COMPONENTS OF HIGH-QUALITY MATHEMATICS CLASSROOMS: ATTENDING TO LEARNING OPPORTUNITIES FOR ENGLISH LANGUAGE LEARNERS

M. Alejandra Sorto
Texas State University
sorto@txstate.edu

Kathleen Melhuish
Texas State University
melhuish@txstate.edu

Eva Thanheiser
Portland State University
evat@pdx.edu

Katty Zied
Texas State University
kattyzied@txstate.edu

Christina Koehne
Texas State University
crz7@txstate.edu

Amanda Sugimoto
Portland State University
asugimo2@pdx.edu

Autum Pham
Portland State University
autpham@pdx.edu

Simon Byeonguk Han
Portland State University
byeonguk@pdx.edu

Sharon Strickland
Texas State University
strickland@txstate.edu

In response to the call for research on integrating best general practices in teaching with those that promote equity and access, we present a two-part study focused on instructional strategies that may remove learning barriers for English Language Learners[1]. We theoretical developed and empirically explored supplemental components for traditional quality of instruction measures (MQI, Hill, 2014, Math Habits Tool, Melhuish & Thanheiser, 2017). We share results from a quantitative study empirically verifying the effect of suggested ELL-focused instructional strategies (Chval & Chávez, 2011) on ELL learning via the creation of an additional MQI dimension. Based on these results, we then provide theoretical operationalizations of these strategies to integrate into the student-and-teacher interaction tool: the Math Habits Tool (Melhuish & Thanheiser, 2017) as means to concretize these strategies for researchers and practitioners.

Keywords: Classroom Discourse, Instructional Activities and Practices, Inclusive Education

In general, the mathematics education field has endorsed certain images of classrooms that are student-driven and supported by instructional practices that serve to focus on student thinking and support students in engagement in productive discussion (e.g., Jacobs & Spangler, 2017). Recent work, however, has revealed that simply engaging in such practices without attention to issues of equity and access can produce classrooms where not all students have opportunity to learn (e.g., Johnson, Andrews-Larsen, Keene, Melhuish, Keller, & Fortune, in press). As a result, researchers have called for attention to equitable teaching that can support the engagement of all students in mathematical practices (Bartell, Wager, Edwards, Battey, Foote, & Spencer, 2017).

Our work addresses this call in the context of English Language Learners (ELLs.) This growing population (National Academies of Sciences, Engineering, and Medicine, 2018) frequently incur barriers to fully engage in learning opportunities in the classroom. While there is literature about teaching practices that may support this population (Barwell, Moschkovich, & Setati Phakeng, 2017), little work has been done at-scale to empirically explore what instructional strategies may support these students. We share results from a quantitative study identifying the effect of suggested instructional strategies on the mathematical achievement gains of middle grades ELLs. We pair this work with a theoretical, qualitative analysis serving to operationalize these results in a way that is congruent with other best practices for instruction

(Melhuish & Thanheiser, 2017), and can serve as grounds for teachers to concretely work on their practice.

Literature Background

There is general consensus in the mathematics education literature that high quality mathematics classrooms contain certain attributes. These classrooms are ones in which student voices are heard and orchestrated, and student thinking is leveraged as the means to move instruction forward (e.g., Ball, 1993; Jacobs & Spangler, 2017; Nasir, & Cobb, 2006; Schoenfeld, 2011; Turner, Dominguez, Maldonado, & Empson, 2013). However, teachers hold an essential role in orchestrating opportunities that encourage and support such forms of student participation to promote active student learning and engagement (Franke, Kazemi, & Battey, 2007; Gresalfi, 2009; Jacobs & Spangler, 2017). Students are not just passive receivers of mathematics, but active participants in sense-making including through the use of justifying and generalizing mathematics (e.g., Boaler & Staples, 2008). Such qualities have influenced a number of quality of instruction tools including the summative Mathematical Quality of Instruction (MQI) instrument (Hill, 2014) and the formative Math Habits Tool (Melhuish & Thanheiser, 2017).

In alignment with this research, standards reform movements have also increasingly called on students to use language in a variety of ways to increase their mathematical understandings, e.g., explaining their thinking or discussing connections between multiple representations (National Governor's Association Center for Best Practices & Council of Chief School Officers, 2010). However, merely providing tasks and discussion support focused on mathematical practices may conceal the complexities of students in the classroom (Bartell et al., 2017). In particular, students such as ELLs, may encounter barriers to full participation and engagement if they are not able to access the mathematical discourse and content in the classroom (Moschkovich, 2002, 2007).

Scholars have explored pedagogical strategies to bypass such barriers, including: (1) exposing students to mathematical content and instruction in multiple modalities, e.g., gestures, mathematical representations, and manipulatives, and (2) exploring the meanings and multiple meanings of words used in the mathematical register (Bartell et al., 2017; Barwell et al., 2017; Campbell et al., 2007; Moschkovich, 2002, 2007; Shein, 2012; Turner et al., 2013). These strategies are designed to develop students' understanding of mathematical content while simultaneously acquiring higher levels of language proficiency (de Araujo et al., 2018; Bartell et al., 2017; Barwell, et al., 2017; Campbell et al., 2007; Moschkovich, 2002, 2007; Turner et al., 2013). In order to provide students with opportunity to learn, instructors need to attend to both general best practices in instruction, but also to the particular needs of their students.

Theoretical Orientation

In our work, we take the viewpoint that mathematically productive classrooms are ones in which each and every student has access to high quality mathematics. High quality mathematics is mathematics in which students regularly have opportunities to make sense of big mathematical ideas via opportunity to justify, generalize, and leverage mathematical structure (e.g., Boaler & Staples, 2008). Further, these classrooms leverage high cognitive demands tasks (e.g., Stein & Smith, 1996), and support students in productively engaging in mathematical discourse (e.g. Jacobs & Spangler, 2017), in order to develop mathematical habits of mind (Cuoco, Goldenberg, Mark, 1996). Such classrooms reflect a micro-community where teachers and students interact in tandem to develop norms around mathematical activity (Takeuchi, 2016; Staples, 2007; Yackel & Cobb, 1996).

In order to operationalize these interactions, we identify core classroom components in the Mathematically Productive Classroom Framework: *mathematically productive teaching routines* that are extended routines enacted by teachers (such as selecting and sequencing student ideas), *catalytic teaching habits* are in-the-moment teaching moves to support students in mathematical reasoning (such as prompting a student to justify), and *mathematical habits of mind* and *interactions* which are ways that students engage with mathematics and each other around mathematics (habits such as justifying, using representations, or engaging in critique and debate) (Melhuish & Thanheiser, 2017). Through this lens, mathematically productive classrooms can be analyzed through a series of connections across these interaction types. However, we acknowledge that the foci of such analysis may obscure the complexities of attending to mathematical access where different members of a classroom community bring individual lenses, backgrounds, and knowledge to engage with mathematics.

As such, we have aimed to complement traditional quality of instruction frameworks with nuanced attention to the qualities of instruction that can promote mathematical learning and engagement for students from linguistically diverse backgrounds. In particular, we focus on the discursive moves that provide students, and specifically ELLs, with *opportunity to learn* (Gresalfi, 2009; Jackson et al., 2013; Takeuchi, 2016). As noted in the recent commentary by Cai, Morris, Hohensee, Hwang, Robison, and Hiebert (2017), one of the most robust results from the research literature is that students learn best when provided opportunity to learn. Identifying moves that can support learning opportunities is imperative. As an initial grounds of analysis, we leverage the elements of Chval and Chávez's (2011) research-based instructional strategies to support ELLs. Our adaptions of these strategies can be found in Table 1.

Table 1: Instructional Strategies for Linguistically Diverse Classrooms (ISLD)

Instructional Strategy	Description
Connections of mathematics with students' life experiences	Teachers reference the mathematics found in daily life by students.
Connections of mathematics with language	Teachers reinforce a mathematical representation with its meaning.
Meaning and multiple meanings of words	Teachers and students explore the meaning of mathematical words and objects through speech and other forms of expression.
Use of visual aids or support	Teacher supplements instruction with powerful visual media that enhance comprehension of mathematical concepts.
Record of written essential ideas and concepts on board	Teacher makes careful and conscientious use of the board or any visual display media, and students have access to pertinent information throughout instruction.
Discussion of students' mathematical writing	Teachers use student work as an instructional tool and point of discussion.

See Sorto and Bower (2017)

Methods

We leverage several video-banks of lessons from grades K-8 to explore the integration of access to learning opportunities in partnership with instruments such as MQI and the MHT. In this report, we share results from two phases of our research: (1) empirically establishing the impact of the instructional strategies for linguistically diverse classrooms through adjoining a qualities of linguistically diverse classroom dimension to the standard MQI instrument and (2) theoretically developing teaching routines, catalytic teaching habits, and student habits of mind/interactions to operationalize what this looks like in the classroom.

To address the research aim in phase 1, we share the results of coding 99 mathematic lessons from a sample of 34 sixth-eighth mathematic teachers and their 4,522 students representing all of the 11 middle schools in a large district in the southwest United States. The majority of the teachers (75%) taught in classrooms with at least half of students classified by the school district as ELLs. Lessons were coded using the MQI and the augmented new dimension, ISLD, with each 7.5-minute segment assigned a code of Not Present, Low, Medium, and High (0 – 3). All videos were coded by two coders independently (average alpha=0.824) with disagreements settled via discussion.

To measure the effect of the set of strategies, as a whole, on teachers' student learning gains and in particular on their ELL students, a multivariate statistical analysis was conducted. We hypothesized that the presence and quality of implementation of the routines, teachers' knowledge, and general quality of instruction, might impact student achievement differently depending on students' language status. We developed the following Hierarchical Linear Model (HLM) in which the outcome variable of student achievement, Y, for the ith student of teacher n in school j was seen as a function of a vector of students' background variables, $\boldsymbol{X}$, and individual teacher quality measures, T:

$$Y_{inj} = \beta_X' \boldsymbol{X}_i + \beta_T T_n + (u_n, u_j) + \varepsilon_i$$

The model includes fixed effects at the school level, u_j, and random effects at the teacher level, u_n. Student achievement was measured as a standardized gain score by using the difference between their state test result at the end of the study year and the previous year's result. The data were pooled across all three middle school grades, and the control variables for students included measures for current grade, grade repetition status, economic status ("disadvantaged" is the label adopted by the state to designate students who qualify for free and reduced-price meals), and their baseline (prior year) mathematics test score. Because of potential collinearity the modes were estimated separately (one by one) for each individual teacher quality measure.

To address the research aim in phase 2, we leverage the empirical results to both identify existing components of the Mathematically Productive Classroom Framework and develop new components to best account for the types of teacher and student interactions that may provide access to mathematical learning opportunities. This process involved theoretically analyzing the existing framework, operationalizing new habits and routines, then testing and refining these habits and routines through qualitative analysis of videos. The goal of this theoretical contribution is to provide a testable theory of in-the-moment interactions, and provide a means to traverse the holistic research results in order to concretize the work in an actionable way.

Results

In this section, we share first the empirical evidence around instructional strategies to support ELLs. We then put these results in communication with the Mathematical Productive Classroom Framework to concretize the teaching work involved.

Phase 1: Empirical Evidence of Effectiveness for ELLs

Table 2 presents the results of a side-by-side comparison of the variables in separate regression models restricted to ELL and Non-ELL.[2] Table 3 provides a breakdown of the individual elements (measured on a 0-3 scale). The augmented MQI dimension, *Instructional Strategies for Linguistically Diverse Classrooms*, was only a significant variable in the ELL student model validating the hypothesized instructional strategies support ELL students above and beyond *Mathematical Quality of Instruction*. We unpack these results further below.

Table 2: Teacher Knowledge and Quality of Instruction Effect Sizes (Standard Deviations of Gains in Mathematics' Achievement Scores) by ELL/non-ELL Students

Independent Variable	ELL	Non-ELL
Mathematical Knowledge for Teaching	0.03 (0.60)	0.30* (2.40)
Instructional Strategies for Linguistically Diverse Classrooms	0.10* (2.36)	0.16 (1.43)
Mathematical Quality of Instruction	0.07† (1.87)	0.25**(2.74)

† $p < .10$; * $p < .05$; ** $p < .01$

Note: t-statistics in parentheses

Table 3: Mean and Standard Deviation of Teaching Strategies for Linguistically Diverse Classrooms

	Mean (0-3)	Standard Deviation
Connections of mathematics with students' life experiences	0.28	0.27
Connections of mathematics with language	1.32	0.41
Meaning and multiple meanings of words	1.16	0.40
Use of visual aids or support	0.70	0.65
Record of written essential ideas/concepts on the board	2.32	0.39
Discussion of students' mathematical writing	0.97	0.51

As expected, the variable of MQI is positive and significant for both samples, but effects were much larger in the non-ELL sample. If we take the MQI measure as a proxy for 'good teaching', this result may imply that the common phrase 'good teaching benefits *all* students' is supported by this data, with the caveat that 'it more greatly benefits the non-ELL'. It further suggests that the gap of learning opportunities still remains and that teachers' pedagogical actions and moves may not reach students in an equitable manner. Additionally, the measure of MKT was only significant for non-ELL, and with a strong effect size. On the contrary, the only variable that was positive and statistically significant for ELLs, but not for non-ELL, was the overall measure of the instructional strategies for linguistically diverse classrooms (see Table 1 for the list of strategies). An increase of 1 point on the 0-3 point scale corresponds to an estimated gain of 0.1 standard deviation in achievement score. This result may be interpreted as a necessary aspect of instruction (going 'beyond good teaching') to help close the learning opportunity gap. These results imply that effective teachers of ELLs need to provide quality mathematics instruction in general, and that they also need to incorporate teaching routines that attend to providing access to learning opportunities, in particular reinforcing the connections between mathematics representations and their meaning, exploring the meanings of mathematical words and objects through speech and other forms of expression, and making conscientious use of the board or any visual display media so that students have access to pertinent information throughout instruction.

Furthermore, we also verified that the overall score for the augmented dimension of the MQI, *Instructional Strategies for Linguistically Diverse Classrooms* was significantly and positively associated with the overall score of MQI (r = 0.592; p = 0.0002) and with the Mathematical Knowledge for Teaching (MKT) survey (r = 0.311; p = 0.0776). These results reflect that anticipated relationships with outside variables hold, providing validity evidence for the measure.

Phase 2: Theoretically Deconstructing Instructional Strategies into Teacher and Student Interactions

In this section, we share examples of how we have adapted and augmented the Mathematically Productive Classroom Framework to operationalize the above aspects of classrooms. We focus on teaching routines that promote learning opportunities through engagement with mathematical tasks and contexts (*Providing access to mathematical tasks and terminology*), and through engagement with one another's mathematical ideas (*Working with public records of students' mathematical thinking*). The first routine aligns with the instructional strategy *Discussion of Students' Mathematical Writing* from the prior section. The second routine incorporates components of the instructional strategy *Meaning and Multiple Meanings of Words*. The following two excerpts provide brief exchanges to illustrate the interconnected nature of teacher and student interactions, and how specific interactions can facilitate potential learning opportunities. In particular, we focus on key triangles of an extended routines into (*Access to Learning Opportunity* (ALO) routines), embedded with a catalytic teaching habit to support students in a particular habit of mind or interaction (see Figure 1).

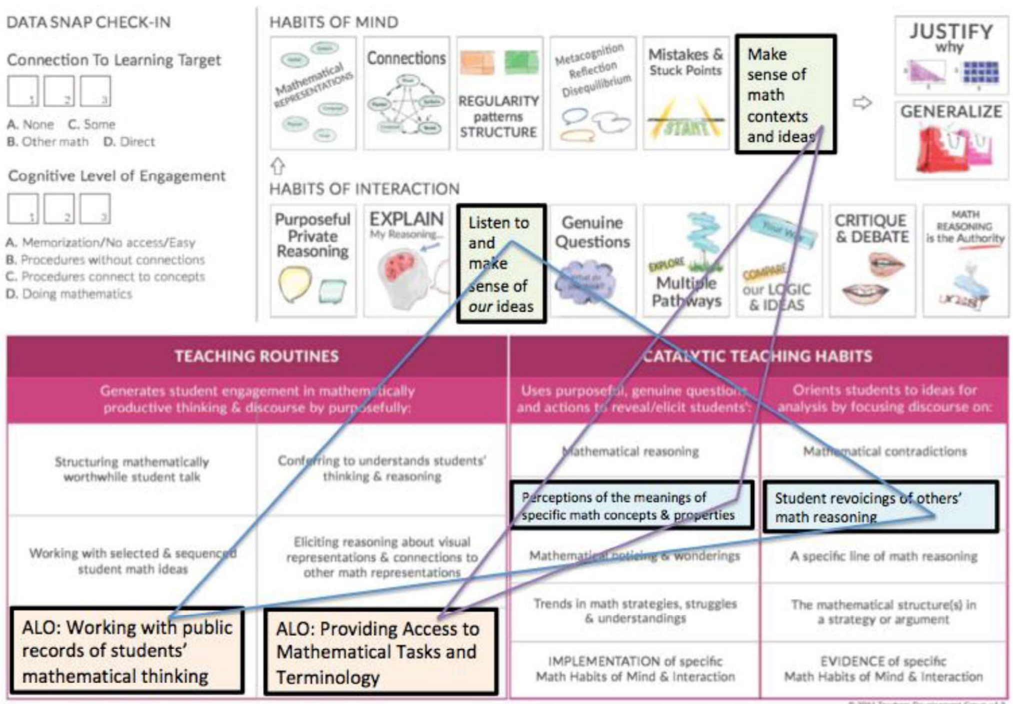

Figure 1: Mathematically Productive Classroom Framework -Connection Between Teaching Routines, Catalytic Teaching Habits, and Student Habits of Mind and Interactions

In this first clip, a pair of 7th grade students from a middle school located in the US southern border is engaged in a task related to the circumference of a circle. However, one of the partners' (identified as ELL by the district) is becoming familiar with words that describe the task.

T: (to S1) Can you explain to her what is circumference? S1: Umm Sure. … S1: Well *(Spanish) Circumference is like the area of a circle.* T: *(Spanish) The distance.* S1: *(Spanish) The distance.* T: [inaudible] good. *(Spanish) Did you understand?* S2: *(Spanish) Yes.* T: (to S2) *(Spanish) Show me the circumference of the circle? What did she do?* S2: *(Spanish) [inaudible]* (tracing the circle)	Routine: Access to Learning Opportunities: Providing Access to Mathematical Tasks	CTH: Uses purposeful genuine questions and actions to reveal/elicit students' perceptions of the meanings of specific math concepts & properties. Math Habit: Make sense of mathematical contexts and ideas CTH: Orients students to ideas for analysis by focusing discourse on student revoicing of others' math reasoning Math Habits: Make sense of mathematical contexts and ideas, Reason with mathematical representations

Figure 2: Student Teacher Interaction on Circumference Analyzed with the Mathematically Productive Classroom Framework

This excerpt (Figure 2) illustrates explicit attention to the mathematical meaning of terms where the teacher spends extended attention (that goes beyond the scope of the brief excerpt above) having students engage with the mathematical context. Within this routine, the teacher prompts for the perception of the meaning of a specific mathematical idea (circumference) (CTH), a student makes sense of this idea (habit of mind), the teacher prompts for revoicing (CTH), and the second student makes sense and reasons with a representation (habit of mind) to illustrate circumference.

P: …yeah- it's greater than- cause it's above the half of 42. T: So, that was a little bit- that was a lot of information for us. I'm going to ask N to revoice, but before N revoices I'm going to ask one other person to revoice. I want a total of two students to revoice P's thinking. So if what P did made sense to you, you can talk about it... Let's have J revoice, and then N, I'm going to ask you to revoice after J. So J, if you need to stand up here, go on. I actually do want you to, so we can hear you better. J: She said she divided 42 divided by two and she got 21. And since 24 is greater than 21, than it's over- the half. It's greater than half.	Routine: Access to Learning Opportunities: Working with public records of students' mathematical thinking Terminology	Math Habit: Explain my reasoning CTH: Orients students to ideas for analysis by focusing discourse on student revoicing of others' math reasoning Math Habit: Listen to and make sense of our ideas

Figure 3: Student Teacher Interaction on Fraction Comparison Analyzed with the Mathematically Productive Classroom Framework

The second excerpt (Figure 3) comes from a fourth-grade classroom where the teacher has students share their strategies publicly for comparing 24/42 to ½, providing a written record of their ideas. We see Student P's conclusion in the excerpt (preceded by a long explanation). The teacher then leverages a revoicing prompt to engage students in analyzing and making sense of the idea proposed by student P. J then makes sense of P's thinking, through revoicing. As in the prior exchange, teachers and students engaged in interactions within the larger routine, this time the routine was working with public records of students' mathematical thinking' By prompting for revoicing, the teacher engaged in a CTH that catalyzed students in listening and making sense of mathematical ideas (Math Habit of Interaction.)

Due to page limit constraints, we provided shortened excerpts to highlight the primary routines supporting ALO including the pre-existing public records routine, and the newly developed task-focused routine. High quality mathematics classrooms do more than just provide contexts for students to engage in valued mathematical activity (such as justifying and generalizing), but also attend to establishing common ground (e.g., Staples, 2007) where students have opportunities to access the mathematics embedded in tasks and in one another's ideas. We can make sense of classroom interactions through attention to the ways in which routines, catalytic teaching habits, and math habits of mind and interaction co-occur.

Discussion

The work presented above stems from a major effort to bring knowledge of teaching to support ELLs in communication with standard mechanisms of best practices in mathematics teaching. As such, we presented dual prongs of work focused first on empirically establishing that literature-suggested supports do in fact support ELLs in their mathematical learning, and second on putting such work in communication with a framework designed to support researchers in parsing student-teacher interactions and support teachers formatively as they grow in their practice.

We posit that research focused on equitable teaching practices must be both empirically grounded, and operationalized in a way to bridge the research practice divide. Through cycles of empirical exploration, theoretical operationalization, and refinement, we have worked to develop actionable teacher routines and catalytic teaching habits that may remove barriers providing access for ELLs' learning opportunities. We acknowledge that our work is limited to the particular lenses we have adopted and certainly cannot attend to all aspects of mathematical productive classrooms, nor are we exhaustive in the teaching practices to support ELLs. Rather, we focus our work in the context of mathematical reasoning first and what types of instructional routines may engage students deeply in mathematics regardless of language background. We see our work as having implications for research (in the creation of operationalized in-the-moment interaction patterns that can be explored in relation to important outcome variables) and for practice (in the creation of actionable routines and catalytic teaching habits that can be incorporated into practice.)

Endnotes

[1]We use this terminology to align with majority of literature; however, we acknowledge this terminology privileges the English language and has potential deficit connotations.

[2]We choose to only report on selected variables, for effect size on other variables related to teacher preparation and education see Sorto, Wilson, & White (2018).

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