TRANSFORMING TEACHERS' KNOWLEDGE FOR TEACHING MATHEMATICS WITH TECHNOLOGIES THROUGH ONLINE KNOWLEDGE-BUILDING COMMUNITIES

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Mathematics teacher educators are faced with designing teacher in-service professional development experiences for developing and transforming Technological Pedagogical Content Knowledge (TPACK) towards integrating digital technologies as mathematics learning tools. Online environments provide opportunities to a broad range of teachers, yet, the asynchronous nature presents communication and collaboration challenges. A researcher-conjectured, empirically-supported learning trajectory guides this online TPACK program for engaging teachers in knowledge-building communities. Three online technology education courses provide teachers with experiences as students, learning about the technologies while confronting challenges to their thinking about teaching with the technologies. The fourth course provides the teachers with key experiences through blended instruction. Through online explorations and discourses in their communities, they examine reform-based instructional strategies for teaching with technologies. Concurrently, they design, implement, analyze and reflect on their teaching experiences through their designed five-day unit in their mathematics classrooms. Four TPACK components reveal how this experience in knowledge-building communities transforms their TPACK.

Keywords: Learning Trajectory, Teacher Education-Inservice/Professional, Teacher Knowledge, Technology

Introduction

Mathematics teachers are challenged to actively engage students with current, more effective technologies as learning tools. A recent handbook chapter (Roschelle, Noss, Jackiw & Blikstein., 2017) highlights three important research-based categories of effective learning digital tools: tools like graphing calculators that can do some of the detail work and students can focus on concepts; tools for providing guidance and feedback to students as they practice mathematics; and tools that help students visualize concepts and develop understanding. With the rapid pace of the development of digital tools, teachers cannot fully realize the value of these tools without teacher professional development. While teachers may have heard about the technologies, this simple knowledge is not sufficient for guiding students in learning mathematics with the technologies. Now recognizing twenty-first century learning, teachers must engage their students in developing four key skills (the 4 C's) in preparation for effectively connecting with a global society: Critical thinking, Communication, Collaboration and Creativity (Partnership for 21st Century Learning, 2015; Thoughtful Learning Organization, 2016). Through these skills, students are prepared to effectively engage in the more complex social, cultural, and educational environments that depend on the advantages offered through the reliance on multiple technological resources. Taking advantage of these 4C's, students participate through various thinking and engagement strategies as they concurrently learn mathematics:

- Critical thinking: Students use analysis as they engage in problem solving where they make comparisons, contrast ideas, analyze ideas, categorize data, and evaluate results of trials.
- Creative thinking: Students engage in open-ended invention and discovery of myriad solution possibilities as they design, improvise, innovate, problem solve and ask questions.
- Communication: Students interact with others, connecting through multiple modes (e.g., text, social media, cell phones, email, Internet and other avenues) where they examine the messages with respect to the purpose, sender, receiver, medium and the context of various communications.
- Collaboration: Student work together toward a common goal, brainstorming ideas, making decisions as a group, delegating, evaluating, goal setting, managing time, resolving conflicts, and team building.

As mathematics teachers consider integrating technologies in their instruction, they must not only determine how the technologies support learning the mathematics, they must consider which pedagogical strategies effectively engage students in learning the mathematics with the tools as they incorporated the 4 C's (Roschelle & Leinwand, 2011). Teachers must identify, orchestrate, and manage different pedagogical strategies and learning tasks for integrating the technologies in new and perhaps different mathematical topics. The challenge involves far more than their understanding of the mathematics content. The experience ultimately challenges their technological pedagogical content knowledge and reasoning (TPACK) with an array of technologies. "Quality teaching requires developing a nuanced understanding of the complex relationships between technology, content, and pedagogy, and using this understanding to develop appropriate, context-specific strategies and representations" (Mishra & Koehler, 2006, p. 1029).

Today's mathematics teachers must continue learning about teaching mathematics beyond their learning in their pre-service teacher preparation programs and typically this learning happens as they are actively teaching. Transforming in-service teachers' TPACK requires more than participation in short-term professional development experiences. Teachers need experiences to actively engage them in the process of "working toward a more complete and coherent understanding," otherwise referred to as knowledge-building experiences (Scardamalia & Bereiter, 1993, p. 39). Knowledge-building communities (e.g., Scardamalia & Bereiter, 1993, 2003; Bereiter, 2002) integrated with classroom teaching experiences are more likely to engage teachers in relearning, rethinking, and redefining teaching and learning to take advantage of new and emerging technologies and methods for teaching mathematics. Through such communities combined with practical teaching experiences, teachers have opportunities to confront their current pedagogical conceptions for integrating technologies as useful learning tools in their content areas (Loughran, 2002) in the process of developing reformed understandings for teaching in the twenty-first century with multiple technologies.

In this paper, we specifically consider the potential benefits but also the potential challenges of providing teachers with learning experiences in online or blended formats that intend to establish online knowledge-building communities. On one hand, online or blended learning is an increasingly recognized educational setting for teachers' professional learning experiences. It can allow teachers more choice about how, when and where they learn, reduce cost, and provide increased access for many more teachers across a broader geographical area. Yet, the primarily asynchronous nature of online learning poses additional challenges for the design of these needed educational experiences (Means, Toyama, Murphy, & Bakia, 2013).

When considering these points, mathematics teacher educators are confronted with a primary and critical question in the design of knowledge-building communities for transforming teachers' TPACK:

What experiences are essential for guiding in-service teachers as they learn about the technologies as well as about teaching mathematics with the technologies?

Building on this broad question, important sub-questions in the design of online professional development instructional programs emerge:

1. What are the key features for online learning for guiding teachers in reframing their current teacher knowledge?
2. What online learning trajectories are not only useful for engaging teachers in knowledge-building communities but also for providing them with an understanding of the pedagogical challenges in their classrooms?
3. How might teachers gain classroom-based learning experiences for applying their theoretical ideas about teaching with technologies?

Through this paper, we report on the first author's research and development project, addressing these questions in the process of developing ad analyzing the outcomes of a new online in-service teacher TPACK program containing four graduate courses. The effort used a researcher-conjectured and empirically-supported learning trajectory (Niess & Gillow-Wiles, 2013, 2014) to frame the experiences and pedagogical strategies to engage the in-service teacher participants in online knowledge-building communities blended with practical teaching experiences when teaching mathematics with technologies. The second author adds comments in the Discussion section.

Theoretical Framework

Teachers' knowledge for teaching with technologies requires far more than just understanding the subject matter. It ultimately necessitates a strong pedagogical knowledge merged with the knowledge for teaching mathematics using a vast array of technological innovations. This task calls for Technological Pedagogical Content Knowledge (Angeli & Valanides, 2009; Mishra & Koehler, 2006; Niess, 2005), otherwise referred to as TPACK (called 'tee-pack', Thompson & Mishra, 2007). TPACK, as shown in Figure 1, describes this teacher knowledge through the intersection of content knowledge, pedagogical knowledge and technological knowledge for guiding their strategic thinking of when, where, and how to guide students' learning of the content such as mathematics with technologies.

TPACK is the composite of the intersecting multiple domains (technological knowledge (TK), pedagogical knowledge (PK), content knowledge (CK), technological pedagogical knowledge (TPK), pedagogical content knowledge (PCK), technological content knowledge (TCK) and TPACK) within the Contexts. Further, the center subset is also described as TPACK. Mishra and Koehler (2008) described this center subset as:

> The representations of concepts using technologies; pedagogical techniques that apply technologies in constructive ways to teach content in differentiated ways according to students' learning needs; knowledge of what makes concepts difficult or easy to learn and how technology can help redress conceptual challenges; knowledge of students' prior

content-related understanding and epistemological assumptions; and knowledge of how technologies can be used to build on existing understanding to develop new epistemologies or strengthen old ones. (p. 3)

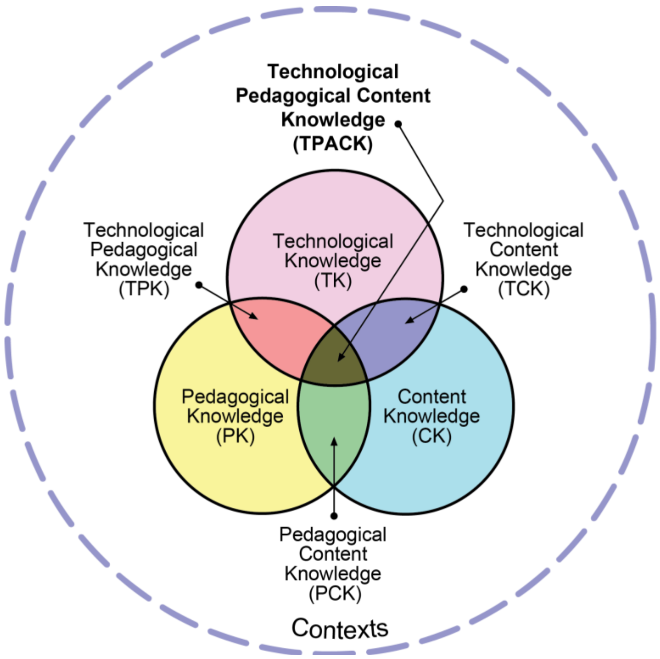


Figure 1. Representation of Technological Pedagogical Content Knowledge (TPACK) as teachers' transformed knowledge. Reproduced by permission of the publisher, © 2012 by tpack.org

Elaborating on TPACK

The TPACK construct is recognized and supported by extensive research and scholarly work. Elaborating on TPACK, Niess (2005) extended from Grossman's (1989, 1991) description of the four components of PCK. The four TPACK components incorporate the influence of technology in teachers' instruction, considering the teachers':

1. Overarching conceptions about the purposes for incorporating technology in teaching mathematics topics;
2. Knowledge of students' understandings, thinking and learning in mathematics topics with technology;
3. Knowledge of instructional strategies and representations for teaching and learning mathematics topics with technologies;
4. Knowledge of curriculum and curricular materials that integrate technology in learning and teaching mathematics topics.

Building on these components, Niess, Sadri and Lee (2007) described TPACK development as a process in transforming teachers' TPACK. They linked Rogers' (1995) five-step process in the ultimate decision of whether to accept or reject a particular innovation with the analysis of extensive observations of teachers' learning about spreadsheets and how to integrate spreadsheets as learning tools in their mathematics classrooms. Through this effort, they found the teachers at different stages in their TPACK transformations:

1. Recognizing (knowledge), where teachers are able to use the technology and recognize the alignment of the technology with mathematics content yet do not integrate the technology in teaching and learning of mathematics.
2. Accepting (persuasion), where teachers form a favorable or unfavorable attitude toward teaching and learning mathematics with an appropriate technology.
3. Adapting (decision), where teachers engage in activities that lead to a choice to adopt or reject teaching and learning mathematics with an appropriate technology.
4. Exploring (implementation), where teachers actively integrate teaching and learning of mathematics with an appropriate technology.
5. Advancing (confirmation), where teachers evaluate the results of the decision to integrate teaching and learning mathematics with an appropriate technology. (Niess et al., 2009, p. 9)

Here's a scenario: Mr. D is a middle school mathematics teacher with a degree in mathematics who was excited as he learned to design dynamic spreadsheets for exploring algebraic problems. However, he was constrained in his acceptance of students using spreadsheets for exploring algebraic changes when thinking about constants and dependent versus independent variables. He believed that students needed to create multiple graphs with paper and pencil in order to identify changes in the constants and variables of different functions. He was willing to use the spreadsheet for his summarization of the results, where he would demonstrate the changes for the students; however, he was not willing to engage the students in spreadsheet explorations. While he was considered to be at the *recognizing* level, even with more work with spreadsheets, he resisted the idea of adding spreadsheets to his mathematics classes.

In a contrasting case, Mrs. A, a teacher with a mathematics education degree, was excited with the ease and visualizations that resulted as she worked with her students in designing dynamic spreadsheets in their explorations. She wanted her students to have this experience as they worked in groups to explore changes in constants and variables. She felt that this more visual approach helped them gain a better understanding than if they had to individually graph each of the problems. As she worked with her students with the spreadsheets, she envisioned additional experiences for using spreadsheets in her mathematics classes and was thus viewed at the *exploring* stage.

These stages were proposed as an iterative process in the development of TPACK rather than a strictly linear process. In essence, some aspects of what is learned about teaching a particular topic with one technology may provide a disposition toward the acceptance of another technology. But teachers need to explore different topics with each new technology, considering its applicability for supporting learning mathematics with that technology

Developing In-service Teachers' TPACK

Reconstructing in-service teachers' knowledge to reflect the ideas as described in TPACK requires teacher engagement in systematic inquiries about teaching, learning, subject matter and curriculum, and schooling, much like that described in Cochran-Smith and Lytle's (2001) conception of "knowledge-of-practice" as a "transformed and expanded view of what 'practice' means" (p. 276). Such a reformed conception assumes that knowledge is "socially constructed by teachers who work together and also by teachers and students as they mingle their previous experiences, their prior knowledge, their cultural and linguistic resources, and the textual resources and materials of the classroom" (p. 280). Transforming teachers' knowledge suggests their involvement in an inquiry knowledge-building community where reflection is a central component (Cochran-Smith & Lytle, 2001; Loughran, 2002; Schön, 1983).

Borko (2004) describes a process for identifying high–quality teacher change programs as called for in transforming teachers' TPACK, as beginning with studies in a single site, exploring relationships between teachers as learners in a specific learning trajectory. Confrey and Maloney (2010) expand this thinking, describing the need for identifying a learning trajectory as a "researcher-conjectured, empirically-supported" description of an "ordered network of experiences" where teachers as students move from "informal ideas, through successive refinements of representation, articulation, and reflection, towards increasingly complex concepts over time" (Confrey & Maloney, 2010, p. 968). In this manner, teachers are engaged in instruction designed to move from informal ideas through successive refinements toward a transformed knowledge for teaching with technology. Through this purposeful learning trajectory, they develop knowledge through their experiences as they are engaged in instructional strategies that ultimately model teaching with technologies.

Teachers as learners are, thus, charged with becoming aware and critical of their own and others' assumptions about teaching to achieve a paradigm shift that transforms their thinking and actions toward the ideas embedded in TPACK. McGonigal (2005) outlines five conditions and processes for fostering such a transformative learning experience for enhancing teachers' TPACK:

1. Teachers need an activating event to expose the limitations of their current knowledge.
2. Teachers need opportunities to identify and articulate underlying assumptions in their teaching knowledge.
3. Teachers need to engage in critical self-reflection, specifically considering the origin of underlying assumptions, and how these assumptions have influenced or limited their understandings about teaching.
4. Teachers need to engage in critical discourse with other teachers and the adult teacher educator in the process of examining alternative ideas and approaches.
5. Teachers need opportunities to test and apply their new perspectives.

Online TPACK Learning Trajectory

We investigated an educational setting where in-service teachers' professional learning experience is provided through online programs. The first author's research group designed an approach based on Niess and Gillow-Wiles (2013, 2014) empirically-supported learning trajectory, to frame online TPACK learning experiences to engage teachers in knowledge-building communities designed toward transforming the teachers' TPACK. The trajectory recognized key instructional strategies through a social metacognitive constructivist instructional framework that identified key tools and processes for organizing the TPACK content development in online asynchronous, text-based inquiry learning experiences.

Tools. Two tools support the online professional development: (1) a community of learners and (2) reflection. With the challenge for establishing connections among the learners and the instructor in online learning, establishing a community of learners provides an important tool for supporting the learners in communicating and interacting through discussions about the tasks and ideas being developed. This tool provides a ***social*** presence such that the community functions as a knowledge-building community. Through this community, the construction of knowledge is a "social activity, with new information and ideas brought into the discourses of a community that shares goals for knowledge advancement and recognizes contribution" (Scardamalia & Bereiter, 1993, p. 38-39). Such a social presence establishes community member participation and educational experiences for meaningful learning, open communication, and group cohesion as the learners engage in active roles to make sense of new information and ideas (Bereiter, 2002; Garrison, Anderson, & Archer, 1999; Garrison & Cleveland-Innes, 2005; Hill, Song, & West, 2009; Kinsel, Cleveland-Innes, & Garrison, 2005; Rourke, Anderson, Garrison, & Archer, 1999; Scardamalia & Bereiter, 1993; Sung & Mayer, 2012; Swan & Shih, 2005).

Reflection was the second important tool in the online learning trajectory. Critical reflection supported the cognitive presence in the online learning trajectory. Learner engagement through reflection happened in multiple ways, such as having learners prepare content reflective essays, reflective essays on the community engagement and peer reviews of another learners' work. The online portfolio provided a consistent way for each teacher to capture, reflect on and share their progress. Basically, the community of learners dynamically integrated the social, cognitive, and teaching presences in the online environment, supporting higher order learning through the reflective actions that result in deep approaches to learning (Garrison & Cleveland-Innes, 2005).

Processes. The online learning trajectory also includes two key processes for incorporating the tools: (1) shared/individual knowledge development and (2) inquiry learning. As the learners participates in the community of learners' activities, they share their understandings of how they are interpreting the ideas. As the discussions evolve through their interactions, the learner's individual knowledge matures. The learners move between group and individual knowledge-building so as to ultimately create an understanding that more clearly reflects a world view with respect to the learning experiences (Dunlap & Lowenthal, 2014; Rienties, Tempelaar, & Lygo-baker, 2013; Swan, 2001). As a result, the learners' individual knowledge expands beyond that which they were able to develop independently.

The second key process with the knowledge-building communities relies on inquiry-based activities to provide the learners with tasks, opportunities and experiences where they negotiated their understandings of the content. The inquiry process immerses them in constructing their understandings, where they take ownership of their learning, beginning with questions and their explorations where they investigated worthy questions, issues, problems or ideas. They ask questions, gathers and analyzes information, generates solutions, make decisions, and justifies their conclusions. The resulting actions interweaves multiple technologies, instructional approaches, and content topics through multiple units. Throughout the process, the participants consistently engage in thinking and reflecting about the dynamic interactions among content, pedagogy and technology that emerges from the tasks in their online learning experiences (Roberts, 2002; Wheatley, 1992).

An Online TPACK Instructional Program

Our program for transforming mathematics teachers' TPACK for teaching with technologies developed four new university graduate level courses. Three of the courses are fully online and focused on technology education. These courses are combined with a blended course that incorporates teachers' practical experiences where they implement their newly developed technological knowledge in their own classrooms with an online experience through a community of learners' inquiry and discourse about reformed-based pedagogical strategies.

Technological Education Courses

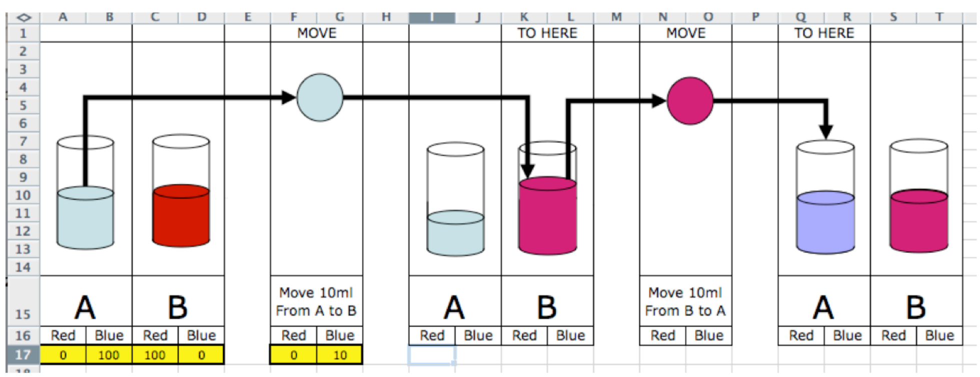
Three courses (SED 520, SED 521, and SED 522) engaged the teachers in experiences as students where they learn about and with some technologies in ways that challenge and advance their thinking about learning mathematics with these technologies. During these experiences, the teachers interact in small communities of learners in discourse, responding to the experiences as students and then as teachers. They use critical reflection in considering the dilemmas, their personal experiences and their discussions to inform their TPACK understandings.

For each unit in these courses, *inquiry* tasks challenge the teachers to examine and explore specific technologies. For example, in SED 520 one unit challenges them to examine and explore the temperature probe, a technology with which they are typically unfamiliar. They are provided with 10 laboratory experiments to guide their explorations as they learn how to collect temperature data and use the accompanying software to analyze the data, both graphically and numerically. As they work with the experiments, they also engage in questioning and discussion with their assigned small community of learner groups, discussing key questions as they examine the readings and their experiences in learning about and with the probeware. The unit emphasizes higher order thinking and inquiry learning to shape their ideas for integrating the technology as a learning tool in mathematics. To conclude the unit, the students prepare a critical reflection on their experiences with the technology, on the key questions and the inquiry processes and how their shared/individual experiences through the community of learner's discussions as well as their individual experiences form their understandings in this unit.

SED520. Integrating Technology and Literacy in Learning Mathematics (SED 520) is a course focused on multimedia technologies for twenty-first century mathematical literacy incorporating the 4 C's in teaching and learning mathematics. The course is arranged in five, two-week unit experiences. The technologies are purposefully selected and organized with the content. Unit 1 begins with presentation software, a technology with which teachers typically have experience. They are charged with using a Google presentation in a cooperative, collaborative experience to inquiry and examine new Web 2.0 technologies (Diigo, Wordle, Blabberize, Glogster, Voki, and Popplet for example). Each class member has responsibility for creating one slide that describes one specific technology term and how it can be used in education and a second slide that introduces them as new members of the class. Unit 2 promotes the teachers as learners exploring a technology with which they are unfamiliar – the temperature probe, as previously described. This experience engages them as students, where they must become familiar with the technology and consider how it might be used in learning mathematics; additionally, in their roles as teachers they consider what instructional strategies are needed to engage students in learning with this technology. The Web Inquiry Unit 3 pairs teachers in the design of a web presentation to guide students in specific mathematical explorations. This unit uses a framework similar to that in WebQuests (http://webquest.org/) to guide students in mathematical inquiries through web experiences (see http://webinquiry.org/ for sample inquiries). Unit 4 expects the teachers to work individually in designing three lessons that require use of a specific web application for learning mathematics. Teachers typically consider applications such as those available through the National Library of Virtual Manipulatives (http://nlvm.usu.edu/en/nav/vlibrary.html) or other similar libraries of interactive mathematics experiences. Finally, Unit 5 organizes the teachers as small cooperative groups for group analysis and writing experiences. These last two units expect the teachers to think about the comparison of knowledge gained as individuals (in Unit 4) versus that gained through cooperative and collaborative sharing with technologies (in Unit 5). The culminating product for this course is a web portfolio that presents the various products produced in each of the units of the course, demonstrating their knowledge of multimedia mathematical literacy.

SED 521. The second course, SED 521 (Teaching Mathematics With Digital and Video Technologies), uses inquiry experiences with digital images and videos to engage the teachers as students in higher order thinking and inquiry in mathematics. The major course difference between this course and SED 520 is the organization of the technology experiences into two major units, one focused on teaching mathematics with digital images and the second on incorporating videos as mathematics learning tools. Weekly, specific inquiry tasks confront the teachers. For example, during the first unit, they are challenged to gather digital images that afford opportunities for engaging students in twenty-first century learning of mathematics. The teachers are challenged to engage in their communities of learners, exploring these images to respond to this question: How might students' selection, acquisition, and presentation of digital images be beneficial for their learning of mathematics topics? The teachers engage in discourse with their assigned communities of learners and complete critical reflections through this question and the accompanying tasks and experiences with multiple digital images. Over the course of all the units they discuss the key questions and how their shared/individual experiences as well as their individual experiences influence their developing understandings. The culminating course product is a collection of digital images and videos with lessons for using them as mathematics learning tools. The collection also includes a video that portrays an example of what students might develop to communicate their higher order thinking when solving mathematics problem.

SED 522. The third course (SED 522), Dynamic Spreadsheets as Learning Tools in Mathematics, provides teachers with opportunities to explore algebraic reasoning when engaging students in learning with spreadsheets. Throughout the 10 units, inquiry tasks that involve developing skills with spreadsheets, the teachers' understanding for designing dynamic and dependable spreadsheets as a mathematics learning technology develops. The units model specific problems for gaining knowledge through access to spreadsheets. For example, Figure 2 presents a problem the teachers as students are to solve using relative cell referencing.



One beaker (A) has 100 ml of a blue liquid and the other beaker (B) has 100 ml of red liquid. First move 10 ml from A to B. Then move 10 ml from B to A. How many exchanges are needed before the liquid in both bottles contains the same concentration of blue and red liquids?

Figure 2. Mixture spreadsheet problem for SED 522

After designing a solution, they must develop a graphical representation (Figure 3) of their design and engage with their communities of learners' knowledge-building communities to explore multiple TPACK-related questions:

1. Where might this problem be useful in the mathematics curriculum?
2. What spreadsheet skills do students need for designing such a spreadsheet solution?
3. How should the students be organized for solving this problem?

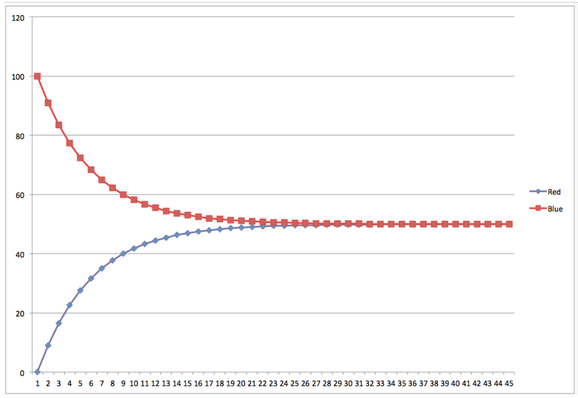


Figure 3. Graphical representation of the solution

In completing this spreadsheet course, the teachers' final project is a portfolio containing their collection of mathematics problems with solutions, curriculum plan for integrating spreadsheets as tools for exploring mathematics problems, and a final reflection on preparing their students to use the spreadsheet as a mathematical tool.

Blended Online and Practical Course: SED 594

Given these new online courses, the program designers questioned whether the experiences actually translated to the teachers' classrooms. For this reason, a fourth course, SED 594 (Advanced Teaching Strategies in Mathematics), was added to the TPACK program. As a blended course, this course combines online discussions and explorations about reform-based instructional strategies with practical teaching experiences in the teachers' own classrooms. Through the online explorations and discussions in their knowledge-building communities, they examine reform-based instructional strategies. Concurrently with their communities of learners' discussions, the teachers individually design, implement, analyze and extensively reflect on their personal, practical teaching experiences in their five-day unit in their mathematics classrooms. Essentially, they engage in action research about their own teaching as they gather artifacts to describe their instructional strategies, tools, and processes for engaging their students in learning mathematics with the selected technologies.

They incorporate extensive reflections in their electronic portfolios. These portfolios incorporate critical reflections from throughout the course to demonstrate how they implement their knowledge for teaching in their classrooms as they integrate technologies in teaching/learning mathematics. They gather two videos of their classroom instruction. They gather student products, examining whether the students demonstrate a strong understanding, average understanding or weak understanding of the concepts or processes. They complete multiple critical reflections throughout their instructional experiences (on the lesson designs before teaching, after teaching the lessons, about their students' work in the lessons, after watching videos of their instruction, and at the completion of the instruction). Weekly, they also reflect on their community of learners' explorations and discussions about the reformed-based instructional strategies. They peer review another teachers' portfolio, providing recommendations for enhancing the communication of the ideas. And, they complete a final in-depth analysis and reflection of the experiences they have had in their classroom teaching and their community of learner's discussions and interactions.

While they engage in the practical teaching experiences, they use the online community of learner groups to explore and discuss different instructional strategies, tools, and processes for teaching with technologies. Throughout their discussions, they cooperatively explore and

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examine reform-based instructional strategies – visible thinking, student discourse, grouping structures, and multiple representations for motivation and engagement. They peer-review another's electronic portfolio with the goal of improving the communication of the events and thinking. This review prepares them for the final critical reflection on the entire blended course experience, including the instructional experiences, video analyses, plans for improving their future instruction, and discourse in their community of learners where they discuss and consider instructional strategies for teaching with technologies. This final critical reflection reveals how their TK, TPK, and TCK merges with their PCK in the process of transforming their TPACK.

Teachers' TPACK Transformations

To investigate the influence of the online TPACK instructional program on teachers' knowledge transformations, we analyzed observations using the teachers' artifacts, expressions and reflections describe. The analysis reveals their knowledge, thinking and reflection with respect to the four TPACK components (Niess, 2005). Three representative cases (using pseudonyms Janis, Judy, and Lucy) were purposefully selected to display the patterns in the diversity of classroom situations and teaching levels. Janis, an elementary teacher, designed a fraction unit combining concrete and virtual manipulatives and games and activities she found on various websites for building students' understandings for representing, comparing and ordering fractions. Judy, a ninth-grade mathematics teacher chose to incorporate graphing calculators as a technology for her geometry course as they explored properties of transversals and the created angles, parallel lines, and perpendicular lines. Lucy, a high school mathematics teacher, incorporated the temperature probe with spreadsheet software in her pre-calculus class to engage her students in examining exponential, logarithmic and logistic functions.

Overarching Conceptions

As the teachers used technology in their own classrooms in the culminating SED 594 course, they were challenged to evaluate the value of the mathematical learning experiences for their students. In essence they were confronted with the question of whether or not the technology would support student learning in a purposeful and useful way.

Janis had previously taught her unit using only concrete manipulatives to build the students' understanding of fractions. "Using the virtual manipulatives was new and did cause me to change the way that students completed some of the tasks from previous years. The virtual manipulatives were nice because they didn't take up space like the concrete ones, and students were able to work with partners more instead of their whole table group." Janis saw the virtual manipulatives as adding to learning in an important way: "It allowed the students to make an instant connection between the manipulative and the mathematical concept because the symbolic notation was also shown on the screen." Reflecting on the value of the virtual manipulatives, she revised her original conceptions since the technologies provided a larger variety of learning experiences that she found important for student understanding of the concepts.

Judy's conception of graphing calculators as learning tools was intertwined with the importance of having students in working groups. She concluded that having the students work in groups led to solving problems and exploring ideas through technology tasks and deepened their learning because they were developing their own plans for solving the problems.

> When students worked in the groups with the graphing calculators, they helped each other when they were stuck. The groups also allowed students to explore many different equations of lines at the same time. They were exposed to many more possibilities of equations of lines than they would have been if I were leading the whole group.

Lucy designed her instruction to integrate technology and science in her mathematics instruction "The learners were supported in their learning through the computers and the Go!Temp Probe which collected data modeling a logistic curve. They were able to make real-world connections to their learning." She concluded "The connection to science makes the learning richer and fuller."

Students' Understandings, Thinking and Learning

Another concern for the teachers involved the question of how the technology aided in students' understandings, thinking and learning of specific mathematics topics. They were confronted with assessing the students' understandings and whether that understanding and thinking supported a stronger knowledge.

Janis noted that her students struggled to represent a given fraction three different ways but with the addition of the virtual manipulatives and other applications, she noted that when they built their fractions "on the fraction bar site they were forced to re-examine their ideas." She often talked about students' thinking and how their understandings and learning were impacted:

> Many students are visual learners, and it would not be enough for them to simply listen to another students' [explanation]. The combination of visual and auditory increased the number of students…engaged during group sharing and discussions and [increased] the chances that students [would] effectively process and retain the information in a meaningful way.

Judy gained an appreciation for the use of small group work with technologies for understanding students' thinking and understandings. During a Think-Pair-Share activity, she observed many conversations and heard many students mention slope. "It helped that they did not have to share among the larger class first ... more students [were] willing to share than usual, and I feel it is due to sharing with a neighbor first." She noted how using technology in the group work helped her understand the students' thinking. "Students were very engaged within the group work today. Perhaps the most engaged I have seen them in [the] group work. I feel the use of technology was a key factor."

Lucy realized her students' knowledge and thinking in one class did not necessarily transfer to other content areas. After the second day of her unit, she reflected that even though students had used microscopes in their biology class, they had a hard time translating that knowledge into the mathematics classroom. With a little guidance, "they got on track nicely and I anticipate this task to go faster as the days go on."

Instructional Strategies and Representations

Since the teachers were expected to design and implement a five-day unit, they were confronted with the task of identifying and scaffolding multiple instructional strategies to support students in learning the mathematics topics. They recognized the additional challenge for also scaffolding the instruction in such a manner that it supported student engagement with the technologies.

Janis connected the online discussions with her community of learners with her ideas for instructional strategies: "I incorporated some of the research-based instructional strategies that we have been discussing...Primarily, I used a lot of discourse, questioning, and collaborative learning…I tried to provide multiple representations…whenever possible, allowing students to connect their ideas with a visual." She gained a deeper understanding of strategies for integrating technologies through her video analysis. She saw the need to provide more direct instruction with the technologies. "What went really well was engaging students in the lessons and giving them opportunities to share their thinking as they worked collaboratively on the

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tasks." She also recognized that with students' ages and the novelty of using the technology, "I had to make sure that the tasks we did in the computer lab were very structured."

For Judy, the emphasis in the online discussions and her experiences with her instruction aligned her thinking about instructional strategies with grouping, problem solving, questioning and student discourse because these strategies "allow[ed] the learning environment to be centered on students who are actively engaged." From her work with these various strategies, she indicated that "Overall I want my classroom to be more student-centered and less teacher-centered."

The strategies Lucy implemented included discourse, questioning, inquiry, motivation, multiple representations, compare and contrast, cooperative and collaborative learning, providing feedback, and technology and science integration. Throughout the various science lab lessons, she was a guide. "I did not measure any pH, nor did I tell them which number it best matched. They asked each other and worked very collaboratively during this lesson…I know the more consistent I am with being a guide rather than the sole provider of information, the sooner it will become a habit."

Curriculum and Curricular Materials

A critical concern for the teachers in this final work was to identify a topic that was best suited for the technologies they wanted to include. They each want to make sure they were teaching the expected curriculum in order to support students as they advanced to future topics.

Janis' knowledge and thinking about her curriculum and curriculum materials changed as she reflected on the experiences in the lessons and what she saw in her lesson videos. She saw clear evidence about the importance of allowing students time to learn about and become familiar with the technology tools. "The students really enjoyed using the virtual manipulatives and they were motivated to complete the relevant tasks. However, I simply had not factored in enough time to let them explore with the manipulatives so that they could effectively utilize them during the lessons."

Judy indicated she needed to look for more ways to integrate technology: "Technology can aid in their learning and provide opportunities for them to be engaged." However, she noted that adding technologies to the learning required extending the curriculum and the need to "find activities and tasks" that would support students in learning with the technology.

A main objective in Lucy's unit was to "connect mathematics and science with the help of technology." She claimed that a promising practice might be for team teaching with the science teacher, where they would co-teach, and she would have the responsibility for the mathematics. "We plan to team teach using real-world data for mathematics…[making] an even better connection [of] math, science, and technology."

In Sum: Teachers' TPACK Transformations

After designing the online TPACK instructional program, our research effort focused on identifying the influence on the teachers' TPACK. The challenge was to identify how the teachers' thinking was transformed as a result of their engagement in their experiences. The most significant shift in their knowledge and thinking for all the teachers was a shift in their thinking about instruction. The majority of the teachers' primary approach prior to this program involved teacher-centered instruction. Yet, through the program, they shifted to valuing student-centered instructional strategies for teaching with technologies. This shift was reflected in the examination of the TPACK components.

Their overarching conceptions of the purposes for incorporating technology in teaching mathematics expanded, identifying the importance and value of multiple technologies, considering more than just content and thinking about the pedagogical strategies and the technological features. For example, they now considered technologies for communication, collaboration, and inquiry as important technologies for displaying mathematical ideas. In other words, their overarching conceptions shifted to a broader range of technologies for teaching in ways that were more student-centered.

Their reflections about student-centered strategies caused them to think more about student thinking and understanding. They reflected more on students' thinking with the technologies and what strategies were more supportive of their learning of mathematics with the technologies. They reflected on how students needed time to practice and work with the technologies before moving to more in-depth problems. They discussed and reflected on the scaffolding that they needed to do in their lessons as they integrated the technologies. They needed to allow students opportunities to discover and explore ideas with the technologies. Based on their experiences of working in groups and engaging in discourse with their communities of learners, they tested more student-centered strategies as they taught their lessons. After observing their videos, they saw how they needed to be "a guide rather than the sole provider of information."

The expanded vision on technologies in mathematics instruction also influenced their thinking about the curriculum. They shifted to the importance of expanding the curriculum to include teaching about the technology before expecting students to automatically gain understanding of mathematics immediately from working with the technology. During their work with various probeware technologies, they were engaged as students; through these experiences they identified the importance of learning about the technologies within content-specific explorations. As they designed and taught their lessons, they tried to incorporate explorations of mathematical ideas during the time in which their students were learning about the technologies.

In essence, the most obvious transformation in the teachers' TPACK was toward student – centered instructional strategies and the integration of multiple technologies for teaching in their classrooms. As teacher educators consider the design of programs to support the transformation of teachers' TPACK, they need to remember the importance of practical experiences. This work also identified the importance of combining practical experiences with opportunities for the teachers to engage in discourse through the knowledge-building communities of learners and extensive critical reflection on teaching in their classrooms. The combination provides teachers with opportunities to rethink, unlearn and relearn in ways that result in changing, revising and adapting their mathematical content and pedagogical strategies in light of the affordances of the multiple technologies – a TPACK transformation that reveals a deeper understanding of the role of technology and meaningful integration into their mathematics classrooms.

Discussion

From an outside perspective (that of the second author), this paper is a case study of how ambitious and laudatory teacher professional development goals can be achieved in an online or blended environment. The dual challenge it focuses on – of helping teachers learn to use technology effectively, through the medium of online and blended coursework – is also a key challenge of our time. Here I reflect on what general lessons and open issues emerge from thinking about this program as an example of where we should head in the future.

Transforming, Not Only Translating

Studies of successful online and blended learning highlight that when it works, it is not simply a matter of putting existing courses online (Bakia, Means & Murphy, 2004). Here, the dramatic shift is from a set of *courses* to a *knowledge building community.* Clearly, essential content from the courses was important to the firm foundations of the knowledge-building community. A knowledge-building community, however, implies a long-term structure of engagements, activities and supports that is much different from a typical course. For example, the engagements may start much more from teachers' own problems of practice. The activities may involve elements closer to that of lesson study (Hurd & Lewis, 2011) than to case studies in conventional courses. The supports involve more peer reflection and coaching, and less feedback from an authority. The concept of a knowledge-building community is also aspirational and would go beyond the results reported here. This report emphasized progress on individualized problems of practice, but a yet-richer knowledge-building community would also define community-level challenges and community-level knowledge about them.

From Courses to Competencies

The shift in this case also evokes a broader shift latent in professional learning more widely, a shift from counting course credit hours to recognizing new competencies (Nodine, 2016). Clearly, courses provided initial structure to the online experience. And yet, it is not clear that giving course credit is the most appropriate way to give teachers' recognition – course credit is normally tied to how many hours a teacher spent in activity and completion of assignments, but course credit much less frequently recognizes what a teacher now can actually do in a classroom. The focus here is nicely balanced, with clear opportunities to both shape teacher's experience based on course content but also to focus on and recognize classroom practice changes – not just seat time and assignment completion.

An emerging framework that would recognize this shift is the framework of educator micro-credentials (Berry, Airhart & Byrd, 2016). In this framework, complex shifts in teaching practice are broken down into smaller component competencies, and teachers have an opportunity to be recognized as they demonstrate the competencies. Importantly, there is no requirement to spend a fixed number of hours learning the competency, or to follow a set path in doing so. It seems in this example, and in many to follow in the future, online and blended learning will offer teachers lots of choices in how they learn and develop – and teachers clearly are not only showing a commitment in time and assignment completion, they are also reporting what they really do differently in their classrooms. This challenges us to think harder about how we will recognize and reward teachers for their learning, and the micro-credential movement gives one clear alternative (still much in development) to think with.

Coherent and Cumulative Frameworks, Leveraging Personal Experiences

A positive feature of online and blended learning is that the asynchronous and non-linear nature can make learning more relevant and timely given immediate challenges. It is clear these features can help teachers too, but if not balanced might lead to fragmentary knowledge. For example, the specific cases give a good sense of how this environment allowed teachers to effectively pursue personal goals. At the same time, this case also makes clear the value of a framework that helps organizing teacher learning to be coherent and cumulative – not just a reversion to the feels good, low calorie "take away" that old fashioned workshops have been critiqued for. In this case, the framework is TPACK, and the case shows how TPACK can organize the kinds of advances in individual teachers' practice into a more coherent whole. For example, TPACK can help us to see when teachers are making advances on many fronts – in their content knowledge, their technology knowledge, and pedagogical knowledge, and also in the relationships among these. Clearly, we need teachers who are learning on all these fronts.

And yet, TPACK is far from the only high-quality framework available. What TPACK appears best at – at least in this paper, is organizing qualitative observations of what teachers learned into categories. It is less obvious that TPACK is powerful in giving teacher's a sense of coherence and direction in their journeys. For example, if the journey is primarily towards student-centered learning, one might like a broad theoretical frame to organize the why, how, and what of that transformation. What might be the role of a framework like the one from *How People Learn* (National Research Council, 2000), which describes the best learning environments as learner-centered (one of the characteristics here), but also knowledge-centered, assessment-centered, and community-centered? There are also recommendations like the TRU framework (Schoenfeld and the Teaching for Robust Understanding Project, 2016), which is more specific to mathematics, and focuses on content, cognitive demand, equitable participation, agency, and formative assessment – and a framework like this might give coherence and direction to what teachers are learning overall. Overall, TPACK is clearly a valuable guiding framework, but for the field as a whole, it is not clear if focusing on the categorical differences among types of knowledge gives enough of a normative of what great math teaching with technology looks like to direct and make coherent teachers' long-term growth.

Further Challenges of Digital Professional Development Futures

One huge advantage that is apparent in this case is that the presenting problem is that teachers do not have enough prior experience as *digital learners.* And thus, it seems highly appropriate that professional development position the teachers themselves as digital learners. For example, as the case shows, teachers can more directly experience learning with the types of virtual manipulatives, online spreadsheets, and algebraic tools that their students can use. This is valuable because as mathematics is represented in digital form, how conceptual understanding arises shifts – new paths to learning emerge. For example, in the case of Dynamic Geometry, construction becomes more powerful in digital form than with compass and straight edge and a fuller counterpoint to proof in the learning experience (De Villiers, 2004). Social learning is also transformed through digital possibilities, such as the possibility to contribute to share mathematical constructions and experiences (Stroup et al., 2002). Teachers need to experience this first hand and be supported to reflect on their experience as learners and what implications this can have for their teaching, in a process of *Instrumental Genesis* (Drijvers et al., 2010).

Finally, challenges of access and equity arise constant in a digital world, and careful work is needed to understand who participates and how the online experience can produce beneficial or harmful experiences. We cannot pretend naively that online experiences will be safe and positive for all. There are also likely major challenges of data sharing as teachers share their classroom experience not just in a closed course classroom, but also now in an online environment where there is no limit to where the content they share may end up. There are also the positive and potentially negative implications of teachers' online activities producing data. On the positive side, we can potentially track and learn much more about the trajectories of teacher professional growth by studying the trajectories of their online activities (through the data those activities leave behind). Rather than a single university course progression, we can learn in detail about the multiple varied paths that teachers take towards professional knowledge; the different resources they use; the nature and range of social interactions that support them. But on the challenging side, there is always the possibility of such data being exploited in inappropriate ways, and the need to develop guards against this.

Conclusion

As more and more technologies become available as mathematics learning tools, teacher educators will be challenged to identify professional development avenues for supporting teachers in transforming their skills for teaching with the technologies. The online TPACK instructional program provides only one example of an online professional development program

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focused as a knowledge-building community to engage teachers in experiences that influence how their TPACK is transformed as they think about and learn from their experiences while learning as students and as they are implementing their accumulated knowledge in classroom practices. The combination of online learning with practical experiences provides a context within which these experiences were provided. The impact supports them in recognizing and valuing shared knowledge for expanding and enhancing their individual knowledge about learning with technologies. A significant teacher knowledge shift from their personal experiences found a change in their beliefs about their primary instruction to actively engaging students for learning with technologies – toward more cooperative and collaborative inquiry activities where students engage in discourse and reflection. The impact of the collective transformative learning experiences provides an important direction for in-service teacher professional development in the twenty-first century.

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