PROBABILITY AND INDEPENDENCE: A COMPARISON OF UNDERGRADUATES

Karen Zwanch
Virginia Tech
kzwanch@vt.edu

Probability and independence are difficult concepts, as they require the coordination of multiple ideas. This qualitative research study used clinical interviews to understand how three undergraduate students conceptualize probability and probabilistic independence within the theoretical framework of APOS theory. One student's reasoning was consistent with a process conception, one student with at least an object conception, and one with a schema conception of probability and independence. Differences in students' thinking are analyzed, with a specific focus on intuition and simultaneously occurring events.

Keywords: Probability; Post-secondary Education

Review of Literature

"Students have difficulty just sorting out the mathematics of whether events are statistically dependent or independent in probability problems" (Shaughnessy, 2003, p. 221). The sentiment of this quotation resounds throughout the literature on probabilistic independence, and it has been well documented for students at varying ages. Considering the definition of independence, for two events A and B, $P(A) = P(A|B)$, there is a clear relationship between conditional probability and independence which students must come to understand. In recognition of the need for students to coordinate these concepts, Tarr and Jones (1997) constructed a framework for understanding how middle school students conceptualize independence and conditional probability. When students' understanding of these two concepts were analyzed within the four levels of this framework, it was determined that students' levels of reasoning regarding conditional probability and independence tend to be the same; this finding advances the apparent connection between conditional probability and independence, and furthermore, implies that students may construct these concepts in tandem.

Undergraduate students' struggles with probabilistic independence have been well documented. Kelly and Zwiers (1988) delineated three common misconceptions undergraduates have with independence: determining whether events are (in)dependent, understanding that dependence does not imply cause, and understanding that (in)dependence is not reliant upon time. Plaxco's (2011) findings support this third struggle. In interviews with undergraduates, Plaxco found that all students alluded to a temporal conception of independence and two of the three students included an element of time in their definition of independence. Students' inabilities to separate an element of time from their understanding of independence is clearly problematic, as the conditioning event may occur at the same time as or after the second event. In response to some of the difficulties students experience in conceptualizing independence, Keeler and Steinhorst (2001) call for the improvement of instruction in undergraduate probability courses, and specifically call instructors to capitalize on students' intuitions. They posit that an inquiry-based learning environment facilitates students' understanding of probability by building upon students' intuitions. Abrahamson (2014) also encourages probability instruction that "guide[s] students to appropriate the cultural resource as a means of supporting and empowering their tacit inference" (p. 250). His findings indicate that students' tacit inferences are a powerful instructional tool that instructors can leverage by linking it to formal mathematics. These researchers build upon the work of Fischbein (1987), who defines students' intuitions as primary or secondary. Primary intuitions are those students develop prior to instruction, whereas secondary intuitions are developed through formal mathematical experience. Secondary intuitions are not right and primary intuitions wrong; rather, secondary intuitions replace primary intuitions in situations where primary intuitions fail. Thus, for both Keeler and Steinhorst, and for Abrahamson, it seems that helping students construct secondary intuitions to complement primary intuitions regarding independence may be a powerful instructional approach.

On the other hand, Ollerton (2015) found that although undergraduate students have an intuitive understanding of independence, they struggle to conceptualize independence in a mathematically appropriate manner. Thus, it remains to be seen to what extent students' intuitions of probabilistic independence are beneficial in a mathematical context. Therefore, the purpose of this qualitative research study is to construct a more comprehensive understanding of how undergraduate students conceptualize both conditional probability and probabilistic independence, including how students use intuition to understand probabilistic independence.

Theoretical Framework

APOS theory (Dubinsky, 1991) utilizes reflective abstraction as the mechanism for learning mathematics, and specifically delineates the process by which individuals mentally construct mathematical schemas by progressing through action (A), process (P), and object (O) conceptions of mathematical concepts. These progressions ultimately formulate a schema (S), which is a "person's own cognitive framework which connects in some way all of the ideas that the individual either consciously or subconsciously views as related to the piece of mathematics" (Mathews & Clark, 2003, p. 3).

An *action* conception is the most rudimentary, in that mathematical tasks are completely external (Mathews & Clark, 2003); therefore, it is necessary for the student to carry out the actions of solving. Once an individual has interiorized the mathematical actions, they are able to act with a *process* conception (Dubinsky, 1991), enabling the individual to perform processes internally and to "reflect on, describe, or even reverse the steps of transformation" (Mathews & Clark, 2003, p. 2). As one's mental constructions become more powerful, processes can be encapsulated into *objects* (Arnon et al., 2014), which are static entities in and of themselves to which actions can be applied (Arnon et al., 2014). Ultimately, a *schema* conception, or a collection of thematized mathematical objects (Mathews & Clark, 2003), is the most powerful, and allows for the coordination of objects either within or between mathematical concepts.

Probability and Independence

The following is a preliminary genetic decomposition (Arnon et al., 2014) used to facilitate the analysis of students' reasoning. Within APOS, students operating with an action conception of probability are likely to calculate probabilities by applying a formula, or by relying upon physical manipulatives or a drawn sample space. As a process, students are able to anticipate the result of a probability problem without physical manipulations, and can therefore compare and reverse probabilities without actions. Once probability is encapsulated into an object, students are able to conceptualize compound and conditional probabilities because simple probability is now a static entity that can be combined (compound) and nested (conditional). As a schema, students can coordinate encapsulated objects within probability, including coordinating multiple representations of compound and conditional probability.

Probabilistic independence is a component of probability, and thus, its construction will be linked with that of probability (Tarr & Jones, 1997). With an action conception of probability

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and independence, determining independence will be reliant upon a formula with no justification of its use. Anticipation of the results of probability, indicative of a process conception, may allow students to develop an intuition for independence, without explicit calculations. As probability is encapsulated into an object, students begin to conceptualize conditional probability, which facilitates reasoning with regard to the definition of probabilistic independence. Finally, as a schema, multiple representations of probability and independence can be coordinated; this includes contingency tables, formulas, and definitions, to name a few.

Methods and Analysis

The participants in this qualitative study include three undergraduate students at a large, research university in the southeastern United States. The participants were recruited because they were at least 18 years of age and had taken a probability course. All participants were individually interviewed once, for approximately 40 minutes, and were assigned a pseudonym. Interviews were video recorded for the purposes of retrospective analysis. During the interview, each student completed nine questions meant to elicit their understandings of conditional probability and probabilistic independence. Questions six and seven are adapted from Manage and Scariano (2010; Appendix A) and question nine is from the research of Tversky and Kahneman (1980; Appendix A). Additional questions are described in the results section.

The data were analyzed using APOS theory. Accordingly, students' responses to each task were compared to the preliminary genetic decomposition described above. Note that it is possible for responses of students who have constructed more sophisticated conceptions of a mathematical concept to align with the reasoning of a less sophisticated conception. However, students with a less sophisticated conception cannot act in a manner consistent with a more sophisticated conception. For example, a student with an object conception of independence may write out all of the steps to completing a task because they believe they are supposed to show work. To an observer, this may seem to indicate independence is external, or an action, for the student; however, it is not a counter-indication of a process or an object conception. Conversely, a student with only an action conception cannot act in a manner consistent with an object. Thus, student responses throughout the interview were taken as a whole, and the most sophisticated conception of probability and independence observed was attributed to the student.

Results

John

The first participant, John, was a sophomore electrical engineering major who had completed a probability course in the previous semester. When asked to find the probability of rolling a six on a six-sided die given the result is even, John answered two related, albeit incorrect, probability problems: What is the probability of rolling a six? And, what is the probability of rolling an even? With some prompting he realized "Ohhh, oh, oh. *Given* that it's even." On its own, this could be interpreted as a misunderstanding, however, John treated conditional probability problems as two related problems, instead of as one problem with a conditioning event, three out of four times throughout the interview. In question nine he went as far as to say, "So this is, like, two parts." I interpret these responses as John's inability to recognize the need to adjust the sample space of the problem as a result of the conditioning event; and therefore interpret this an indicator that John has not encapsulated probability into an object because he does not nest two probabilities.

In the instances when John did solve conditional probability problems correctly, he relied heavily on a formula. On one question in particular, he applied a formula for conditional

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probability and when asked to justify its use, said, “so you find (laughs)… I don’t know how to explain this… it’s just…” An explanation never followed. Later, in response to question nine, John tried to find a relationship between the formulas he had inappropriately applied and the contingency table provided in the problem. He explained:

> I’m thinking this is definitely true (points to his written formula), but I’m thinking maybe some of those (pointing to numbers in the contingency table) could actually give me the answer. … doesn’t (as he checks one number in the contingency table against his formula), doesn’t (checks a second number), doesn’t (checks a third). No, it doesn’t.

John’s conclusion is that the numbers in the contingency table were unrelated to the formula. This speaks to his inability to conceive of the relationship between the two events within the task, which is a counter-indication of constructing an object conception of probability and independence because he does not justify the nesting of two probabilities. This is also a counter-indication of a schema conception because he does not relate multiple representations of conditional probability.

Although John relied heavily on formulas to reason about conditional probability, when reasoning about independence, he used formulas to justify his responses, not the other way around. In a question about the probability of being dealt two different sets of cards, he indicated that all cards dealt were independent of one another, and later used calculations to explain this. John went as far as to say, “just by intuition, they’re the same,” meaning the two hands are equally likely to be dealt. What John termed intuition, I consider to be his anticipation of the result, and therefore, evidence of at least a process conception of probability and independence. An area in which John continued to struggle with respect to independence was when two events were occurring simultaneously. In question seven, events C and D occur on a single roll of one die. John intuited these to be dependent events (incorrect), however, in justifying this response with a formula, he found them to be independent (correct). This was the only question regarding independence on which John’s intuition led him astray. This was also the only question in which one event could not be interpreted to occur before the other event. Ultimately, John explained, “I believe in the formula. The results of the formula, they can’t be false. … [but] if you just read it, not thinking about the formula, it would seem like they’re kind of dependent.” In this situation John struggled to discern independence from dependence as a seeming result of temporal reasoning; this limitation indicates that John has not coordinated conditional probability with independence in situations involving simultaneous events.

I attribute to John a process conception of probability and independence; that is to say, he has not yet encapsulated probability into an object upon which he can perform actions. John was able to apply formulas for probabilistic independence as a justification for his reasoning, not as a substitution for his reasoning. This is evidence of a process conception. However, John did not consistently adjust the sample space to account for the nesting of probabilities in conditional probability problems, which is a counter-indication of him having constructed an object conception. Furthermore, he did not demonstrate an understanding of the relationships between contingency tables and conditional probability, nor could he coordinate conditional probability with independence when events occurred simultaneously. These limitations are counter-indications of John having constructed a schema conception of probability and independence.

Dan

The second participant, Dan, was also a sophomore electrical engineering major, and was in the final three weeks of his probability course. In all conditional probability questions, Dan immediately recognized the need to adjust the sample space to account for the conditioning event. When asked to find the probability of rolling a six given the result is even, Dan explained, "If it's even it's going to be one-third because there's one possibility and there are three times it can be even – two, four, six. You want it to be six, so the number is one over three." This cogent justification implies that for Dan the probabilities of rolling a six and rolling an even are static objects that he acts upon by nesting them; this indicates at least an object conception of probability and independence.

He also coordinated his understanding of conditional probability with contingency tables. In question nine, for instance, he visually demonstrated this apparent coordination when he circled one column on the contingency table and said, "So it's gonna be this right there." He proceeded to verbalize the relationship between the table and the formula for conditional probability. This is evidence that Dan has constructed a schema conception of probability because of the coordination between multiple representations within the mathematical concept.

With regard to probabilistic independence, Dan relied upon his intuitions to anticipate the results of events within a set without performing calculations. In response to question six, he said, "I mean, there's only three possible outcomes. I'm just thinking about it … so that's the same for all of these … they can't be independent." When asked if he was comparing the probabilities of each event occurring, Dan repeated that he was "just thinking about it." This is evidence of his use of intuition, rather than mental calculations, which reinforces the indication of at least a process conception of probability and independence.

Dan's anticipation of probabilistic independence failed him, however, when responding to question seven. His intuition led him to believe that the simultaneous events occurring on a single die should be dependent, but upon calculation, he determined them to be independent. Reflecting on his calculations, he said, "It doesn't make sense. I'm just trying to picture this as a single event. It's not like you're saying this happened after the other event. … I just feel like it shouldn't be independent because they're happening at the same time." Dan spent several minutes trying to think of and explain other situations of independence as a means of eliminating his cognitive conflict, but was unsuccessful; he remained perturbed by the idea of independent events resulting from the roll of a single die. This is evidence of Dan not having coordinated objects of independence and probability in the case of simultaneous events, which is a limitation of his conception, and a counter-indication of him having thematized these objects into a schema. Considering probability tasks alone, it is possible to attribute to Dan a schema conception. However, when considering in tandem his reasoning with regard to probabilistic independence, it seems inappropriate to attribute to him the same conception. Based on this evidence, it is appropriate to attribute to Dan at least an object conception of probability and independence.

Aaron

The third student, Aaron, was a sophomore mathematics and economics major; he had completed AP Statistics in high school and received college credit for the course. Aaron calculated conditional probabilities correctly on all appropriate tasks, and did so largely mentally. In each of these questions he adjusted the sample space correctly in response to the conditioning event. In question nine, for instance, he explained that the sample space was changing from 1,000 taxis to 290 taxis by saying, "You add up how many times he actually says blue, which is the 290, and then of those times it's actually blue only 120. … That's (points to 120/290) the probability that the cab is blue given that he said blue." This excerpt exemplifies Aaron's ability to operate on one probability nested within another. When asked to explain his response to question nine, Aaron also constructed a tree diagram, and stated it was better organized than the table. He also clarified, "This is just what the table was telling me." As a

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whole, this is evidence that Aaron can coordinate formulas, contingency tables, and tree diagrams in reasoning about conditional probability.

In response to an independence task in which a set of people's behaviors and socio-economic statuses were given in a contingency table, Aaron calculated P(A|B) in determining whether event A depends on event B. When asked a follow-up probe (is event B dependent upon event A), he said "Maybe that's something where it's always if it goes one way [A depends on B] then it goes the other [B depends on A]. No, that isn't right." When asked how he arrived at this conclusion, he did not describe performing any calculations; instead, he described reasoning that P(A|B) is not equivalent to P(B|A), which is evidence of using a mental structure to anticipate the results of conditional probability. Aaron's ability to fluently solve conditional probability problems, to compare and reverse the conditioning events, and to reason using multiple representations, are all indicators of at least an object conception of probability. Moreover, his ability to compare and reverse conditioning events indicates he is likely operating on probabilities as static entities in their own right; for this reason, it seems likely that Aaron has constructed a schema conception of probability.

To further examine the schema attribution, it is important to consider his means of operating with regard to probabilistic independence. Aaron began the interview by explaining that probabilistic independence "means that the knowledge of one of them [events] happening shouldn't affect the likelihood of the other one happening." His metaphor of "dependence as knowing" was recurrent; I interpret his use of this metaphor as a mental structure that Aaron engaged to anticipate the independence of events. He used this metaphor again in question six: "you know that if you win, you don't lose and so the information of winning would help you figure out whether or not you lose. … they are dependent." Here, his metaphor of dependence as knowing includes the idea that "information" about winning provides knowledge, and therefore, implies dependence. Aaron's metaphor of dependence as knowing facilitates the coordination of independence with conditional probability, as demonstrated on question seven:

> If you know you rolled C [1 or 3], it doesn't actually help me know how likely it is that you rolled D [1 or 4], it just lets me know that it's more likely that you rolled a one… that means these would be independent… And then it goes the other way. If I knew about event D I wouldn't know any more about event C happening.

Aaron reasons about independence by explaining that knowing event C occurred doesn't give him more knowledge about whether event D occurred. Aaron does not struggle with the idea of events C and D occurring simultaneously because he coordinates conditional probability with probabilistic independence by applying his metaphor; this is evidence of a schema conception.

Discussion

John, Dan, and Aaron demonstrated different levels of reasoning; John's being consistent with a process, Dan's with at least an object, and Aaron's with a schema. A major difference between John's and Dan's conceptions is that Dan had encapsulated probability into an object upon which he could act, making it possible for him to appropriately interpret conditional probability. Dan understands the conditioning event to require an adjustment of the sample space, and justifies the use of probability formulas in his reasoning. John could not nest probabilities. Fischbein and Gazit (1984) found that difficulty reconstructing sample space in conditional probability is widespread, and this was experienced by John. Fischbein and Gazit's findings may be explained through the lens of APOS, as it appears that students must objectify probability to appropriately reconstruct sample space. This is an area for future research.

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The main difference between Dan’s and Aaron’s reasoning was regarding independence. Aaron was able to coordinate probabilistic events as static entities and act upon these events to determine independence regardless of a temporal connection. The task on which this difference was the most apparent was task seven, in which two dice are rolled simultaneously. On this task, Dan’s object conception did not allow him to coordinate multiple objects (probabilities) occurring simultaneously, whereas Aaron’s schema conception did. It is possible that Dan was relying on primary intuitions (Fischbein, 1987) in this situation, whereas Aaron had constructed secondary intuitions that allowed him to intuit results in the situation of simultaneously occurring events. However, the role of simultaneously occurring events in students’ constructions of independence requires further empirical consideration; specifically, why this difficulty limits students’ coordinations of probability and independence, and how a perturbation, such as Dan’s, can be exploited in instruction to engender the construction of more sophisticated conceptions. Abrahamson (2014) indicates that probability instruction should link students’ intuitions to analytic reasoning and empirical activities with regard to probability. Perhaps by helping students construct these links, they will be engendered to construct secondary intuitions to supplement their primary intuitions; without secondary intuitions, it seems as though students may be restricted to thinking about independence as reliant upon time.

Interestingly, Aaron had the most sophisticated conception of probability and independence among this group of students, and was the only student to rely on a metaphor for determining probabilistic independence. Sfard (1994) argues that “reification is, in fact, the birth of a metaphor which brings a mathematical object into existence and thereby deepens our understanding” (p. 54). With this understanding, it stands to reason that Aaron’s metaphor of dependence as knowing was born out of his thematization of probability and independence into a schema, thereby allowing him to more meaningfully intuit independence. Aaron’s use of metaphor was an unexpected result in this research study, and it remains to be seen whether reasoning with metaphor with regard to probabilistic independence advantages students’ reasoning in some way over the intuitive reasoning engaged by the other two students.

Conclusions

As is frequently noted in the literature, students’ abilities to determine the independence of probabilistic events is problematic. Although these undergraduates were all STEM majors with formal instruction in probability and independence, each demonstrated a different conception. With a process conception, John was significantly limited; moreover, his intuitions regarding independence were primary (Fischbein, 1987). Similarly, Dan relied on primary intuitions in the situation of simultaneous events. Although Dan had constructed at least an object conception of probability, he had not constructed secondary intuitions regarding independence involving simultaneous events. John’s and Dan’s intuitions advantaged their reasoning in different ways, demonstrating that not all intuitions are equally beneficial. Future research should examine the conception of probability and independence that supports students’ constructions of secondary intuitions, specifically with regard to temporal reasoning.

The preliminary genetic decomposition utilized in this research is preliminary in that it requires refinement, and the varying reasoning of these students can direct these revisions. First, intuition was included as an indicator for the construction of a mental structure for independence, and thus, a process. While this was appropriate for John, Dan’s intuitions were more sophisticated. As a result, intuition needs to be more thoroughly examined within the APOS framework. Furthermore, these results indicate that even primary intuitions can be beneficial to students’ reasoning, which supports leveraging students’ intuitions in instruction. By capitalizing

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on primary intuitions, perhaps researchers can identify the mental constructs that support the construction of secondary intuitions and schemas, and instructors can begin to engender these constructions in their classrooms.

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Appendix A

6) Assume a competitive game can end for team A in a win (P(win) = 0.4), loss (P(loss) = 0.5), or tie (P(tie) = 0.1). Are these events pairwise independent or dependent?

7) Roll a single, fair, four-sided die once and observe its upper face. Define two events: "C: Rolling either a 1 or a 3" and "D: Rolling either a 1 or a 4." Are these events independent or dependent?

9) A cab was involved in a hit and run accident at night. There are two cab companies that operate in the city, a Blue Cab company and a Green Cab company. It is known that 85% of the cabs in the city are Green and 15% are Blue. A witness at the scene identified the cab involved in the accident as a Blue cab. The witness was tested under similar visibility conditions and made the correct color identification in 80% of the trial instances. What is the probability that the cab involved in the accident was a Blue Cab rather than a Green one?

**note that a contingency table was also provided to participants for question 9.*