematical results, which are interpreted in terms of the real model. The results are validated as they are checked against the real-world conditions and constraints. This process iterates until a satisfactory model is obtained (Figure 1).

Greefrath (2011) proposed an extension to Blum's modeling cycle (Figure 1) by adding technology as a bridge between the mathematical model and mathematical results. Greefrath proposes that in addition to being able to directly solve a mathematical model to arrive at mathematical results, the modeler can also build a computer model based on their mathematical model. Running a simulation is the process of executing the computer program developed to implement the mathematical model, and the outputs are denoted as computer results. The computer results are then translated back to mathematical results. The next steps are again similar to the original model, being the mathematical results and real results.

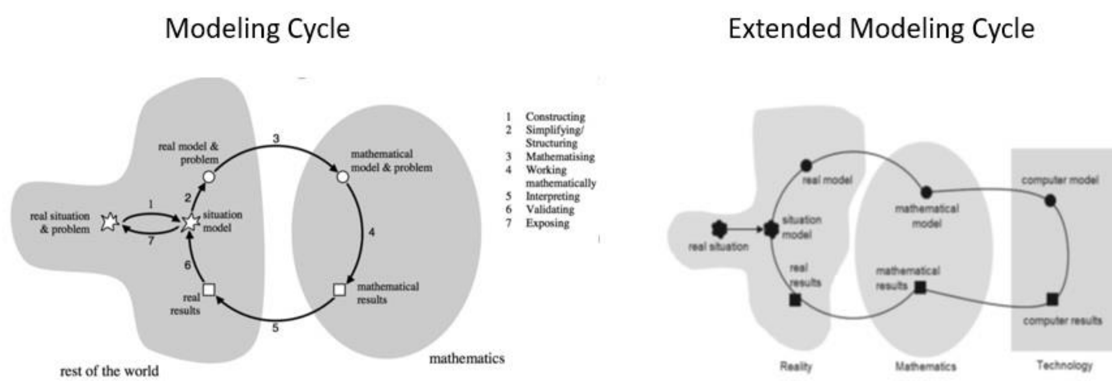


Figure 1: Modelling Cycle (Blum and Leiß 2006) and Modeling Cycle with Added Computer Model (adapted from Greefrah, 2011)

The extended cycle is advantageous as its steps do not need to be followed in order, allowing transitions between any of its stages. We used Greefrah's (2011) extended modeling cycle to track the modeling path of the students as they worked on a simulation.

Methodology

Participants

A semi-structured task-based interview (Maher and Sigley, 2014; Goldin, 2000) was used to study how 3 students examined and defined relationship among different variables as they attempted to predict specific outcomes associated with modeling a problem involving rate of change. The participants were fifth-grade students enrolled in an elementary school in the Midwest. All three were female and representing different levels of mathematical knowledge.

Procedure

Each participant (Marry, Nikki, and Tina) was interviewed three times. Each interview lasted approximately 50 minutes. During the first interview students were asked to solve The Three Runners problem (Table 1), requiring them to compare the distance between two runners, each runner's distance to the finish line, and given their speed determining which one would reach the finish line first. Participants' responses to this question provided baseline data on their interpretations of the task, procedures they used and factors they considered when doing so.

During the second interview participants were introduced to a computer simulation environment depicting a running scenario which paralleled the task used in the first interview. Following a free play time, they were asked to solve the same problem using the interactive simulation. During the last interview session they were asked to solve the task they had considered during the first interview without using the interactive simulation. The purpose of the last interview was to trace any shifts in their thinking as the result of exposure to the simulation.

Table 1: Interview Questions (Three Runners Problem)

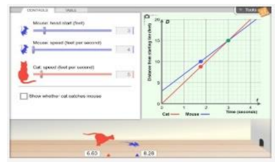
	Three Runners Problem
Step 1	Three runners are racing: a. Runner 1 is 18 meters away from the finish line, and is running towards it with the speed of 6 meters per second. How long does it take the runner to reach the finish line? b. Runner 2 is 14 meters away from the finish line, and is running towards it with the speed of 2 meters per second. How long does it take the runner to reach the finish line? c. Which runner reaches the finish line sooner? Why? d. Now suppose Runner 3 is 15 meters away from the finish line, and is running towards it with the speed of 3 meters per second. How long does it take the runner to reach the finish line? e. This time which runner reaches the finish line first? Runner 1 or runner 3? Why?
Step 2	If students' answer to part (c) or part (e) is runner 1, it means runner 1 passes runner 2 or runner 3 at some point. Following questions are: f. Can you tell when this happens? g. Can you determine where this happens?
Step 3	Irrespective of their answers to parts (c) and (e) students were asked to: h. Draw a graph of the motions of runners 1,2, and 3.

Simulation

We used the "cat and mouse" simulation during the second interview. This simulation is a part of Gizmos platform, including different interactive math and science simulations, which are designed for students in grades 3–12 aligning with the National Science Educational standards (Cholmsky, 2003). Gizmos mimics the real-world phenomena and allows the users to control several important factors while presenting information in a way that is easy to manage.

In this simulation (Table 2), students are able observe the evolution of a system over time. They explore how different objects move through time and if they satisfy a certain condition at a certain time. They also learn about acceptable parameter regions and how they should use constraints to solve the problem. Additionally, the distance between the cat and mouse, the speed of the cat, and the speed of the mouse are the parameters that students can tune. The simulation provides a continuous graph of the movement of cat and mouse running, as well as their location on the x-t plane. Table 2 summarizes the objectives and the content areas addressed in the environment.

Table 2: Modeling Simulation

Simulation	Cat and Mouse (https://www.explorelearning.com/index.cfm?method=cResource.dspDetail&ResourceID=108) 
Associated Task (Modeling Problem)	A small mouse plays on the floor, unaware of the cat creeping up on it from behind. The cat springs and the mouse desperately runs away. Will the mouse reach its hole in time to escape the cat?
Variables	Distance between cat and mouse, cat speed, mouse speed
Objectives	Inferring the effect of variables on two objectives: - The indicator of cat catching the mouse - The time it takes for the cat to catch the mouse
Content Area	Algebra-linear system

Data Analysis

Data analysis followed a two stage process. First, videos of each of the interview sessions were transcribed and reviewed to distinguish the different types of comments students made and actions they took as they worked on tasks. These comments and approaches were mapped against the phases of extended modelling cycle (Greefrath, 2011). The participants' interactions with the environment were examined to capture how exposure to the simulation shaped their modeling behaviors. Transitions between the mathematical model to the computer model, the simulation settings, and interpreting of the simulation results were of particular interest to the researchers, which were sought out amongst the data. The frequency of occurrence of each event was tallied to characterize each transition. The transitions were then analyzed in more detail, were used to study how students analyze and interpret the computer results, and were used to assess how the subsequent analyses of students regarding the problem is affected after being exposed to the simulation environment.

Findings

Table 3 offers an overview of the three participants' performance during each of the three interviews according to the answers they provided to the questions asked and their explanation of their thinking. It also describes how each student used the simulation environment and what they seemingly gained from the experience of working on the simulation to respond to a compatible context. In the following each interview session is discussed in more detail.

Interview Session 1

In the first session students answered questions (see Table 1) without using the simulation environment. The goal was to study their understanding of a problem that concerns distance/time travel in presence of rate of change, and what mathematical concepts they referenced or used. This interview served as a baseline for tracing ways that the use of simulation environment may impact their understanding of the problem or their solutions to it.

In the first session Marry showed difficulty representing the physical quantities mathematically even though she was capable of using algebraic tools (i.e., could easily carry out operations). While she correctly completed the questions in step 1, she could not justify her the algorithm she had used for computing answers for instance why she should divide distance by speed to calculate the time. She explained that she did so because she thought it is an "easy" thing to do. In an attempt to responds to questions in step 2, she drew a line to denote the location of each runner on a line, but did not correctly place the runners' locations. Given that runner 1 is 20 minutes away from the finish line and runner 2 is 14 meters away, she put runner 1 at unit distance of the finish line, and struggled to find a way to show that runner 2 has a head start.

Figure 2: Marry's Representation of Runners' Locations on a Number Line

Nikki seemed more comfortable with step 1 questions though part (e) seemed challenging to her. She could justify how her mathematical manipulations helped her compute physical quantities however, in response to part (e) she needed to compute the net effect of speed and distance but instead she only focused on speed and ignored distance. In step 2 she used geometric representations of physical quantities by drawing a line, and analyzed the running process in 1 second intervals to answer the question. Although she could explain how a solution to step 2 questions meant in physical quantities (for instance a runner passing another means at some time the runner is behind and in the next second is ahead), she did not know how to express this event mathematically.

Tina seemed more comfortable with algebraic operations. She completed step 1 questions easily. She correctly answered part (e) and justified her answer. She also correctly explained that in answering step 2 question she needed to check if there existed a time for which both runners were at the same distance from the finish line. She correctly used her understanding to compute the location of each runner at each time, moving in increments of 1 second, and successfully answered where and when the two runners would meet, or never meet at all. Interestingly, she did not use geometric representations, such as drawing a line, to answer these questions. She was the only students who moved to step 3 in session 1. She successfully identified the axes of the graph, correctly denoted runners' initial location, and identified the point denoting the location of each runner after one second of movement. Although she computed the location of each runner at each given time, she did not realize that the location versus time graph is a line, and did not complete the task. Finally, she seemed more comfortable using algebra to solve the problems rather than drawing figures or graphs to explain her answers.

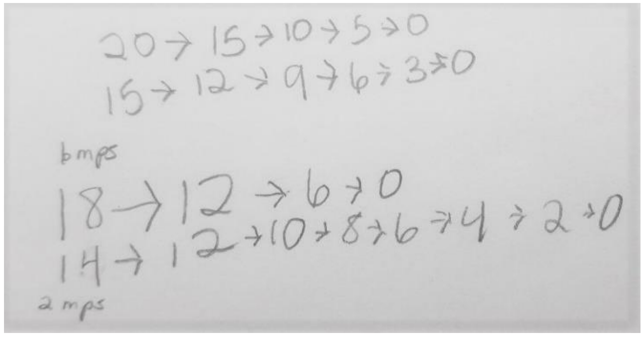


Figure 3: Tina's Computation on Runners' Locations in Time to Determine When and Where They Meet in Interview Session 1

Interview Session 2

In session 2 students were introduced to the simulation. They were given some time to explore the environment prior to the interview questions, initially all students seemed to “play” with the simulation settings. However, gradually they became more purposeful with their setting selection. In the course of the participants’ interactions with the simulation they showed a tendency to test extreme values (i.e., largest and smallest values of each variable, and moderate values) to discover the possible behaviors of the simulation with the least number of simulation trials.

Marry initially focused on the animation generated by the simulation environment, of the cat chasing the mouse, and how the location versus time graph represented this process. She spent a big chunk of her time on these outputs and used them to explain how she could represent locations, such as the head start of the mouse, on a line denoting the location to the finish line.

Nikki focused mostly on the graphical representation provided by the media and following several trials concluded that the cat catching the mouse meant that they were both at the same location at the same time. She further explained that by scaling all variables by the same value, say doubling them, the output, (i.e., if the cat catches the mouse and the time it happens) remains unaffected.

Tina seemed to follow a different approach to the use of the simulation. She immediately started with putting variables at extreme values and gradually changing them to learn about the environment. Instead of focusing on the animation or the graph, she generated ta able in the next tab explaining how this table would inform her about where the cat and mouse were at each given time point. She elaborated on how she could use this information to deduce answers. She then went back to the graph and explored how different settings affected it. She noticed the linear structure of the graph, and she extracted information from it so answer questions.

Interview Session 3

In this session students answered the same question they had encountered in session 1. After working with the simulation environment Marry correctly represented the locations on a line, distinguished the scaling of the problem, and accurately interpreted units of distance and time.

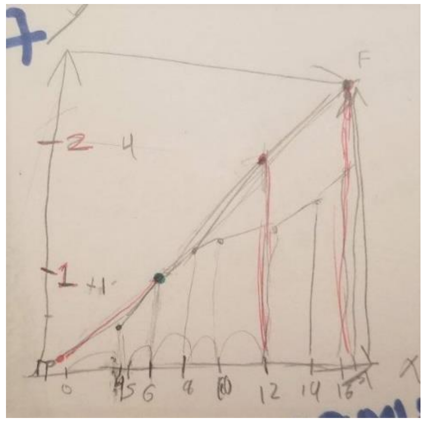


Figure 4: Marry's Representation of the Time Versus Location Graph

She successfully answered step 2 questions, and used a number line to determine if and where the runners met, similar to Tina's solution in session 1. She also completed step 3 and could explain her thinking; however, she was not as comfortable working with a graph. During the first session Nikki could only determine if the wo runners met and approximated the location where this meeting occurred. During the third interview she computed the exact values for time and location of the runners' intersecting. She also drew the location versus time graphs and explained her answers. Tina completed the graph which she could not produce in session 1, and explained how it would be affected by changing the variable values.

Table 3: Summary of Students' Performance and Use of the Simulation

	Marry	Nikki	Tina
Interview 1	-Completed step 1 but not did not correctly answered steps 2 and 3 - Mathematically representing/comparin g the physical quantities	-Completed step 1 but partially answered step 2 and did not answered step 3 -Geometric representation, good understanding of the physical concepts -Taking advantage of algebraic tools to simplify and speed up the calculations	-Completed steps 1 and 2 but partially answered step 3 -Good grasp of physical quantities and mathematically expressing the relations. Using algebra to solve the problem -Geometrically and physically interpreting the results
Interview 2	-Completed steps 1, 2, and 3. In particular, successfully drew the location versus time graph.	-Completed all steps successfully. In particular, took advantage of algebraic methods as well as geometric methods.	- Successfully completed all tasks.

Interview 3	-Visualize physical quantities and study their evolution in time. Afterwards, we could mathematically describe and manipulate them	-Learn how to mathematically manipulate quantities, in particular, algebraically describe the problem and solve it using mathematical operations rather than geometric representations	-How to connect mathematical relations to physical quantities and learn about their connections.
Shifts	-Learns how to compare different distance values	-Used to draw figures and compute locations in steps of one second to arrive at a solution. Afterwards, she used mathematical manipulations without drawings to solve.	-Could not describe the graph initially, with the help of simulation gained a better understanding of the problem as a process

Discussion

At the initial stage of exposure to the simulation students shifted between the computer model and computer results for an extended period of time as a means to discover the impact of change in various variable values on the outcomes depicted on the screen. They tended to design a simulation setting, run it, and compare the computer results with what they had computed mathematically, confirming their initial ideas. By repeating this procedure, they seemed to form a more refined understanding of the problem leading to development of more precise descriptions. For instance, Nikki was comfortable using abstract algebraic methods to solve a problem in fewer steps, rather than relying on her visualization skills. On the other hand, Tina seemed to have made a connection between her abstract formulations of key physical patterns, and how they corresponded to a representation or a graph. All three participants, irrespective of their background knowledge, benefited from interacting with the simulation environment as evident in how they solved the task during the third interview. Each student used the simulation environment differently, and tested settings that helped them learn about specific aspects of the problem with which they had struggled the most. In particular, they used the simulation environment to visualize the dynamical system and its evolution, how different quantities relate to each other, how they can be represented mathematically, and if their solution is correct.

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