Preschoolers' Broad Mathematics Experiences with Parents During Play

Keywords: Parent-child interactions; early math; preschool; number; patterns; space

Erica L Zippert, Ph.D. (corresponding author)

Ashli-Ann Douglas

Michele Smith

Bethany Rittle-Johnson, Ph.D.

Vanderbilt University

Zippert, E., Douglas, A., Smith, M., & Rittle-Johnson, B. (2020). Preschoolers' broad mathematics experiences with parents during play. *Journal of Experimental Child Psychology*, *192*. doi: 10.1016/j.jecp.2019.104757

Author Note:

Department of Psychology and Human Development, Vanderbilt University.

Research supported by Heising-Simons Foundation grant #2016-093 to Bethany Rittle-Johnson and Erica Zippert, and by the Institute of Educational Sciences grant # R305A160132 to Bethany Rittle-Johnson. The authors thank Danielle Bice, Katherine Gross, Haley Rushing, Joyce Hwang, Briana Thompson, and Zejian (Grace) Guo for their assistance with data collection and coding as well as the staff, teachers, and children at A. Z. Kelley Elementary School, Hull Jackson Montessori School, Shayne Elementary School, McNeilly Center for Children, Blakemore Children's Center, and Holly Street Daycare for participating in this research.

Address correspondence to Erica Zippert, Department of Psychology and Human Development, 230 Appleton Place, Peabody #552, Vanderbilt University, Nashville TN 37203, USA. Email: Erica.l.zippert@vanderbilt.edu.

Abstract

The current study broadens our understanding of preschoolers' early math experiences with parents, recognizing that math knowledge and experiences are inclusive of numeracy as well as non-numeracy domains. Parents and preschoolers ($N = 45$) were observed exploring three domains of early mathematics knowledge (i.e., number, space, and pattern) during play in three activities (playing cards, building with blocks, and stringing beads, all with activity suggestions). Children were administered a broad math and numeracy measure, and individual measures of spatial and patterning skills concurrently and 7 months later. Dyads explored math broadly across most activities, but emphasized number more than space or patterning. In addition, there was more overall math exploration during card and bead play than block play, with the greatest parent support during card play. Parent support was not linked to children's skills, although children's exploration of space and patterns related moderately to their concurrent spatial and pattern skills. Overall, parents and young children explored a variety of early math domains in guided play contexts, with an emphasis on numeracy. Future work should aim to increase the breadth and rigor of individual concepts that parents and preschoolers explore during play.

Keywords: Parent-child interactions; early math; preschool; number; patterns; space

Preschoolers' Broad Mathematics Experiences with Parents During Play

Children's math knowledge prior to formal schooling is important to their future achievement. Specifically, math knowledge at school entry varies substantially (Starkey et al., 2004) and predicts later math and reading skills (Nguyen et al., 2016). Further, better math knowledge is associated with higher incomes, more prestigious careers, and better healthcare decisions (Lipkus & Peters, 2009; Ritchie & Bates, 2013; Shapka et al., 2006). One source of individual differences in children's math knowledge is their early informal and playful math experiences with parents (Levine et al., 2010; Pruden et al., 2011; Ramani et al., 2015). However, the majority of this research has focused solely on experiences with one specific domain of math (e.g., numeracy), even though math is defined more broadly than numeracy alone (National Council of Teachers of Mathematics, 2006; National Research Council, 2009; Sarama & Clements, 2004). Further, the little work that has taken a broad approach to measuring early home math experiences utilizes parental report. Thus, research has yet to observe early parent-child math experiences broadly or identify conducive contexts for broad math exploration.

The current study had two aims. The first was to better understand whether observations of parent-preschooler joint math experiences are indeed as broad in nature as parents report, and whether support of certain domains is more prevalent than for others. We did this by observing parents and preschoolers during play with common math activities chosen to elicit exploration of multiple math domains (number, pattern, and space). The second aim was to explore whether the frequency of math experiences in each domain was related to children's general math knowledge and specific math skills. To do so, we examined concurrent and predictive relations between observed playful broad math experiences and preschoolers' general math knowledge and number, pattern, and spatial skills.

Math Knowledge and Skills Defined Broadly

Past research and attention have focused on young children's number skills. Basic numeracy includes knowing the number words and their order, counting and labeling quantities of objects, and naming numerals (Gelman & Gallistel, 1978; Le Corre & Carey, 2007; Seo & Ginsburg, 2004; Wynn, 1992). More challenging early number skills include the ability to solve number problems, such as magnitude comparison and simple arithmetic (Ramani et al., 2015; Skwarchuk et al., 2014).

However, broader perspectives highlight that math *includes* but is not *limited to* numeracy (National Research Council, 2009; Sarama & Clements, 2004). First, spatial skills are theorized to be necessary for mathematical reasoning and problem solving (Mix & Cheng, 2012; Verdine et al., 2017). Empirically, young children's spatial skills predict their later math achievement (Gunderson et al., 2012; Verdine et al., 2017). In particular, spatial visualization (transforming mental images of objects) is a particularly sophisticated skill that begins to develop in the preschool years (Frick et al., 2013), and is the spatial skill most consistently and strongly associated with concurrent and later math knowledge (Mix & Cheng, 2012). Visual-spatial working memory skill (mentally holding and transforming a sequence of locations) also develops in preschool and relates to concurrent and later math knowledge (Bull et al., 2008; Mix & Cheng, 2012).

Second, pattern skills are thought to support finding and generalizing rules and regularities that are core to mathematics (National Research Council, 2009; Rittle-Johnson et al., 2015). Indeed, young children's pattern skills predict their later math achievement (Rittle-

Johnson et al., 2015, 2017, 2019). Pattern skills for young children focus on repeating patterns, linear arrays that have repeating units, such as ABBABB (National Association for the Education of Young Children, 2014). Children systematically develop more sophisticated pattern skills, first *creating* and *duplicating* repeating patterns, next *extending* patterns (continue an existing pattern by at least one unit of repeat), then *abstracting patterns*—recreating a model pattern using a different set of materials (Rittle-Johnson et al., 2015; Sarama & Clements, 2009).

For a complete understanding of early math development, we need to consider a variety of early math skills. We chose to focus on preschoolers' repeating pattern and spatial skills in addition to numeracy because much stronger evidence links these skills at this age to later math performance than other math concepts (e.g., measurement and shape; Rittle-Johnson et al., 2017).

Early Informal Math Experiences with Parents

Parents provide some support for early math knowledge, and research has begun to study preschoolers' home math experiences more broadly. Most have focused on only one math domain, so we review evidence for each domain separately.

Number. First, consider parent-child early shared experiences with numbers. Observational studies indicate that preschoolers explore numbers with parents as early as their first year (Durkin et al., 1986; Levine et al., 2010). By the preschool years, parents report counting (rote and object counting), identifying numerals, and labeling cardinal values of sets with children multiple times a week to daily (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009), and observations confirm that talk about these concepts comprises the majority of parent-child number exploration during play (Ramani et al., 2015). Parent-preschooler dyads infrequently explore arithmetic and magnitude comparison during informal interactions, based on observations and survey reports (Ramani et al., 2015; Skwarchuk, 2009; Thompson et al., 2017; Zippert & Ramani, 2017).

Parent-child number exploration likely varies by context, especially during guided, explicitly number-related activities. For example, parents informed of ways to focus on numeracy were observed talking about numbers with their preschoolers more frequently than dyads who did not receive any guidance (Vandermaas-Peeler, Boomgarden, et al., 2012; Vandermaas-Peeler, Ferretti, et al., 2012; Zippert et al., 2016). In addition, the type of activity used during play may influence number exploration. For example, parents and preschoolers engaged in more joint number talk during play with a number board game than a number puzzle or book (Ramani et al., 2015). Numbers likely served a critical role in board game play, but could be easily ignored in the other contexts.

Spatial. Parents and young children also engage in exploration of spatial concepts during play. Observational studies of spatial talk suggest that parents use spatial words frequently with their infants and preschoolers, including talk about locations and directions (e.g., up, down), shapes, dimensions (e.g., big, small), features and properties (e.g., edges, corner), and orientations and transformations (e.g., upside down, flip, rotate; Ho et al., 2017). Survey research also indicates that many parents talk to their 3- to 8-year-old children multiple times a week to daily about spatial topics, and engage with children monthly in spatially relevant activities, such as block and puzzle play (Dearing et al., 2012; Hart et al., 2016; Missall et al., 2015; Verdine et al., 2014).

Context likely matters for parent-child spatial exploration as well. For example, parents and preschoolers who were given detailed information about what they could build or discuss while building talked about spatial concepts (e.g., location words and shape words) more frequently than dyads who did not receive such guidance (Ferrara et al., 2011; Polinsky et al., 2017). Additionally, play with blocks elicits greater spatial talk from parents and their preschooler than non-building block activities (Ferrara et al., 2011). However, more work is needed to determine if specific non-building contexts can promote parent-preschooler spatial exploration.

Pattern. Research has begun to examine children's early home experiences with patterns (Rittle-Johnson et al., 2015; Zippert & Rittle-Johnson, 2018). Parents of preschoolers report monthly to weekly engagement in activities like extending patterns and making and duplicating them (Rittle-Johnson et al., 2015; Zippert & Rittle-Johnson, 2018). Observational studies have not considered parent-preschooler experiences with patterns at home, but suggest that preschoolers spontaneously engage in pattern exploration during free play in their classrooms, suggesting that they may also do so at home with parents (Ginsburg et al., 2003),

Parent survey research has also explored the contexts of children's early pattern experiences with their parents. For example, parents of preschoolers reported watching television shows and reading books which involved patterns weekly to monthly (Rittle-Johnson et al., 2015; Zippert & Rittle-Johnson, 2018). Little is known, however, if some contexts are better than others for promoting parent-preschooler pattern exploration, or if prompting parents and children to engage in pattern-relevant activities increases the quantity or quality of their pattern exploration.

Broad math. Finally, research has begun to study children's early math experiences across multiple math domains, primarily via parent report. Only one survey study comprehensively considered home spatial, pattern, and numeracy experiences. Parents reported home numeracy experiences with their preschooler occurring on a weekly basis, significantly more frequently than spatial and pattern activities, both of which occurred monthly on average (Zippert & Rittle-Johnson, 2018). Other parent-report studies examining two components of early home math experiences also found that parents, preschoolers, and elementary schoolers jointly explored multiple math domains, but reported engaged in number activities more often than spatial or pattern activities (Hart et al., 2016; Huntsinger et al., 2016; Missall et al., 2015; Rittle-Johnson et al., 2015).

Limited observational research also suggests parents and preschoolers engage in number exploration more than other math domains. For example, during a play session, parents and children engaged most often with numeracy, some with geometry and measurement, and even less with algebra (e.g., it is unclear if patterns were included as part of the algebra coding Skwarchuk, 2009). A similar trend was observed in another study of parent-preschooler dyads during play (Missall et al., 2017). While the two studies differed in play contexts, the results suggest that playful parent-preschooler math exploration may incorporate number more than spatial topics; however, which activities elicited math exploration of each type was not reported, nor did either study clearly code for pattern or spatial exploration. Observations of free play in preschools also suggests that this age group engages in more numeracy exploration than spatial exploration, and more spatial than pattern exploration (Seo & Ginsburg, 2004).

More observational research on parent-child broad math exploration is needed for two reasons. First, it must confirm findings from parent reports because parents may have difficulty accurately recognizing or remembering the frequency of support they provide. Indeed, parental reports do not correlate significantly with experimenter observations of parents' math support in past studies (Missall et al., 2017; Mutaf Yildiz et al., 2018). Second, research must identify which contexts support exploration of specific math domains.

Thus, our first research question was: How much do parents and preschoolers explore number, pattern, and spatial concepts within a play session with math-relevant activities, and is there more focus on numeracy than on pattern and space? We hypothesize that parents and children do explore math broadly during play, but focus more on numbers overall (Hart et al., 2016; Missall et al., 2015; Skwarchuk, 2009; Zippert & Rittle-Johnson, 2018). We evaluated this question in the context of parent-child engagement in three playful math-relevant activities: card games, building with Lego© Duplo blocks, and stringing beads. Number games are better for eliciting parent-preschooler number exploration than less number-centric activities (Ramani et al., 2015); block building activities, especially those involving copying a model, elicit more spatial exploration than non-block-building activities (Ferrara et al., 2011); and stringing beads likely encourage pattern creation and comparison. Our goal was to examine what math concepts parents and preschoolers explore when given commonly used materials along with suggestions to engage with the materials in ways that naturally elicit exploration of different math concepts. Families were not told that the goal of the study was to examine their math exploration, and we did not provide suggestions to directly emphasize specific math concepts. Thus, as with other studies with similar methodologies (Casey et al., 2018; Elliott et al., 2017; Lombardi et al., 2017; Ramani et al., 2015), we considered parents' math support and children's math exploration as spontaneous.

Relatedly, we investigated whether parents and preschoolers explored multiple math domains within a single activity rather than just the corresponding one. We hypothesized that within an activity (e.g., number activity), the corresponding domain would be explored most often (e.g., number), but that both non-corresponding domains (e.g., pattern and space) would be explored as well, albeit less frequently.

Relation Between Early Math Experiences and Children's Knowledge and Skills

Interest in early home math experiences is based on the claim that it supports children's academic performance. Although this is broadly true, evidence for a relation between early math experiences with parents and children's math knowledge is mixed, as reviewed below. Further, parents adjust their support based on children's knowledge and skills (Rogoff, 1990; Vygotsky, 1978), so evidence of relations between parents' support and children's academic skills may reflect parent sensitivity to children's skills. Additionally, theorists suggest that children apply their math skills during their everyday experiences (Sarama & Clements, 2009); thus, we might expect links between children's math exploration and their math knowledge and skills. Below, we review research on links between the frequency of broad early math experiences and children's general math knowledge and specific math skills.

Number. First consider research on relations between parent-child early number experiences and children's math knowledge and skills. Some research supports a positive relation. In at least three studies, parental reports of number experiences with their preschooler was related to children's concurrent number skills (Blevins-Knabe & Musun-Miller, 1996; Skwarchuk, 2009; Zippert & Ramani, 2017). Observational research examining both parent number support and child number exploration in this age group mirrors parental report findings concurrently (Ramani et al., 2015) and longitudinally (Susperreguy & Davis-Kean, 2016). In contrast, some studies have found no relation between parent-reported numeracy support and preschool children's numeracy skills concurrently (Blevins-Knabe et al., 2000) or longitudinally (Zippert & Rittle-Johnson, 2018). Very little research has considered links to math knowledge beyond numeracy skills, but one study found null relations between parent-reported numeracy support and preschoolers' concurrent broad math knowledge, pattern or spatial skills, and later broad math knowledge (Zippert & Rittle-Johnson, 2018). Studies using parent reports used varied parent survey measures and varied child outcome measures, making it difficult to identify the source of inconsistent relations, although low power may explain the inconsistencies. More often, existing observational studies of free play with math-relevant activities or in-home contexts have reported positive relations between parents' number support, children's number exploration, and children's number skills (Casey et al., 2018; Elliott et al., 2017; Levine et al., 2010; Ramani et al., 2015; Susperreguy & Davis-Kean, 2016).

Spatial. Emerging research suggests that the frequency of early spatial experiences is positively linked to spatial skills and math knowledge in some studies but not others. Frequency of observed parent spatial talk from 14 to 46 months during everyday interactions predicted children's spatial skills in preschool (spatial visualization and analogies), in part because it predicted more spatial talk by their children (Pruden et al., 2011). A large-scale parent report study found that children aged 3- to 8 who played with blocks, puzzles, and board games "often" had higher concurrent spatial visualization scores than children who only played with these activities "sometimes", though the effect size was small (Jirout & Newcombe, 2015). However, in a smaller study, parental reports of building with construction toys and playing with puzzles with their first grade girls was not associated with children's concurrent spatial skills (Dearing et al., 2012). Another parent survey study also found non-significant relations between home spatial activities and children's concurrent spatial skills (Zippert & Rittle-Johnson, 2018).

One study also suggested that early spatial experiences may contribute to broad math knowledge. Specifically, the quality of parents' spatial support observed during a play session (dyads were given Duplo blocks and a 2-D diagram of a structure that could be built with the blocks) when their child was 36-months-old predicted children's broad math knowledge at age

4½ and first grade (Lombardi et al., 2017). However, a parent survey study failed to find links between home spatial activities and children's concurrent and later math knowledge (Zippert & Rittle-Johnson, 2018). To our knowledge, no research links observations of children's spatial experiences in the final preschool year to beginning- and end-of-the-year spatial skills or math knowledge.

Pattern. Even less research has examined the links between early parent-child pattern experiences and children's skills. Rittle-Johnson et al. (2015) found that the frequency of parent-reported pattern experiences with their child at the end of preschool was moderately but marginally related to their child's concurrent pattern skills. However, a larger parent-report study failed to replicate these results with children earlier in preschool (Zippert & Rittle-Johnson, 2018). No research to date has examined the links between observed parent-child pattern experiences and children's concurrent and later math knowledge and pattern skills.

Broad math. Finally, a few studies have explored the relations between early parent-child broad home math experiences and children's broad math knowledge, with mixed findings. Parental reports of engagement in general math and numeracy activities, block play, and making patterns during art activities was related to preschoolers' concurrent broad math knowledge (DeFlorio & Beliakoff, 2015). Similar results were found with elementary schoolers (Huntsinger et al., 2016). In contrast, parent-reported broad home math experiences (numeracy, geometry, pattern activities) were not related to preschoolers' concurrent numeracy knowledge (Missall et al., 2015; Skwarchuk, 2009). Observational research has not considered whether broad math support is related to children's math knowledge.

Summary. There is some evidence that the early home math environment relates to children's broad early math knowledge. Most evidence shows a relation within the same component (e.g., numeracy support and numeracy skill), although a few studies have found no relations. There is some evidence that the frequency of observed spatial support is related to broad math knowledge and some evidence that the broad home math environment is related to concurrent broad math skills (DeFlorio & Beliakoff, 2015; Huntsinger et al., 2016). There seems to be more consistent links between observed, rather than parent-reported, early number and spatial experiences and children's knowledge and skills, and for concurrent, rather than longitudinal, relations, but the research literature is quite limited.

Thus, our second research question was: Are observed parent-child math experiences related to preschoolers' broad math knowledge and number, pattern, and spatial skills? We hypothesized that observed early math experiences during play with math-relevant activities would be positively related to concurrent and end-of-preschool broad math knowledge, and that individual types of the math exploration would correlate with corresponding skills.

Methods

Participants

Forty-eight preschoolers and a parent participating in a larger study (Rittle-Johnson et al., 2019) on early mathematics development were recruited from six preschool programs (two private, three public, and one Head Start center) in a Southeastern U.S. state. Families were recruited from the larger study by indicating on the consent form their interest in being contacted about participating in the separate play session, and 62% of the full sample were successfully recruited. Three parent-child dyads were dropped from the study because their play sessions could not be coded due to technical issues (n=2) or because a language other than English was used in the session (n=1). Thus, the final sample consisted of 45 parent-child dyads. Most of the parents in the final sample (82%) also reported their math-related beliefs and parent-child engagement in math-related activities via a survey earlier in the Fall of the school year, as reported separately (Zippert & Rittle-Johnson, 2018), although in a small number of cases, the parent who filled out the survey was not the same parent who participated in the play session. Children were 4.57 years old on average (*SD*=.30) and were almost evenly divided by gender (58% male). Over half of children received at least some financial assistance for attending preschool (58%), and about half were racial or ethnic minorities (51%). Parents were mostly mothers (84%) and about half of them identified as racial or ethnic minorities (47%). A majority of the parents had at least an Associate's degree or some college education (95%). Additional sample demographic information is reported in Table S1.

Procedure

Play session. The play sessions were run by an experimenter in a quiet room at each child's preschool during drop-off or pick-up time during the Fall of the school year. The activity was modeled after the Three Bags Task (Vandell, 1979). Dyads were presented with three randomly ordered bags containing separate math-related activities and suggestion cards. The dyads were told that they could interact with the contents of each bag as they would at home, but that they had to open the bags in the order in which they were numbered. Given that providing suggestions can elicit more parent-child math exploration than contexts in which no suggestions are provided (Ferrara et al., 2011; Vandermaas-Peeler, Ferretti, et al., 2012), each bag included a card with optional ways to engage in each activity.

Each activity was selected to encourage parents and their children to explore number, pattern, and spatial concepts through play (see Table S2). The activity chosen to elicit number exploration was a modified deck of cards that included numerals one through ten, and corresponding numbers of dots. Our number suggestion card indicated that dyads could play the card games *War* or *Order Up*, which both involve thinking about numerical magnitudes. As shown in Table S3, many dyads played War or Order Up, and some dyads played other games such as Go Fish, Matching, or engaged in free play. Card games specifically are a common and familiar activity played on a weekly basis by families with young children (Ramani et al., 2015; Skwarchuk et al., 2014). Further, parents in the current study reported that they compared quantities with their child in contexts such as playing card games or serving food for dinner one- to two-times per week. Most parents knew the card game *War*, although they may have been playing the card game for the first time with their child. The pattern task included two sets of beads of different colors and shapes, and two laces. Suggestions included copying the sample patterns provided or making similar patterns using beads of different colors and shapes. Many dyads created at least one pattern, but they usually created their own patterns rather than the suggested patterns, and very rarely tried to abstract a pattern by making the same type of pattern with new materials. Free play was more common with the beads than the other materials. Past research has suggested that families report a median frequency of once a week of copying patterns, talking about what comes next in a pattern, and families in our current sample reported making or copying patterns with objects or sounds such as blocks about once a month. The spatial activity included a set of Lego© Duplo blocks and pictures with suggested structures typically included on containers. This activity was chosen because playing with construction toys is common in families of preschoolers in past research (Ramani et al., 2015), and families in the current study reported engaging in building with construction toys such as building blocks and Legos one- to two-times per week. All of the dyads created a structure, and a majority created at least one of the suggested structures. The experimenter left the room or sat quietly out of sight after providing the instructions and returned every 5 minutes to remind each dyad that they could move on to the next activity. An entire interaction lasted 22.90 minutes on average (*SD* = 5.01). Dyads played with the cards, blocks, and beads for 7.38 (*SD* = 2.88), 8.09 (*SD* = 2.14), and 7.68 (*SD* = 2.57) minutes on average, respectively. Parents were given a $25 gift card to thank them for participating.

Children's knowledge and skills. Children's general math knowledge and numeracy, pattern, and spatial skills were assessed during the Fall (Time 1) of the school year, usually before the play session, and were reassessed in the Spring (Time 2). Measures are described below. Children completed two additional assessments that are beyond the scope of the current investigation in the larger study. Specifically, children's verbal working memory was assessed using the Backward Digit Span and their verbal ability was assessed using the Picture Vocabulary Test from version 1.6 of the *NIH Toolbox app*. Results of all measures are reported separately (Rittle-Johnson et al., 2019).

Broad math knowledge and number skills. We assessed preschooler's broad math knowledge using the Research-Based Early Mathematics Assessment Short-Form (REMA Short-form; Weiland et al., 2012) at both time points. The REMA Short-form assessed children's numeracy skills with 13 items that tested children's rote and object counting, ability to match small numerals to corresponding set sizes, subitizing, number comparison, and nonverbal arithmetic skills. A second section of the measure assessed children's geometric knowledge (six items including shape identification, shape creation, and mental rotation) and pattern skill (one item). The internal consistency in our sample was above .7 for general math and numeracy at both timepoints, similar to past work (Weiland et al., 2012). However, the geometric section's internal consistency was poor (.56 at T1 and .39 at T2). We generated separate IRT ability estimates for the entire math measure and for the numeracy items at both time points by using a partial credit model. We did Empirical Bayes estimation using WinBUGS 1.4.3 to constrain the item parameters (Baker & Kim, 2004; Spiegelhalter et al., 2003).

Pattern skills. Children's pattern skills were assessed using two measures. One of the measures was taken from previous studies (Miller et al., 2016; Rittle-Johnson et al., 2013, 2015) and was only administered at Time 1. It assessed preschoolers' ability to duplicate, extend, abstract, and identify units of repeating visual patterns using nine items. The measure's internal consistency in our sample was good (Cronbach's α= .84). We developed the second pattern measure by using pattern worksheets found on websites for early-childhood educators (Rittle-Johnson et al., 2019; Zippert et al., 2018). The internal consistency for the second pattern measure was also good (Cronbach's α = .83). We estimated children's ability using a Rasch model with a Laplace approximation (Cho & Rabe-Hesketh, 2011) for both measures.

Spatial skills. We administered three assessments of children's spatial skills at Time 1 and re-administered two of the measures at Time 2. To measure form perception, we used The Position in Space subtest of the Developmental Test of Visual Perception–Second Edition (Hammill et al., 1993) at Time 1. The assessment required children to choose the figure that matched a target image from a set of four or more figures. We assessed children's spatial visualization using the Block Design subtest of the Wechsler Preschool and Primary Scale of Intelligence–Fourth Edition (Wechsler, 1991) at both time points. Children used red and white colored blocks to create pictured model block structures. In addition, we assessed children's visual-spatial working memory using an adapted version of the Corsi Block Tapping Task at both time points on an Apple® iPad (available at https://hume.ca/ix/pathspan.html). The assessment required children to touch lily pads in the same or reverse order that a frog jumped.

While both orders were administered at Time 1, only the forward order was administered at Time 2 due to time restraints.

Coding of Play Sessions

Three coding schemes were adapted from previous research to measure parents' and children's number (Ramani et al., 2015), pattern (Rittle-Johnson et al., 2013), and spatial (Cannon et al., 2007) exploration. Each coding scheme was separately applied mutually exclusively and hierarchically within each code type in accordance with the order of the codes in Tables 1-3. Specifically, at each 10-second interval, each type of code was applied for the most advanced number, pattern, and spatial concept explored by the parent and by the child, with coding of parents and children done separately. Thus, each interval for parent and child could receive up to three codes: a number, spatial, and pattern code. To account for variability in the amount of time parents and children spent interacting and language use and engagement, code frequencies were divided by total number of intervals for the entire interaction or for each activity, as appropriate.

The hierarchy of codes was based on past theory and research on the difficulty of different tasks for young children and the order in which they typically develop. The pattern hierarchy was based on a validated construct map (e.g., identifying the pattern unit is more difficult than pattern extension; Rittle-Johnson et al., 2015; Sarama & Clements, 2004) as well as research on increasingly sophisticated ways to communicate patterns using gestures and verbal labels (Fyfe et al., 2015; Rittle-Johnson et al., 2008). The number hierarchy was based on research on the difficulty of different number tasks (e.g., object counting develops before labeling the specific cardinal value of a set (e.g., object counting develops before labeling the specific cardinal value of a set; Seo & Ginsburg, 2004; Wynn, 1990) as well as research on the degrees of complexity for preschool children's exploration of quantity and magnitude. Specifically, approximately describing the magnitude of a single set is considered less advanced than comparing quantities relatively. Also, exploring which set of two objects is relatively more is thought to be less complex than doing so precisely through arithmetic or other advanced operations (Seo & Ginsburg, 2004). The spatial hierarchy was based on past research suggesting that mentally imagining the rotation of object orientations in space is a more advanced skill for preschoolers, given that they do not perform above chance on these tasks until the latter half of their fourth year (Levine et al., 1999). In contrast, visual spatial working memory skills, requiring attention to specific locations and directions, are likely easier because they develop earlier in the fourth year (Bull et al., 2008; LeFevre et al., 2010).

Reliability. Videos were coded by 5 students and twenty percent of the videos were double coded. Kappa coefficients were calculated for each pair of coders, with good reliability for parents' number (.87-.93), pattern (.87-1.00), and spatial (.79-.93) support and children's number (.80-.93), pattern (.90-1.00), and spatial (.80-.88) exploration.

Results

Math Exploration

Parent math support. First, we describe parents' math support during the entire interaction (see descriptive statistics for individual codes in Tables 1-3 and domain types in Table 4). For parents' number-specific math support, which constituted a third of the total interaction, over two-thirds of this support involved comparing magnitudes, identifying numerals, and labeling cardinal values during the play session. Additionally, parents made general statements about quantity (e.g. used statements like "a lot") in 15% of their numeracy support. Of parents' spatial support, which accounted for another 20% of total intervals, the majority of this support was related to naming locations and directions of objects in space (60%). Parents also provided input on spatial dimensions (i.e. the size of objects, people, and spaces) and the orientations of objects in 20% of their spatial input. Finally, parents' pattern support, which comprised 11% of the total interaction, primarily included labeling patterns and talking about which item should come next in a pattern (55% of pattern support). Parents also labelled the items in patterns and created patterns (36% of pattern support).

Next, in line with our first aim, we compared the frequencies of each type of math exploration (number, pattern, and space) for the entire play session and for each activity. We conducted a repeated-measures ANOVA with domain type and activity type as within-subject variables. There was a main effect of math domain type, $F(2, 43) = 122.84$, $p < .001$, $\eta_p^2 = .74$. Post hoc pairwise comparisons with a Bonferroni correction indicated that, as hypothesized, parents supported numeracy significantly more often than space and patterns, and provided spatial support significantly more frequently than pattern support (see Figure 1a, Table 4).

The repeated measures ANOVA also revealed a significant main effect of activity, $F(2, 43) = 33.46$, $p < .001$, $\eta_p^2 = .43$. Post hoc comparisons with a Bonferroni correction indicated that parents provided more overall math support during card play than block or bead play, and more overall math support during bead play than block play (see Table 4).

There was also a significant interaction between activity and domain of math support $F(4, 41) = 160.65$, $p < .001$, $\eta_p^2 = .78$. Univariate analyses indicated a significant main effect of activity on the frequency of each type of parent math support $F_{Numeracy}(2, 43) = 215.27$, $p < .001$, $\eta_p^2 = .83$, $F_{Spatial}(2, 43) = 11.75$, $p < .001$, $\eta_p^2 = .21$, $F_{Pattern}(2, 43) = 136.89$, $p < .001$, $\eta_p^2 = .76$. Results of post hoc comparisons suggested that each activity primarily elicited its intended type of math support (see Figure 1b, Table 4). Specifically, numeracy support was greatest during the card game, spatial support was greatest during block play, and pattern support was greatest during bead play. However, parents still provided some support for space during card play, number during block play, and both number and space during bead play (15-18% each).

Correlational analyses were conducted to further explore relations among parents' broad math support (the summed proportions of their number, pattern, and spatial support out of total 10-second intervals in which dyads engaged in each activity) across the three activities. Parents' broad math support during bead play was related to their broad math support during play with blocks, $r(45) = .31$, $p = .038$, and cards, $r(45) = .31$, $p = .039$. However, their broad math support during block and card play were not significantly related, $r(45) = .23$, $p = .121$.

Child math exploration. Children's numeracy exploration comprised 23% of total intervals and focused primarily on identifying numerals and comparing magnitudes (54%). Children's spatial exploration, comprising 11% of total intervals, focused mostly on locations and directions (66% of spatial exploration). Additionally, children examined spatial dimensions in 15% of their spatial exploration. Lastly, children's pattern exploration, which comprised 12% of total intervals, mainly involved creating patterns and discussing which item should come next in a pattern (73% of pattern exploration). Children also labelled the items in patterns and labeled object sequences as patterns during 12% of their pattern exploration.

A parallel repeated measures ANOVA was conducted for child math exploration. There was a main effect of math domain type, $F(2, 43) = 45.22$, $p < .001$, $\eta_p^2 = .51$. Post hoc pairwise comparisons with a Bonferroni correction indicated that, as hypothesized, children explored number more often than space (see Figure 1a, Table 4).

There was also a significant main effect of activity, $F(2, 43) = 49.72$, $p < .001$, $\eta_p^2 = .53$. Post hoc comparisons with a Bonferroni correction indicated that children explored math more often during card and bead play than block play (see Table 4).

There was also a significant interaction between activity and domain of math exploration, $F(4, 41) = 180.32$, $p < .001$, $\eta_p^2 = .80$. Univariate analyses indicated a main effect of activity for child exploration of each domain $F_{\text{Numeracy}}(2, 43) = 168.20$, $p < .001$, $\eta_p^2 = .79$, $F_{\text{Spatial}}(2, 43) = 15.26$, $p < .001$, $\eta_p^2 = .26$, $F_{\text{Pattern}}(2, 43) = 212.84$, $p < .001$, $\eta_p^2 = .83$. As with parents, results of post hoc comparisons with Bonferroni corrections suggested that each activity primarily elicited exploration of its intended domain (see Figure 1c, Table 4). Also, as with parents, children explored some space during card play, some number during block play, and both number and space during bead play (7-17% each).

Correlations were also conducted to explore relations among children's broad math exploration (the summed proportions of their number, pattern, and spatial exploration out of total 10-second intervals in which dyads engaged in each activity) across the three activities. Children's broad math exploration during bead play was significantly related to this exploration during card play, $r(45) = .44$, $p = .002$, but not with their exploration during block play, $r(45) = .20$, $p = .183$. However, their broad math exploration during block and card play were related, $r(45) = .39$, $p = .008$.

In summary, as hypothesized, parents and children explored math broadly during play but focused most on number. Further, there was more overall math exploration during card and bead play than block play, with the greatest parent support during card play. Finally, each activity primarily elicited parent-child exploration of its corresponding math domain, though multiple domains were emphasized during each activity.

Linking Math Exploration with Child Knowledge and Skills

To address our second research question, partial correlations were conducted between parent-child math exploration and children's math knowledge and skills concurrently and at the end of preschool, controlling for age at test time (see Table 5). A power analysis was conducted to assess whether we had sufficient power to detect a two-tailed effect of .4 at the $p < .05$ level. The obtained power (.79) suggested that we had sufficient power to detect significant relations using effect size estimates from past research (e.g., Ramani et al., 2015) with our final sample of 45 parent-child dyads. Due to the large number of correlations, we also note whether correlations were significant at the $p < .01$ level. Contrary to our hypothesis, parents' observed number, pattern, and spatial support were not significantly related to children's corresponding skills or broad math knowledge at either time point, *r*s = -.24 to .15. Likewise, parents' broad math support (summed proportions of their numeracy, patterning, and spatial support) was not significantly related to children's number, pattern, spatial, or broad math knowledge, *r*s = -.17 to .00.

Similarly, children's broad math exploration (summed proportions of their numeracy, patterning, and spatial exploration) was not significantly related to their number, pattern, spatial, or broad math knowledge, *r*s = -.18 to .19. Likewise, children's observed number, pattern, and spatial exploration were not significantly correlated with their corresponding skills or their broad math skills at either time point, *r*s = -.36 to .28, with one exception. Specifically, children's spatial exploration was negatively related to their concurrent visual-spatial working memory, $r(41) = -.36$, $p = .019$. Additionally, a few of the correlations suggested medium effect sizes (i.e. around .3) worth consideration in future research. First, children's pattern exploration was moderately and positively correlated with one of two concurrent measures of their pattern skill,

$r(41) = .28$, $p = .067$. Second, children's spatial exploration was negatively related to their concurrent spatial visualization skills, $r(41) = -.29$, $p = .056$.

To further explore relations between parents' and children's math exploration and children's math skills, we created summed proportions of parents' and children's math exploration of specific concepts based on their complexity and examined relations to children's broad math knowledge and corresponding skills, controlling for child age. However, very few of the correlations were significant, and the few that were significant were negative relations between the complexity of children's spatial exploration and children's skills, and they were difficult to interpret (see Table S4 for correlations and S5 for a written description of the findings).

Discussion

The current study broadens our understanding of children's early math experiences with parents, recognizing that they encompass more than numeracy alone. Indeed, parents and preschoolers explored number, pattern, and spatial concepts during play, and number exploration was most frequent. Activity type influenced parent support and child exploration of math concepts, and a given activity often elicited exploration of more than one math domain. Overall, little evidence was found for the links between broad early math experiences and children's broad math knowledge and skills. Parent math support during guided play with math-relevant activities was not related to children's knowledge and skills concurrently or predictively, though moderate effects were found between child exploration and a few concurrent skills. We discuss parent-preschooler broad math experiences during guided play and how they relate to children's broad math knowledge and corresponding skills. Throughout, we discuss suggestions for future research and practical recommendations for supporting preschoolers' broad math experiences.

Parents and Preschoolers Explore Math Broadly

An important contribution of the current study was that it explored whether parents and preschoolers can be observed exploring math broadly during a single play session. Some work has begun to acknowledge the broad nature of children's early math experiences by surveying parents about the frequency of both number- and space-related home activities (e.g., Dearing et al., 2012; Hart et al., 2016), with only one study comprehensively examining parent reports of number-, space-, and patterning-related home activities (Zippert & Rittle-Johnson, 2018). However, past observational approaches have only examined parent-child math exploration of a single domain at a time, typically with activities chosen to elicit exploration of a specific math domain (Ramani et al., 2015; Vandermaas-Peeler, Ferretti, et al., 2012) with few exceptions (Missall et al., 2017; Skwarchuk, 2009). Further, our study was the first to observe how parents and preschoolers explore patterns specifically during play, as past work had only examined parent patterning support via parent report (Zippert & Rittle-Johnson, 2018). The lack of a correlation between observed and parent-reported early home math experiences in past studies (Missall et al., 2017; Mutaf Yildiz et al., 2018) suggests that these two methodological approaches likely tap into different aspects of the home math environment, and thus are both important for capturing children's early math experiences.

We found that children's early math experiences can be broad in nature. In addition to number exploration, such as naming written numerals and labeling groups of objects, parents and preschoolers also explored locations and directions of objects as well as their dimensions, orientations, shapes, and spatial features. In addition to their focus on these spatial concepts, parents and children also engaged in pattern creation, predicting what comes next in their pattern, discussing patterns more broadly, as well as labeling the individual items within a given pattern.

We were able to capture a much wider range of parent-child math exploration in our study than in past work because of our broad approach, and this highlights the opportunities for exploration of important math concepts that informal playful parent-child interactions can provide.

While we found evidence in our observations of broad math emphasis during parent-child play, there was significantly more emphasis on number than the other math concepts we examined (i.e., space or patterning). Past parent report research has also suggested that there is a greater emphasis on numbers than space or patterns in homes with young children (Hart et al., 2016; Zippert & Rittle-Johnson, 2018). It is not surprising that parents focus on numbers more than other math domains. Early childhood math content recommendations currently place more emphasis on number than pattern or spatial skills (*Common Core State Standards*, 2010; National Council of Teachers of Mathematics, 2006), and some research has even found that early number skills are stronger predictors of later math achievement than non-numeracy math skills (Nguyen et al., 2016; Rittle-Johnson et al., 2017). With parents' limited time and resources, they may be putting more emphasis on supporting skills that seem especially important. In contrast, parents may not even realize that supporting pattern or spatial skills might support math knowledge.

We also found that parents were observed emphasizing space more often than patterning during play with their preschoolers, which contrasted with prior work suggesting that early spatial and patterning experiences occur at a similarly infrequent rate (Zippert & Rittle-Johnson, 2018). This could be due to a potential tendency for parents to underestimate the frequency of their spatial support, especially as it occurs unintentionally, such as during non-math-explicit conversations with their children. Indeed, well over half of parent and child spatial emphasis involved exploration of locations and directions of objects. Researchers have suggested that talk of this kind, also called *where words*, though still important for children's spatial skill development, is strongly correlated with and frequent during non-spatial conversations (Pruden et al., 2011; Pruden & Levine, 2017). Thus, parents might underreport how frequently they are providing spatial support overall if they are focusing only on their explicit attempts to support children's spatial skills. Thus, it was especially important that we compared observations of parent support of both space and patterning.

Context Matters for Broad Math Exploration

Additionally, in line with our first study aim, we examined how different activities and suggestions were associated with parent and child broad math exploration. Overall, cards and beads and our activity suggestions elicited more broad math exploration and support than blocks, and for parents, the cards elicited more overall math support than beads. Below, we detail how each activity may have encouraged parent-child exploration of different math concepts.

Considering cards first, this activity and its suggestions elicited parent and child exploration of mostly number. Past work has suggested number games as effective contexts for promoting number exploration than number puzzles and books, likely due to the centrality of number problem-solving to the game specifically (Ramani et al., 2015). Our suggested activity of playing number-related card games may have been especially important for parent-child numeracy exploration. As was the case in past studies, parents and children tended to explore numeral names, labeling cardinal values, and counting objects most often during their play in comparison to other numeracy topics (Levine et al., 2010; Ramani et al., 2015). In addition, parents and children spent almost a quarter of their numeracy-focused play time comparing magnitudes. Past studies have found that parents and children spent little to no time focusing on magnitude comparison during their play and regular home routines (Levine et al., 2010; Ramani et al., 2015). Our suggestion to play *War* may have encouraged magnitude comparison exploration because the game requires comparing the quantities on the cards. *War* may be an especially conducive card game for promoting parent-preschooler exploration of numerical magnitudes (Douglas et al., 2019). Further, playing *War* with an experimenter promoted preschoolers' number skills, especially symbolic magnitude comparison (Scalise et al., 2018). It is encouraging that card game play improved magnitude comparison skill and talk, given that it is considered important in mathematics development (Siegler & Lortie-Forgues, 2014). We thus suggest card games, especially magnitude-based card games like *War*, as conducive contexts for exploring a range of number topics, and the potential for such experiences to promote child number skill.

The block activity, and our suggestion to build different pictured objects, promoted more spatial exploration than other activities, in line with past research (Ferrara et al., 2011). Our study further specified blocks as a more conducive context than playing cards and stringing beads. However, spatial exploration in our study was still fairly infrequent and low level due to little parent-child emphasis of spatial concepts beyond locations and directions. Exploring spatial features and properties is also important for later spatial development (Pruden et al., 2011), so parents and preschoolers may need suggestions that more explicitly emphasize how and why to do so, as recent research has done (Borriello & Liben, 2018). Adding more shape variants (i.e., unusual in addition to canonical versions) to standard block sets may be one cost-effective way to promote parent-child exploration of spatial features and properties, as suggested by recent work (Verdine et al., 2019). Additionally, puzzle play may promote exploration of an even wider array of spatial concepts (Levine et al., 2012), given that individual puzzle pieces can differ in their spatial features with others in a set.

Finally, the bead stringing context elicited the most pattern exploration. The current study made an important first attempt to observe parent-child patterning during play in one specific context. Dyads focused mostly on predicting what came next in a pattern, completing patterns without discussion, labeling individual pattern items, or identifying patterns. This is in line with past work suggesting that parents focus more on making, copying, and extending patterns than more sophisticated activities such as abstracting patterns (Zippert & Rittle-Johnson, 2018). This is important because preschoolers can learn to engage in more challenging pattern tasks like pattern abstraction over time and with adult guidance (Fyfe et al., 2015; Rittle-Johnson et al., 2015). Parents were given suggestions for pattern abstraction, but very few chose to do so, perhaps because they did not consider it appropriate to challenge their preschooler in this way. Free play, rather than guided play, was also the most common during the bead activity compared to the other two activities. As in the domain of spatial thinking (Borriello & Liben, 2018), future work could expose parents to information on the development of patterning in preschool to diversify parents' pattern support.

We also found that different contexts promoted exploration of multiple domains. In addition to pattern exploration, bead play encouraged a small amount of exploration of both number and space. Spatial exploration included focusing on where in the sequence a bead should be placed or describing the shapes of the individual beads. Number exploration took place when copying pattern models (e.g., focusing on the number of beads of each color and shape, and how many times the sequence repeated). Similarly, the block building activity encouraged some consideration of numeracy when parents and children assessed how many blocks of each color were needed to duplicate the object in the picture. Finally, playing with the cards encouraged some spatial exploration, such as the spatial orientations of the cards themselves as well as the contents on the card faces (e.g., positioning of the numerals and the spatial arrangement of the non-symbolic quantities). Thus, exploration of a range of math concepts is possible across activities, though this exploration could be further broadened.

Links Between Math Experiences and Children's Knowledge and Skills

Contrary to our hypothesis, we did not find significant links between parent support or child math exploration and children's broad math knowledge and numeracy, patterning, and spatial skills. First consider links with parent support. In past work, associations were not found between parent-reported spatial and pattern support and children's corresponding concurrent skills, although home number support was concurrently related to a composite of preschooler's number skills (Zippert & Rittle-Johnson, 2018). Additionally, observations of parent number support (measured as parent number talk) during free play with number-explicit toys was concurrently related to preschoolers' individual number skills (Ramani et al., 2015). In the current study, by intentionally providing activities and suggestions ranging in their emphasis of number, we likely reduced individual differences in parent support, mirroring past findings with kindergarteners (Elliott et al., 2017). Findings overall suggest that most parents are able to provide broad math support with supportive materials and activity suggestions. Individual differences in parent-child math exploration without explicitly domain-relevant toys and suggestions may be more related to children's skills.

Second, consider links with child exploration. A few of the study's correlations suggested some medium effects for within-domain child exploration and concurrent skills for patterning and space; however, the patterns of associations found could have been spurious due to the number of tests conducted, so further replication is needed to confirm these findings. First consider the link to patterning. Our study was the first to show positive concurrent relations between child pattern exploration and pattern skills. This extends previous work in a smaller sample showing that parent-reported home-based parent-child patterning experiences positively predicts preschoolers' later patterning skills (Rittle-Johnson et al., 2015), but contrasts with a more recent parent-report study with a larger sample that failed to find these relations (Zippert & Rittle-Johnson, 2018). This provides preliminarily encouraging data on the importance of early patterning experiences for preschoolers' patterning development.

Additionally, we found moderately negative relations between children's overall spatial exploration and two concurrent spatial skills of spatial visualization and visual-spatial working memory. In past research, home observations of early child spatial exploration (measured as cumulative child spatial talk during everyday activities) significantly and positively predicted children's spatial skills at 4.5 years (Pruden et al., 2011). The latter study differed from ours in that it examined child spatial experiences earlier in development, in the home across contexts, and over multiple observations (from 14 to 46 months). Further, it excluded analysis of child exploration of locations and directions, and did not include a measure of children's visual-spatial working memory. We suggest that children's exploration of certain spatial concepts at earlier points in development, in everyday contexts, and perhaps over a longer period of time, may positively contribute to their later spatial development, while emphasis of certain spatial concepts later in development in highly spatially relevant contexts may indicate a weaker foundation in children's spatial skill at that time and potentially in the future. While prior work on parents' spatial support has previously shown positive relations to children's skills (e.g., Pruden et al., 2011), researchers have found negative correlations between parent-reported support of certain number topics and concurrent child number skill (Blevins-Knabe & Musun-Miller, 1996). The authors suggested that foundational number concepts are relied upon more when children have lower numerical knowledge, and subsequently less when this knowledge improves. For example, parent-preschooler dyads explicitly explored the connection between symbolic and non-symbolic quantities during card game play more when children had less advanced number skill, likely because children could not yet read the written numerals to determine set sizes, and thus needed to count the objects (E. L. Zippert et al., 2016). This trend is also observed in children's reliance on counting as a strategy in arithmetic problems (Briars & Siegler, 1984). Our study extends these findings to the domain of spatial thinking. Specifically, explicit reliance on and discussion of basic spatial concepts, such as locations and directions, might indicate that children are struggling to mentally manipulate and remember objects in space. Our study was the first to explore the relation between observations of preschool children's spatial exploration and their visual-spatial working memory; thus, more work is needed to replicate this finding.

Limitations

A few methodological limitations should be considered. First, our sample was limited in socioeconomic diversity and size, though representative of other similar studies in this area of research. Thus, these findings may not generalize to families with lower parent education or to other cultural contexts. Our parent-child dyads were also limited to the toys we provided for them, and their behavior was shaped by those toys and our specific suggestions to encourage exploration of specific math domains. This is an important consideration given that parents and preschoolers may not have explored any of our domains given different materials or without specific suggestions. Further, some materials and activities may have been unfamiliar to families, although parents do report engaging in all three domains at home (Zippert & Rittle-Johnson, 2018). It is also important to recognize that our suggestions and activities may have reduced individual differences in parent-child broad math exploration as they relate to child skill. Thus, future work should investigate how parents and preschoolers explore math broadly in other contexts, such as free play during everyday routines (e.g., cooking), and how this exploration relates to child skill in both contexts.

Conclusions

While previous theory and research implied that children's math knowledge was primarily number-based, more recent empirical work considers math more broadly, including pattern and spatial skills as well (Nguyen et al., 2016; Rittle-Johnson et al., 2019; Sarama & Clements, 2004; Verdine et al., 2017). Accordingly, conceptions of home math experiences have increasingly begun to broaden to include activities beyond numeracy (Hart et al., 2016; Huntsinger et al., 2016; Missall et al., 2017; Zippert & Rittle-Johnson, 2018). This study was the first to observe and compare how parents and preschoolers explore math more broadly, including exploration of number, patterns, and space. Results suggest that parents and children do explore math broadly, even during activities thought to support one domain of learning. The beading activity encouraged the broadest range of math exploration, encouraging parent-child emphasis on numbers, patterns, and space. Further, card play was especially conducive to more advanced number exploration (e.g., magnitude comparison). Parent broad math support did not correlate with preschooler's broad math knowledge and skills, but moderate effects were found for child spatial and pattern exploration and concurrent spatial and pattern skills. Future work should aim to increase the breadth and rigor of children's broad early math experiences, and determine if individual differences in parent-child math exploration in strictly naturalistic contexts are more related to children's knowledge and skills.

References

Baker, F. B., & Kim, S.-H. (2004). *Item response theory parameter estimation techniques* (2nd ed.). Marcel Dekker.

Blevins-Knabe, B., Austin, A. B., Musun, L., Eddy, A., & Jones, R. M. (2000). Family home care providers' and parents' beliefs and practices concerning mathematics with young children. *Early Child Development and Care*, *165*, 41–58. https://doi.org/10.1080/0300443001650104

Blevins-Knabe, B., & Musun-Miller, L. (1996). Number use at home by children and their parents and its relationship to early mathematical performance. *Early Development and Parenting*, *5*, 35–45. https://doi.org/10.1002/(SICI)1099-0917(199603)5:1<35::AID-EDP113>3.0.CO;2-0

Borriello, G. A., & Liben, L. S. (2018). Encouraging maternal guidance of preschoolers' spatial thinking during block play. *Child Development*, *89*, 1209–1222. https://doi.org/10.1111/cdev.12779

Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, *20*, 607–618. https://doi.org/10.1037/0012-1649.20.4.607

Bull, R., Espy, K. A., & Wiebe, S. A. (2008). Short-term memory, working memory, and executive functioning in preschoolers: Longitudinal predictors of mathematical achievement at age 7 years. *Developmental Neuropsychology*, *33*, 205–228. https://doi.org/10.1080/87565640801982312

Cannon, J., Levine, S. C., & Huttenlocher, J. (2007). *A system for analyzing children and caregivers' language about space in structured and unstructured contexts*. Spatial Intelligence and Learning Center.

Casey, B. M., Lombardi, C. M., Thomson, D., Nguyen, H. N., Paz, M., Theriault, C. A., & Dearing, E. (2018). Maternal support of children's early numerical concept learning predicts preschool and first-grade math achievement. *Child Development*, *89*, 156–173. https://doi.org/10.1111/cdev.12676

Cho, S.-J., & Rabe-Hesketh, S. (2011). Alternating imputation posterior estimation of models with crossed random effects. *Computational Statistics & Data Analysis*, *55*(1), 12–25. https://doi.org/10.1016/j.csda.2010.04.015

Common Core State Standards. (2010). http://www.corestandards.org/Math/

Dearing, E., Casey, B. M., Ganley, C. M., Tillinger, M., Laski, E., & Montecillo, C. (2012). Young girls' arithmetic and spatial skills: The distal and proximal roles of family socioeconomics and home learning experiences. *Early Childhood Research Quarterly*, *27*, 458–470. https://doi.org/10.1016/j.ecresq.2012.01.002

DeFlorio, L., & Beliakoff, A. (2015). Socioeconomic status and preschoolers' mathematical knowledge: The contribution of home activities and parent beliefs. *Early Education and Development*, *26*, 319–341. https://doi.org/10.1080/10409289.2015.968239

Douglas, A.-A., Zippert, E. L., & Rittle-Johnson, B. (2019). *Supporting early numeracy development with card games: "War" tops the deck* [Poster]. Biennial Meeting of the Society for Research in Child Development, Baltimore, MD.

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number development. *British Journal of Developmental Psychology*, *4*, 269–288. https://doi.org/10.1111/j.2044-835X.1986.tb01018.x

Elliott, L., Braham, E. J., & Libertus, M. E. (2017). Understanding sources of individual variability in parents' number talk with young children. *Journal of Experimental Child Psychology*, *159*, 1–15. https://doi.org/10.1016/j.jecp.2017.01.011

Ferrara, K., Hirsh-Pasek, K., Newcombe, N. S., Golinkoff, R. M., & Lam, W. S. (2011). Block talk: Spatial language during block play. *Mind, Brain, and Education*, *5*, 143–151. https://doi.org/10.1111/j.1751-228X.2011.01122.x

Frick, A., Ferrara, K., & Newcombe, N. S. (2013). Using a touch screen paradigm to assess the development of mental rotation between 3½ and 5½ years of age. *Cognitive Processing*, *14*, 117–127. https://doi.org/10.1007/s10339-012-0534-0

Fyfe, E. R., McNeil, N. M., & Rittle-Johnson, B. (2015). Easy as ABCABC: Abstract language facilitates performance on a concrete patterning task. *Child Development*, *86*(3), 927–935. https://doi.org/10.1111/cdev.12331

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Harvard University Press.

Ginsburg, H. P., Lin, C., Ness, D., & Seo, K.-H. (2003). Young American and Chinese children's everyday mathematical activity. *Mathematical Thinking and Learning*, *5*, 235–258. https://doi.org/10.1207/S15327833MTL0504_01

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2012). The relation between spatial skill and early number knowledge: The role of the linear number line. *Developmental Psychology*, *48*, 1229–1241. https://doi.org/10.1037/a0027433

Hammill, D. D., Pearson, N. A., & Voress, J. K. (1993). *Developmental Test of Visual Perception: DTVP-2*. Pro-ed.

Hart, S. A., Ganley, C. M., & Purpura, D. J. (2016). Understanding the home math environment and its role in predicting parent report of children's math skills. *PLoS ONE*, *11*. https://doi.org/10.1371/journal.pone.0168227

Ho, A., Lee, J., Wood, E., Kassies, S., & Heinbuck, C. (2017). Tap, swipe, and build: Parental spatial input during iPad® and toy play. *Infant and Child Development*. https://doi.org/10.1002/icd.2061

Huntsinger, C. S., Jose, P. E., & Luo, Z. (2016). Parental facilitation of early mathematics and reading skills and knowledge through encouragement of home-based activities. *Early Childhood Research Quarterly*, *37*, 1–15. https://doi.org/10.1016/j.ecresq.2016.02.005

Jirout, J. J., & Newcombe, N. S. (2015). Building blocks for developing spatial skills. *Psychological Science*, *26*, 302–310. https://doi.org/10.1177/0956797614563338

Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, *105*, 395–438. https://doi.org/10.1016/j.cognition.2006.10.005

LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*, 1753–1767. https://doi.org/10.1111/j.1467-8624.2010.01508.x

Levine, S. C., Huttenlocher, J., Taylor, A., & Langrock, A. (1999). Early sex differences in spatial skill. *Developmental Psychology*, *35*, 940–949. https://doi.org/10.1037/0012-1649.35.4.940

Levine, S. C., Ratliff, K. R., Huttenlocher, J., & Cannon, J. (2012). Early puzzle play: A predictor of preschoolers' spatial transformation skill. *Developmental Psychology*, *48*, 530–542. https://doi.org/10.1037/a0025913

Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*, 1309–1319. https://doi.org/10.1037/a0019671

Lipkus, I. M., & Peters, E. (2009). Understanding the role of numeracy in health: Proposed theoretical framework and practical insights. *Health Education & Behavior*, *36*(6), 1065–1081. https://doi.org/10.1177/1090198109341533

Lombardi, C. M., Casey, B. M., Thomson, D., Nguyen, H. N., & Dearing, E. (2017). Maternal support of young children's planning and spatial concept learning as predictors of later math (and reading) achievement. *Early Childhood Research Quarterly*, *41*, 114–125. https://doi.org/10.1016/j.ecresq.2017.07.004

Miller, M. R., Rittle-Johnson, B., Loehr, A. M., & Fyfe, E. R. (2016). The influence of relational knowledge and executive function on preschoolers' repeating pattern knowledge. *Journal of Cognition and Development*, *17*, 85–104. https://doi.org/10.1080/15248372.2015.1023307

Missall, K. N., Hojnoski, R. L., Caskie, G. I. L., & Repasky, P. (2015). Home numeracy environments of preschoolers: Examining relations among mathematical activities, parent mathematical beliefs, and early mathematical skills. *Early Education and Development*, *26*, 1–21. https://doi.org/10.1080/10409289.2015.968243

Missall, K. N., Hojnoski, R. L., & Moreano, G. (2017). Parent–child mathematical interactions: Examining self-report and direct observation. *Early Child Development and Care*, *187*, 1896–1908. https://doi.org/10.1080/03004430.2016.1193731

Mix, K. S., & Cheng, Y.-L. (2012). The relation between space and math. In J. B. Benson (Ed.), *Advances in Child Development and Behavior* (Vol. 42, pp. 197–243). Elsevier.

Mutaf Yildiz, B., Sasanguie, D., De Smedt, B., & Reynvoet, B. (2018). Investigating the relationship between two home numeracy measures: A questionnaire and observations during Lego building and book reading. *British Journal of Developmental Psychology*, *36*, 354–370. https://doi.org/10.1111/bjdp.12235

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics*. National Council of Teachers of Mathematics.

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity*. National Academies Press.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, *36*(3), 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003

Polinsky, N., Perez, J., Grehl, M., & McCrink, K. (2017). Encouraging spatial talk: Using children's museums to bolster spatial reasoning. *Mind, Brain, and Education*, *11*, 144–152. https://doi.org/10.1111/mbe.12145

Pruden, S. M., & Levine, S. C. (2017). Parents' spatial language mediates a sex difference in preschoolers' spatial language use. *Psychological Science*, *28*, 1583–1596. https://doi.org/10.1177/0956797617711968

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? *Developmental Science*, *14*, 1417–1430. https://doi.org/10.1111/j.1467-7687.2011.01088.x

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15–33. https://doi.org/10.1016/j.cogdev.2014.11.002

Ritchie, S. J., & Bates, T. C. (2013). Enduring links from childhood mathematics and reading achievement to adult socioeconomic status. *Psychological Science*, *24*(7), 1301–1308. https://doi.org/10.1177/0956797612466268

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development*, *88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662

Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly*, *31*, 101–112. https://doi.org/10.1016/j.ecresq.2015.01.005

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development*, *14*, 376–396. https://doi.org/10.1080/15248372.2012.689897

Rittle-Johnson, B., Saylor, M., & Swygert, K. E. (2008). Learning from explaining: Does it matter if mom is listening? *Journal of Experimental Child Psychology*, *100*(3), 215–224. https://doi.org/10.1016/j.jecp.2007.10.002

Rittle-Johnson, B., Zippert, E., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly*, *46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006

Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. Oxford University Press.

Sarama, J., & Clements, D. H. (2004). Building blocks for early childhood mathematics. *Early Childhood Research Quarterly*, *19*(1), 181–189. https://doi.org/10.1016/j.ecresq.2004.01.014

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

Scalise, N. R., Daubert, E. N., & Ramani, G. B. (2018). Narrowing the early mathematics gap: A play-based intervention to promote low-income preschoolers' number skills. *Journal of Numerical Cognition*, *3*, 559–581. https://doi.org/10.5964/jnc.v3i3.72

Seo, K.-H., & Ginsburg, H. P. (2004). What is Developmentally Appropriate in Early Childhood Mathematics Education? Lessons from New Research. In D. H. Clements, J. Sarama, & M. DiBiase (Eds.), *Engaging young children in mathematics: Standards for early childhood mathematics education* (pp. 91–103). Earlbaum.

Shapka, J. D., Domene, J. F., & Keating, D. P. (2006). Trajectories of career aspirations through adolescence and young adulthood: Early math achievement as a critical filter.

Educational Research and Evaluation, *12*(4), 347–358. https://doi.org/10.1080/13803610600765752

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives*, *8*, 144–150. https://doi.org/10.1111/cdep.12077

Skwarchuk, S.-L. (2009). How do parents support preschoolers' numeracy learning experiences at home? *Early Childhood Education Journal*, *37*, 189–197. https://doi.org/10.1007/s10643-009-0340-1

Skwarchuk, S.-L., Sowinski, C., & LeFevre, J.-A. (2014). Formal and informal home learning activities in relation to children's early numeracy and literacy skills: The development of a home numeracy model. *Journal of Experimental Child Psychology*, *121*, 63–84. https://doi.org/10.1016/j.jecp.2013.11.006

Spiegelhalter, D., Thomas, A., Best, N., & Lunn, D. (2003). *Winbugs user manual*. https://www.mrc-bsu.cam.ac.uk/wp-content/uploads/manual14.pdf

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*(1), 99–120. https://doi.org/10.1016/j.ecresq.2004.01.002

Susperreguy, M. I., & Davis-Kean, P. E. (2016). Maternal math talk in the home and math skills in preschool children. *Early Education and Development*, *27*, 841–857. https://doi.org/10.1080/10409289.2016.1148480

Thompson, R. J., Napoli, A. R., & Purpura, D. J. (2017). Age-related differences in the relation between the home numeracy environment and numeracy skills. *Infant and Child Development*, *26*. https://doi.org/10.1002/icd.2019

Vandell, D. L. (1979). A microanalysis of toddlers' social interaction with mothers and fathers. *The Journal of Genetic Psychology: Research and Theory on Human Development*, *134*, 299–312. https://doi.org/10.1080/00221325.1979.10534063

Vandermaas-Peeler, M., Boomgarden, E., Finn, L., & Pittard, C. (2012). Parental support of numeracy during a cooking activity with four-year-olds. *International Journal of Early Years Education*, *20*, 78–93. https://doi.org/10.1080/09669760.2012.663237

Vandermaas-Peeler, M., Ferretti, L., & Loving, S. (2012). Playing The Ladybug Game: Parent guidance of young children's numeracy activities. *Early Child Development and Care*, *182*, 1289–1307. https://doi.org/10.1080/03004430.2011.609617

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2017). Spatial skills, their development, and their links to mathematics. *Monographs of the Society for Research in Child Development*, *82*, 7–30. https://doi.org/10.1111/mono.12280

Verdine, B. N., Golinkoff, R. M., Hirsh-Pasek, K., Newcombe, N. S., Filipowicz, A. T., & Chang, A. (2014). Deconstructing building blocks: Preschoolers' spatial assembly performance relates to early mathematical skills. *Child Development*, *85*, 1062–1076. https://doi.org/10.1111/cdev.12165

Verdine, B. N., Zimmermann, L., Foster, L., Marzouk, M. A., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. (2019). Effects of geometric toy design on parent–child interactions and spatial language. *Early Childhood Research Quarterly*, *46*, 126–141. https://doi.org/10.1016/j.ecresq.2018.03.015

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds.). Harvard University Press.

Wechsler, D. (1991). *WISC-III: Wechsler intelligence scale for children: Manual*. Psychological Corporation.

Weiland, C., Wolfe, C. B., Hurwitz, M. D., Clements, D. H., Sarama, J. H., & Yoshikawa, H. (2012). Early mathematics assessment: Validation of the short form of a prekindergarten and kindergarten mathematics measure. *Educational Psychology*, *32*(3), 311–333. https://doi.org/10.1080/01443410.2011.654190

Wynn, K. (1990). Children's understanding of counting. *Cognition*, *36*, 155–193. https://doi.org/10.1016/0010-0277(90)90003-3

Wynn, K. (1992). Children's acquisition of the number words and the counting system. *Cognitive Psychology*, *24*, 220–251. https://doi.org/10.1016/0010-0285(92)90008-P

Zippert, E., Daubert, E. N., Scalise, N. R., Noreen, G. D. * Ramani G. B. (2019). 'Tap space number three': Promoting math talk during parent-child tablet play. *Developmental Psychology* 55, 1605–14.

Zippert, E. L., Schneider, R., Kaplan, D., & Ramani, G. B. (2016, April). *Connecting the dots: Examining the role of preschoolers' numerical knowledge and interest in parent-child number exploration during card game play* [Poster]. National Research & Development Center on Cognition & Mathematics Instruction, Arlington, Virginia.

Zippert, E., Loehr, A. M., & Rittle-Johnson, B. (2018, February). *A new teacher-based assessment of preschoolers' patterning skills* [Poster]. Society for Research on Educational Effectiveness, Washington, D.C.

Zippert, E., & Ramani, G. B. (2017). Parents' estimations of preschoolers' number skills relate to at-home number-related activity engagement. *Infant and Child Development*, *26*. https://doi.org/10.1002/icd.1968

Zippert, E., & Rittle-Johnson, B. (2018). The home math environment: More than numeracy. *Early Childhood Research Quarterly*. Advance online publication. https://doi.org/10.1016/j.ecresq.2018.07.009

Table 1

Frequency of Parent and Child Number Exploration

Code Name	Definition	Examples	Proportion of Numeracy Exploration Intervals *M*(*SD*)[a]		Proportion of All Intervals *M*(*SD*)[b]	
			Parent	Child	Parent	Child
Advanced Operations	Adds or subtracts two numbers or explores other advanced topics (e.g., fractions)	"What's one less than eight?"	.03(.07)	.01(.03)	.01(.02)	.00(.01)
Magnitude Comparison	Compares or matches two numbers/quantities	"Is one bigger than nine?"	.24(.20)	.23(.24)	.08(.06)	.05(.06)
Numeral Identification	Identifies a written numeral	"This is a six."	.23(.19)	.31(.21)	.07(.06)	.07(.06)
Cardinal Values	Labels number of elements in a set or asks about quantity in a set	"Why don't we pick three cards?"	.20(.11)	.14(.12)	.06(.04)	.03(.03)
Counting Objects	Counts objects or discusses counting objects as a strategy	"Count the dots."	.07(.12)	.13(.18)	.02(.04)	.03(.03)
Ordinal Relations	Describes order of numbers, asks before or after questions or mentions "then" relations	"What comes after four?"	.06(.07)	.03(.06)	.02(.02)	.01(.02)
Rote Count	Counts numbers sequentially	"Four, five, six…"	.01(.06)	.01(.04)	.00(.01)	.00(.01)
Number Other	Uses number in a way other than described above	"I hope I get a six!"	.01(.02)	.02(.03)	.00(.01)	.00(.01)
Relative Magnitude	Makes a general statement about quantity or uses quantifying words such as "many"	"We have a lot of blocks."	.15(.08)	.11(.09)	.05(.03)	.02(.02)

Notes. Adapted from Ramani et al., 2015. We did not consider number-ambiguous statements such as "This one" and "that one" as number exploration. Coding was hierarchical (priority given to codes appearing higher in the list in the table) and mutually exclusive. [a] Proportion of total 10-second intervals in which parents and children explored numeracy. [b] Proportion of total 10-second intervals in which parents and children engaged in the play session.

Table 2

Frequency of Parent and Child Spatial Exploration

Code Name	**Definition**	**Examples**	**Proportion of Spatial Exploration Intervals *M*(*SD*)**[a]		**Proportion of All Intervals *M(SD)***[b]	
			Parent	**Child**	**Parent**	**Child**
Orientations & Transformations	Discusses or performs relative orientation or transformation of objects and people in space (e.g. upside down, rotate, flip)	"Let's turn the block this way."	.11(.10)	.09(.12)	.02(.02)	.01(.02)
Spatial Dimensions	Mentions size of objects, people, and spaces including volume, capacity, and measure (e.g. big, little, long, short, tall)	"We need a shorter Lego piece."	.11(.09)	.15(.14)	.03(.02)	.01(.01)
Spatial Features and Properties	Mentions features and properties of 2D and 3D objects, spaces, people (e.g. border, line, round, bent, straight, flat, corner, ends, this side)	"This Lego is flat." "Ends of string"	.09(.10)	.02(.05)	.02(.02)	.01(.01)
Shapes	Mentions standard or universally recognized form of enclosed two- and three-dimensional objects (e.g. square, circle, polygon) and spaces (e.g., hole)	"This is a triangle."	.08(.07)	.08(.10)	.02(.02)	.01(.01)
Locations & Directions	Talks about position of objects, people, and points in space (e.g. underneath, side, on top of, inside of, under, vertical, column, high, low, sideways, end, through)	"And then on top of the yellow, what do we have?"	.60(.14)	.66(.19)	.13(.06)	.07(.05)
Continuous Amount	Discusses amount of continuous quantities (including extent of an object, space, liquid, etc.)	"This part of the castle should be blue."	.00(.01)	.00(.00)	.00(.00)	.00(.00)

Notes. Adapted from Cannon, Levine, Huttenlocher, 2007. Coding was hierarchical (priority given to codes appearing higher in the list in the table) and mutually exclusive. $n = 44$ for Child column because one child did not engage in any spatial exploration.
[a] Proportion of total 10-second intervals in which parents and children explored space. [b] Proportion of total 10-second intervals in which parents and children engaged in the play session.

Table 3

Frequency of Parent and Child Pattern Exploration

Code Name	Definition	Examples	Proportion of Patterning Exploration Intervals *M(SD)*[a]		Proportion of All Intervals *M(SD)*[b]	
			Parent	Child	Parent	Child
Identify Pattern Unit	Explicitly identifies the pattern unit	“This is a green-white pattern.”	.01(.02)	.00(.01)	.00(.00)	.00(.00)
Link Patterns/ Abstracting	Links the individual items from one pattern to another pattern	“Blue in this pattern is like green in that one.”	.01(.08)	.00(.01)	.00(.01)	.00(.00)
Label Items in Order	Says characteristic of at least 2 consecutive items in a pattern (after pattern is made or while making a pattern with materials- as long as it is a pattern)	“Yellow, blue, blue. Yellow, blue, blue.”	.18(.18)	.12(.12)	.02(.03)	.02(.02)
Pattern Identification	Asks what the pattern is or identifies that a pattern is present	“What is your pattern?”	.22(.20)	.12(.13)	.02(.02)	.01(.02)
Pattern Extension	Asking what comes next/first in the pattern or responding with what’s next	“What’s next in the pattern?	.33(.25)	.25(.20)	.04(.04)	.03(.04)
Pattern creation, no verbalization	Creating at least one unit of a pattern without discussing the pattern	Child/parent makes pattern on their own	.18(.27)	.48(.28)	.02(.03)	.05(.04)
Gestures to Pattern	Points to or sweeps over their own pattern but does not provide a verbal explanation	[Points to each bead on their string]	.00(.00)	.00(.02)	.00(.00)	.00(.00)
Pointing out similarities & differences	Determining features that are the same, noticing similarities and differences between objects/images that are present	This doesn’t look like that (in reference to the pictures)	.07(.12)	.03(.06)	.00(.01)	.00(.01)

Notes. Adapted from Rittle-Johnson et al., 2013. Coding was hierarchical (priority given to codes appearing higher in the list in the table) and mutually exclusive. $n = 42$ for Parent column because 3 parents did not engage in any pattern support. [a] Proportion of total 10-second intervals in which parents and children explored of patterns. [b] Proportion of total 10-second intervals in which parents and children engaged in the play session.

Table 4

Math Support and Exploration for the Entire Play Session and by Activity for Parents and Children

	Math Exploration (Proportion of Total Play Session or Activity Intervals) *M*(*SD*)			
	Average of Each Type	**Number**	**Spatial**	**Pattern**
Parent				
Math Support Across Activities	.21(.01)	.31(.09)	.21(.09)	.11(.06)
Card Activity	.26(.07)[a]	.62(.16)[c]	.17(.13)	.00(.02)
Block Activity	.15(.09)	.16(.10)	.28(.17)[c]	.01(.03)
Bead Activity	.22(.07)	.15(.11)	.18(.10)	.31(.17)[c]
Child				
Math Exploration Across Activities	.15(.01)	.23(.10)	.11(.06)	.12(.06)
Card Activity	.19(.06)[b]	.48(.18)[c]	.09(.10)	.00(.01)
Blocks Activity	.09(.05)	.11(.10)	.17(.11)[c]	.01(.02)
Beads Activity	.18(.07)	.10(.09)	.07(.06)	.36(.16)[c]

Note. Math Support Across Activities represents the average proportion of math exploration out of total 10-second intervals across the 3 activities. Math exploration during each activity (e.g., cards) is reported as proportions of the 10-second intervals during which dyads engaged in the respective activity. Math exploration and support did not differ significantly by the level of financial assistance families received to attend preschool.

[a] More math support during card play than bead or block play, and more math support during bead play than block play (*ps* < .01). [b] More math exploration during card and bead play than block play (*ps* < .01).[c] More exploration during play with this toy than play with other toys (*ps* < .01).

Table 5

Partial Correlations Between Child Skills at Two Time Points and Parent-Child Math Exploration, Controlling for Child Age at Test Time

Measures	Math Exploration (Proportion of Total Play Session Intervals)							
	Parent Support Types				Child Exploration Types			
	Broad Math Total	**Number**	**Spatial**	**Pattern**	**Broad Math Total**	**Number**	**Spatial**	**Pattern**
Time 1 Child Skills								
Broad Math	-.11	-.03	-.11	-.02	-.10	.00	-.23	.08
Numeracy	-.17	.07	-.22	-.09	.04	.19	-.16	.04
Pattern (Research-based)	-.00	.09	-.24	.11	.19	.14	-.09	.28
Pattern (Teacher-based)	-.11	-.03	-.16	-.06	.03	.11	-.11	.01
Spatial Visualization	-.03	.04	-.12	.01	-.05	.13	-.29	.03
Form Perception	-.07	-.07	-.18	.06	.03	.05	-.05	.03
Visual-Spatial Working Memory	.00	.13	-.15	-.02	-.18	-.08	-.36*	.08
Time 2 Child Skills								
Broad Math	-.05	-.07	-.07	-.04	.08	.10	-.15	.15
Numeracy	-.00	-.04	-.09	.10	.08	.10	-.17	.19
Pattern (Teacher-based)	-.01	.13	-.17	.07	.05	.20	-.19	.00
Spatial Visualization	-.02	.15	-.11	-.03	.04	.18	-.13	-.04
Visual-Spatial Working Memory	.00	.08	-.13	.05	.00	.10	-.20	.01

Note. One child had missing data for an assessment at Time 1 and all assessments at Time 2, so the analyses excluded this child ($df = 41$). Separate analyses not presented also controlling for financial assistance to attend pre-k yielded comparable results.
* $p < .05$.

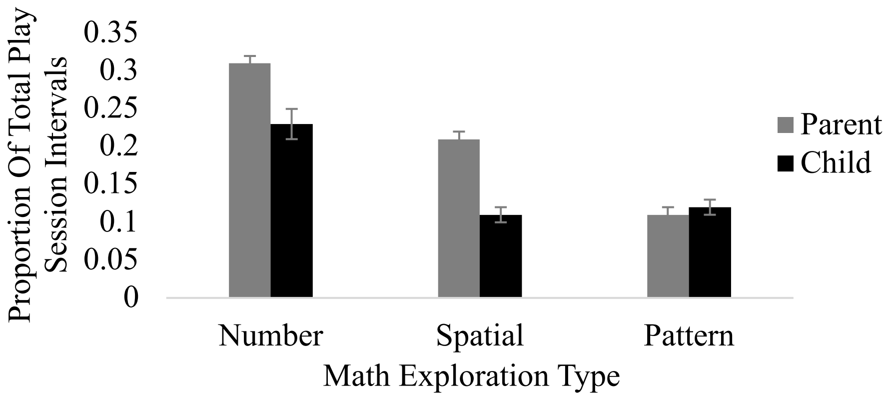


Figure 1a. Parent-Child Math Exploration by Math Domain. Error bars reflect standard errors.

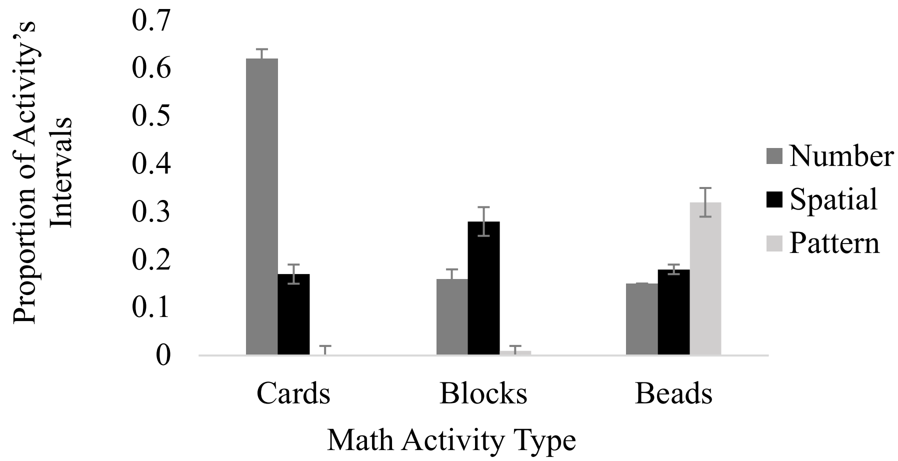


Figure 1b. Parent Number, Spatial & Pattern Exploration by Activity. Error bars reflect standard errors.

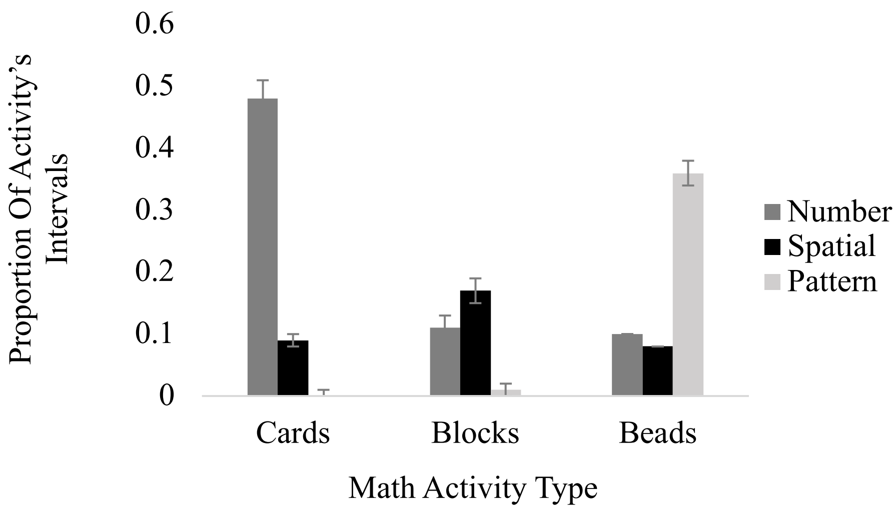


Figure 1c. Child Number, Spatial & Pattern Exploration by Activity. Error bars reflect standard errors.

Supplemental Materials

Table S1

Participants' Demographic Information

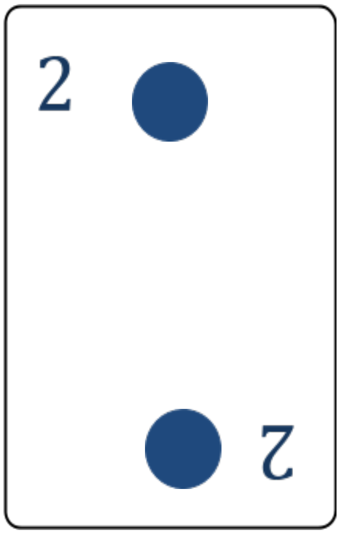
Demographics	Frequency	Percent (%)
Parent Race or Ethnicity[a]		
African-American or Black	11	30.6
Caucasian or White	19	52.8
Asian or Pacific Islander	1	2.8
Biracial or Mixed Race	5	13.9
Parent Identification as Hispanic or Latino[b]	1	2.3
Parents' Highest Level of Education[c]		
High school diploma or GED	2	4.7
Some college or 2-year degree	15	34.9
Bachelor's degree	10	23.3
Some graduate work	2	4.7
Master's, professional, or doctoral degree	14	32.6
Child Race or Ethnicity		
African-American or Black	17	37.8
Caucasian or White	22	48.9
Middle Eastern	2	4.4
Multiple races/ethnicities	4	8.9
Child Financial Assistance		
None	19	42.2
Some	10	22.2
Full	16	35.6
Type of Preschool Attended		
Private	15	33.3
Public	30	66.7
Child Reported as Bilingual	4	8.9
Child Receives Special Education Services	6	13.3

Note. $N = 45$.

[a]Parent race or ethnicity information was missing for 9 parents. [b]Eight parents did not report whether or not they identified as Hispanic. [c]Two parents had missing education level data.

Table S2

Activities and suggestions used for the play session

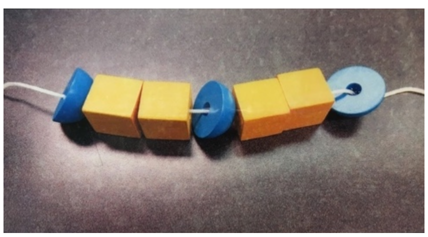


Cards

This bag contains simple playing cards. You may play any card game that you would like. Feel free to use our suggestions below.

Suggestions:

- War: Divide the deck of cards evenly for each player. Each player places his/her stack of cards face down, in front of him/her. Players turns up a card at the same time and the player with the higher card (the card that has more) takes both cards and puts them face-down on the bottom of his/her stack.
- Order Up: Each player orders cards by number. Start with three cards and then increase the number of cards.



Beads

This bag contains beads of different colors and shapes as well as two laces. You can put the beads on the laces to make patterns together. Feel free to use our suggestions below.

Suggestions:

- Use the sample patterns included to make a pattern with one of the plastic bags of beads.
- Challenge each other to make the same type of pattern using beads of different colors and shapes from the second plastic bag.

Lego Duplo Blocks

This bag contains Lego Duplo blocks. You may choose to build anything you would like with the blocks, such as a house. Feel free to use our suggestions below.

Suggestions:

- Build a house like the one in the picture included in the bag.
- Build things around your house like a tree, a car, or a basketball hoop, like the pictures included in the bag.

Table S3

Descriptions of Dyads' Engagement in Each Activity

Activity Type	Number of Dyads (N = 45)			
	First (or only) Activity	Second Activity	Third Activity	Fourth Activity
Card Activity				
War (suggested)	23	2	-	-
Order Up (suggested)	10	2	-	-
Go Fish	3	-	-	-
Matching	5	1	-	-
Counting or Numeral Identification	1	1	-	-
Free Play[a]	3	-	-	-
Block Activity				
Suggested structure	29	14	4	-
Different structure	16	8	6	4
Free Play[b]	-	3	2	3
Bead Activity				
Copy suggested pattern	9	8	1	-
Abstract suggested pattern	2	3	-	-
Create different pattern	21	15	3	1
Free Play[c]	14	10	5	1

[a]Dyads played with the cards provided but did not engage in an activity that featured numbers (e.g. dyads used cards to build tents). [b]Dyads played with the blocks provided but did not build a structure. [c]Dyads played with the beads provided but did not create a pattern.

Table S4

Partial Correlations Between Child Math Skills, Parent-Child Math Support and Exploration, Controlling for Child Age at Test Time

	Type of Math Exploration (Proportion of Total Play Session Intervals)			
	Parent		Child	
	Advanced	**Foundational**	**Advanced**	**Foundational**
Time 1				
Numeracy				
Broad Math	.10	-.22	.10	-.13
Numeracy	.21	-.22	.23	.00
Spatial				
Broad Math	-.01	-.13	.02	-.34*
Spatial Visualization	-.10	-.10	-.11	-.33*
Form Perception	-.04	-.22	.00	-.07
Visual-spatial Working Memory	-.08	-.14	-.19	-.38*
Patterning				
Broad Math	-	-.02	-	.08
Pattern (Research-based)	-	.08	-	.27
Pattern (Teacher-based)	-	-.02	-	.00
Time 2				
Numeracy				
Broad Math	.05	-.20	.14	-.03
Numeracy	.09	-.21	.14	-.03
Spatial				
Broad Math	-.00	-.10	-.02	-.20
Spatial Visualization	.02	-.17	.03	-.20
Visual-spatial Working Memory	-.17	-.06	-.33*	-.05
Patterning				
Broad Math	-	-.05	-	-.14
Pattern (Teacher-based)	-	.04	-	-.01

Note. Advanced numeracy was the summed proportion of exploration focused on number operations (arithmetic and magnitude comparison), numeral identification, and cardinality. Foundational numeracy was the summed proportion of exploration focused on counting, ordinal relations, relative magnitude and other references to numbers. No advanced patterning composite was created as parents and children almost never explored advanced patterning concepts (i.e. identifying pattern units and abstracting patterns). Foundational patterning was the summed proportion of exploration focused on identifying, labelling, creating, and extending patterns as well as exploring similarities. Advanced spatial was the summed proportion of exploration focused on orientations and transformations, spatial dimensions, spatial features and properties, and shapes. Foundational spatial was the proportion of exploration focused on locations and directions since parents and children rarely discussed the other foundational spatial concept that was coded (i.e. continuous amounts). * $p < .05$.

Table S5

To further explore relations between parents' and children's math exploration and children's math skills, we created summed proportions of parents' and children's math exploration of specific concepts and examined relations to children's broad math knowledge and corresponding skills, controlling for child age. First, numeracy exploration focused on number operations, numeral identification, and cardinality were summed to create a measure of advanced numeracy while numeracy exploration focused on counting, ordinal relations, relative magnitude and other uses of number words were summed to create a measure of foundational numeracy. Next, patterning exploration focused on identifying, labeling, creating, and extending patterns as well as discussing similarities were summed to create a measure of foundational patterning exploration. Parents and children very rarely gestured to patterns, so this was excluded from the foundational patterning composite. We did not create a measure of parents' and children's advanced patterning exploration as this occurred very infrequently (see Table 3). Lastly, spatial exploration focused on orientations and transformations, spatial dimensions, spatial features and properties, and shapes were summed to create a measure of advanced spatial exploration. Parents and children rarely discussed continuous amounts; thus, only exploration focused on locations and directions was used as a measure of foundational spatial exploration.

Even after considering the complexity of parents' numeracy, patterning, and spatial support, we did not find significant links between this support and children's broad math knowledge and corresponding skills at either time point, *r*s = -.22 to .11. Similarly, the complexity of children's numeracy and patterning exploration was not related to their broad math knowledge or corresponding skills at either time point, *r*s = -.13 to .27. However, children's foundational spatial exploration (i.e. focused on location and direction) was significantly negatively related to their spatial visualization skills, $r(41) = -.33$, $p = .029$, visual-spatial working memory, $r(41) = -.38$, $p = .012$, and their broad math skills, $r(41) = -.34$, $p = .025$, at the start of pre-K. Their advanced spatial exploration (sum of exploration focused on orientation and transformation, spatial dimensions, spatial features and properties, and shapes) was only significantly related to their visual-spatial working memory at the end of pre-K, $r(41) = -.33$, $p = .031$. We did not account for the number of correlations conducted in these exploratory analyses, so findings should be interpreted with caution and require replication.