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Does initial learning about the meaning of fractions present similar challenges for students with and without adequate whole-number skill?☆

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ABSTRACT

The purposes of this study were to (a) explore whether early fractions understanding at 4th grade is differentially challenging for students with versus without adequate whole-number competence and (b) identify specific whole-number skill associated with difficulty in fractions understanding. Based on initial whole-number competence, 1,108 4th graders were classified as having (a) adequate whole-number competence (n = 775), (b) less severe whole-number difficulty (n = 201), and (c) severe whole-number difficulty (n = 132). At the end of 4th grade, they were assessed on fractions understanding and further classified as with versus without difficulty in fractions understanding. Multi-level logistic regression indicated that compared to students with adequate whole-number competence, those with less severe whole-number difficulty were almost 5 times as likely to experience difficulty with fractions understanding whereas those with severe whole-number difficulty were about 32 times as likely to experience difficulty with fractions understanding. Students with severe whole-number difficulty were about 7 times as likely to experience difficulty with fractions understanding compared to those with less severe whole-number difficulty. Among students with adequate whole-number competence, the pretest whole-number skill distinguishing those with versus without difficulty in fractions understanding was basic division facts (i.e., 2-digit dividend ÷ 1-digit divisor) and simple multiplication (i.e., 3-digit × 1-digit without regrouping). The role of whole-number competence in developing initial fractions understanding and implications for instruction are discussed.

1. Introduction

Competence with fractions is required for success with more complex and advanced mathematics (Booth & Newton, 2012; National Mathematics Advisory Panel [NMAP], 2008). In a longitudinal study examining the types of mathematical knowledge that predict later mathematics achievement in the United States and United Kingdom, fifth-grade fractions knowledge uniquely predicted algebraic performance and overall mathematics achievement in high school, even after controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education (Siegler et al., 2012). In turn, algebra is critical for success in higher education and with careers involving science, technology, engineering, and mathematics (NMAP, 2008). Given the foundational role fractions play in algebra, advanced mathematics, and technological careers, it is unfortunate that learning fractions is often challenging for students.

Difficulty with fractions understanding is often discussed as pervasive and persistent, even among students with competent whole-number skill (NMAP, 2008), and many published studies rely on the National Assessment of Educational Progress (NAEP) as evidence for the pervasiveness of this problem (e.g., Bailey, Siegler, & Geary, 2014; Namkung & Fuchs, 2016; Siegler, Fazio, Bailey, & Zhou, 2013; Siegler & Pyke, 2012; Siegler, Thompson, & Schneider, 2011). For example, according to the 2013 NAEP (U.S. Department of Education, 2013), only 60% of fourth-grade students correctly identified the greatest unit fraction. This reflects a common assumption in the literature: achieving competence with fractions is challenging even for students whose whole-number skill develops with relative ease. However, little is

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known about how early difficulty in developing fractions understanding relates to students' whole-number competence. The primary purpose of this paper was to explore whether early understanding about fractions (at fourth grade, when fractions become a major focus of mathematics curricula) is differentially challenging for students with versus those without adequate whole-number skill. The study's secondary purpose was to identify specific whole-number skill associated with difficulty in fractions understanding.

1.1. Early difficulty in fractions understanding

One source of challenge in learning about the meaning of fractions is the phenomenon referred to as *whole-number bias* (e.g., Cramer, Post, & delMas, 2002; Cramer & Wyberg, 2009; Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004), the overgeneralization of whole-number knowledge to fractions. Fractions first become a major focus of the mathematics curriculum in fourth grade, after students have already consolidated knowledge about whole numbers. So, it is not surprising that students initially rely on their whole-number knowledge to interpret the meaning of fractions (Behr, Wachsmuth, Post, & Lesh, 1984; Ni & Zhou, 2005).

Assimilating incompatible whole-number concepts into understanding about fractions may lead to substantial misconceptions about fractions because fundamental differences exist between whole numbers and fractions (Stafylidou & Vosniadou, 2004). For example, symbolic representation differs for whole numbers (i.e., one number) and fractions (i.e., one number represented by two numerals and a fraction bar). Moreover, in the whole-number system, students can use "counting on" to place numbers in order, whereas no discrete number precedes a fraction, and infinite quantities of fractions exist between any two fractions. Thus, common misconceptions and errors include viewing numerators and denominators as independent numbers, comparing fraction magnitudes based on whole-number knowledge (e.g., $1/5 > 1/2$ because $5 > 2$), and ordering fractions based on whole-number knowledge (e.g., $1/2 < 1/8 < 1/12$ because $2 < 8 < 12$).

Accordingly, a strong foundation in whole-numbers may not necessarily support or guarantee success with fractions understanding, at least when students are forming initial fractions concepts. Instead, students' prior whole-number knowledge may interfere with fractions understanding. In a recent study (Malone & Fuchs, 2017), approximately 65% of errors in ordering fractions at fourth grade were due to whole-number ordering errors, in which students misapplied whole-number properties to understand fractions magnitudes.

Other studies also indicate that fractions understanding is more difficult than understanding the meaning of whole numbers. Young children easily shift their initial logarithmic representations of whole-number magnitudes to build a linear model of the 0–100 number line (Booth & Siegler, 2006, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Laski & Siegler, 2007). Yet, fractions magnitude understanding presents a more major obstacle, with sixth and eighth graders' fractions number line estimation less accurate than first graders' whole-number line estimation (Booth & Siegler, 2008; Siegler & Booth, 2004; Siegler & Pyke, 2012). Even adults (DeWolf & Vosniadou, 2011, 2015; Vamvakoussi, Van Dooren, & Verschaffel, 2012) and expert mathematicians (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013) may regress to whole-number reasoning when they cannot directly and automatically access the meaning of fractions magnitudes. A large literature, therefore, documents that fractions understanding is differentially challenging compared to whole numbers.

1.2. Role of whole-number knowledge in understanding fractions

The individual differences literature provides some insight into the role of whole-number competence in learning about fractions. Findings on this point are, however, mixed. On one hand, in a series of studies, Hecht and colleagues found that skill with simple whole-number calculations (addition and multiplication) does not concurrently predict fractions estimation or word-problem solving at fifth grade (Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2010). Similarly, Namkung and Fuchs (2016) found that whole-number calculation skill (addition, subtraction, multiplication, division) does not contribute to the longitudinal prediction of fractions magnitude understanding (i.e., number line estimation). Results of these studies suggest that proficiency with whole-number principles and operations alone is not sufficient for developing understanding about fractions concepts. Results also suggest that difficulty with fractions may be distinct from difficulty with whole numbers. Alternatively, it is also possible that whole-number calculation skill is still necessary, although not sufficient for fractions understanding.

On the other hand, other studies find that whole-number competence predicts fractions competence. Although Hecht et al. (2003) reported that whereas fifth-grade whole-number calculation skill does not have concurrent predictive value for fractions estimation or word-problem solving, it was a unique predictor of fractions calculation skill. In a similar way, three longitudinal studies found that whole-number calculation skill predicts fractions competence. Bailey et al. (2014) found that simple multi-digit whole-number calculation skill at first grade predicts fractions calculation competence in middle school. Hansen et al. (2015) showed that simple whole-number addition and number line estimation at third grade make unique contributions to fractions calculations and conceptual understanding at fifth grade. Vukovic et al. (2014) found that second-grade whole-number addition, subtraction, multiplication, and division competence contributes to the prediction of fractions calculations at fourth grade.

Yet, with the exception of Hansen et al. (2015), the latter set of studies, in which whole-number calculations predict fractions outcomes, differ from the former set of studies by focusing on fractions calculation outcomes, rather than fractions understanding. They also relied on simple whole-number calculation skill as the predictor. So it is important to consider additional studies, in which complex whole-number calculation skill was employed as the predictor. We located two such studies, in which whole-number long division proved a valuable predictor.

Hansen et al. (2015) showed that fifth-grade whole-number long division predicts knowledge of fractions concepts at sixth grade, and Siegler and Pyke (2012) showed that whole-number long division concurrently predicts fractions calculation competence among sixth and eighth graders. Together, these studies suggest that whole-number long division predicts fractions understanding as well as fractions calculation skill. The relation between whole-number long division and fractions competence may be explained by the fact that fractions can be understood and written as a form of division, with mathematical equivalence (e.g., $2/3 = 2 \div 3$). Siegler and Pyke further explained that whole-number division often yields fraction answers (e.g., $12 \div 5 = 2\ 2/5$), allowing students to understand that 2 2/5 is a value between 2 and 3.

1.3. Present study's purpose

Although prior research suggests widespread fractions difficulty, with students' whole-number skill perhaps not predicting fractions competence, the individual differences literature is mixed on this point and suggests that whole-number skill is more predictive of fractions calculations than understanding. Moreover, although a predictor may have utility within an individual differences framework, the meaningfulness of its effect must be understood in the context how many and which other domain-specific and general variables are included in the model. The small size of this literature, as pertains to fractions, makes it difficult to parse this out and estimate with confidence the importance of whole-number skill to fractions understanding.

An alternative and more straightforward strategy for gaining insight into the role of whole-number competence in fractions understanding involves examining the prevalence of difficulty in fractions understanding among students with versus without strong whole-number competence, while exploring the whole-number skill profiles of students with strong whole-number competence who go on to develop difficulty with fractions. We identified no studies taking this approach.

If having a strong foundation in whole-numbers facilitates the initial fractions understanding, we would expect relatively a few students with a history of adequate whole-number performance to develop difficulty with fractions understanding. This would raise questions about the widespread assumption in the literature that initial learning about fractions concepts creates similar challenges for both students with and without strong whole-number skill. On the other hand, finding that difficulty with fractions is similarly prevalent across students with and without adequate whole-number competence would lend credence to the commonly held assumption that initial fractions understanding is difficult to achieve for many students in general, regardless of their whole-number competence.

Accordingly, in the present study, we assessed (a) whether the proportion of students who experience difficulty in fractions understanding differs as a function of the level of their whole-number competence (those with strong vs. adequate vs. weak whole-number competence whole-number competence); and (b) which whole-number skills discriminate between those who do and do not develop difficulty in fractions understanding among students with adequate whole-number competence. We focused on fourth grade when the school curriculum first includes a strong emphasis on fractions.

Although mixed findings exist in the literature, we hypothesized that students with inadequate whole-number skill would be far more likely to develop fractions difficulty compared to those with strong whole-number skill. Adopting the integrated theory of numerical development proposed by Siegler et al. (2011), who argue that whole-number and fractions understanding develop continuously in an integrated number system, we expected that students' whole-number competence would be linked to fractions understanding. Therefore, it is expected that students who have adequate whole-number competence would be much less likely to experience difficulty with fractions. Additionally, because our fractions measure focused on conceptual understanding of fractions, rather than fractions calculations, we hypothesized that more complex whole-number calculation skill would predict fractions understanding. More specifically, we hypothesized that in line with the empirical evidence from Hansen et al. (2015) and Siegler and Pyke (2012), in addition to the mathematical link between whole-number division and fractions, whole-number long division skill predicts fractions understanding.

2. Method

2.1. Participants

The University's Institutional Review Board approved this study, and the study was conducted in accordance with human subjects guidelines and principles. The data described in this analysis were collected as part of two parent studies, each a randomized-control trial investigating the efficacy of a fraction intervention in fourth grade (Fuchs et al., 2014, 2016). We used data only from control students because the intervention was designed to alter the typical course of fractions learning. Data for the present analysis were based on 1108 students from 46 fourth-grade classrooms in 14 schools in one parent study and from 45 classrooms in 14 schools in the other parent study, both conducted in a southeastern metropolitan school district. The control group fractions instruction relied on Houghton Mifflin Math (Greenes et al., 2005), which focuses on conceptual understanding and procedural calculations and relies heavily on part-whole understanding by using shaded regions and other manipulatives related to the area model. This instruction was delivered in whole-class arrangement and via math centers. Houghton Mifflin Math conceptual lessons included vocabulary instruction, connections across the curriculum (e.g., social studies, music, writing), guided practice, independent work, and links to real life.

The topics covered in the curriculum included reading, writing, and identifying fractions and mixed numbers; finding equivalent fractions and writing fractions in simplest form; comparing and ordering fractions; finding a fractional part of a whole number or finding a fraction of the number of objects in a set; and drawing pictures to solve problems. Calculations with fractions focused on procedures for adding and subtracting. Adding and subtracting proper and improper fractions were introduced first before introducing mixed numbers with like denominators. For proper and improper fractions, denominators did not exceed 12. For mixed numbers, denominators did not exceed 15, and the largest whole number component of the mixed number was 9.

We defined whole-number difficulty as performance on 23 whole-number items on a broad-based calculations assessment (Wide Range Achievement Test-4 Math Computation [WRAT-4]; Wilkinson, 2008) falling below or at the 25th sample-based percentile scores. Students with whole-number difficulty were further specified as severe or less severe using the 10th sample percentile as the criterion. These cut-offs (25th and 10th percentile) reflect an empirical basis found in the mathematics cognition literature indicating that the groups (i.e., low achievement versus mathematics learning disabilities) formulated with these criteria have differential response to intervention and distinctive cognitive profiles (e.g., Murphy, Mazzocco, Hanich, & Early, 2007). Thus, 1108 students were categorized as having (a) adequate whole-number competence (> 25th sample percentile; n = 775), (b) less severe whole-number difficulty (> 10th and ≤ 25th sample percentiles; n = 201), and (c) severe whole-number difficulty (≤ 10th sample percentile; n = 132).

Students' overall raw scores on WRAT-4 across the three groups averaged 27.47 (SD = 3.24) with a standard score mean of 96.06. Table 1 shows age, and raw score and standard score means, and standard deviations on WRAT-4 (Wilkinson, 2008) at the start of fourth grade and on released NAEP fraction items at the end of fourth grade, as a function of students' whole-number competence. (The percentage in each group does not exactly equate to the percentiles used to create each group due to the lack of variability in our sample, i.e., multiple students had the same score.) As expected, students with adequate whole-number competence scored reliably higher than both less severe and severe groups, with students with less severe whole-number difficulty scoring higher than those with severe difficulty.

2.2. Measures

We describe the subset of measures on which we report data. WRAT-4 (Wilkinson, 2008), which was used to assess calculation competence at the start of fourth grade, include mathematical concepts items such as counting (oral administration) as well as calculations (written administration). No participants scored low enough on the

Table 1
Means and standard deviations by risk status.

	Adequate (n = 775)		Less Severe (n = 201)		Severe (n = 132)	
	M	*SD*	*M*	*SD*	*M*	*SD*
Age	9.52	0.38	9.60	0.44	9.55	0.46
Pretest						
WRAT-4 (raw)	12.93	1.52	9.58	0.50	6.52	1.73
(standard)	101.90	8.02	87.20	3.95	75.50	9.34
Posttest						
NAEP fractions	16.95	4.34	13.23	4.24	9.41	3.62

Note. Adequate = Students with adequate whole-number competence; Less Severe = Students with less severe whole-number difficulty; Severe = Students with severe whole-number difficulty; WRAT-4 = Wide Range Achievement Test: Math Computation; NAEP = National Assessment of Educational Progress.

written section to require the oral component. Students have 15 min to complete calculation problems of increasing difficulty. Because WRAT-4 includes non-whole-number items (e.g., multi-digit decimal multiplication with regrouping, multiplying three fractions, addition three mixed number fractions), we used the subset of 23 whole-number items to formulate categories of whole-number competence. Of 23 whole-number items, 18 items focus on calculations with four operations (e.g., basic addition and subtraction facts with single digits, multi-digit addition and subtraction with regrouping, multi-digit by single-digit multiplication without regrouping, multi-digit multiplication with regrouping) and five items involve whole-number calculations, but do not target the four calculation operations (e.g., finding the mean of five numbers, finding a square of a number). Alpha on the sample was 0.72.

To assess difficulty in fractions understanding, we administered 18 released fraction items from 1990 to 2009 NAEP: items classified as easy, medium, or hard from the fourth-grade assessment and items classified as easy from the eighth-grade assessment. Testers read each item aloud. Eight items assess part-whole interpretation, which focuses on understanding a fraction as a part of one entire object or a subset of a group of objects (e.g., identifying a fraction of the figure shaded, identifying a picture that shows 3/4, identifying a fraction for the group of umbrellas closed, shading 1/3 of a rectangle); eight assess measurement interpretation, which focuses on understanding a fraction as a number with a magnitude (i.e., identifying a fraction on a number line, placing 3/4 on a number line, identifying a fraction that is closest to 1/2, identifying a correct reason for why 4/5 is > 2/3 by comparing their magnitudes in relation to 1); one requires subtraction with like denominators (i.e., 4/6–1/6 =); and one asks how many fourths make a whole.

Students select an answer from four choices (11 problems); write an answer (3 problems); shade a portion of a fraction (1 problem); mark a number line (1 problem); write a short explanation (1 problem); or write numbers, shade fractions, and explain the answer (1 problem with multiple parts). The maximum score is 22. Alpha on the sample was 0.86.

2.3. Procedure

We trained testers to administer the WRAT-4 and NAEP fractions, which were administered in a large-group session in September and April, respectively. All sessions were audiotaped; 20% of tapes were randomly selected for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 98%.

2.4. Data analysis

Data analysis proceeded in three steps. First, students in each whole-number competence category were classified as with or without fractions difficulty based on their spring NAEP scores. The 25th sample-based percentile cutoff was used to denote difficulty in fractions understanding, consistent with the cutoff used for whole-number difficulty. Table 2 shows the raw frequency and percentage of students with April fractions difficulty by September whole-number competence.

Table 2
Raw frequencies of students with difficulty in fractions understanding by strata.

	Fractions difficulty	
	Yes (%)	No
Adequate	142 (18.32)	633
Less severe	95 (47.26)	106
Severe	111 (84.09)	21

Note. Adequate = Students with adequate whole-number competence; Less Severe = Students with less severe whole-number difficulty; Severe = Students with severe whole-number difficulty.

Next, to determine whether whole-number difficulty predicted difficulty in fractions understanding while accounting for nesting in the data, we estimated a three-level random (students nested within classrooms, classrooms nested within schools) intercept logistic regression model using the GLIMMIX procedure in the SAS/STAT software Version 13.2 (SAS Institute Inc., 2014). The model was specified as follows:

$$\log\left(\frac{\pi_{ijk}}{1-\pi_{ijk}}\right) = \gamma_{000} + \gamma_{100} LSWND_{ijk} + \gamma_{200} SWND_{ijk} + \gamma_{010} Cohort_{jk} + u_{0jk} + v_{00k}$$

where π_{ijk} is the probability that the i^{th} student in the j^{th} classroom in the k^{th} school has fractions difficulty; LS_WND_{ijk} and S_WND_{ijk} are dummy variables indicating less severe whole number difficulty and severe whole number difficulty, respectively, with adequate whole number competence as the reference group; $Cohort_{jk}$ is a dummy variable indicating cohort (i.e., parent study) with the first parent study as the reference group; γ_{000} is the fixed intercept and γ_{100}-γ_{010} are the fixed main effects; and u_{0jk} and v_{00k} are the random classroom and school intercept effects, respectively. Maximum likelihood with Laplace approximation was used for estimation of model parameters. Sandwich variance estimators with a small sample correction (Morel, Bokossa, & Neerchal, 2003) were used to obtain standard errors.

Finally, to determine which September whole-number calculation items distinguished students who did versus did not develop difficulty in fractions understanding, we selected only those who started the year with adequate whole-number competence. Additionally, because we were specifically interested in whole-number calculation items with four operations (+, −, ×, ÷), we excluded five items that involved whole-number calculations, but did not target the four calculation operations (e.g., finding the mean of numbers, finding a square of a number). Then, we estimated the same multilevel logistic regression model as before but using 18 individual whole-number calculation items to predict difficulty in fractions understanding.

3. Results

Table 3 shows the results for the logistic regression analysis examining the effect of whole-number difficulty on difficulty in fractions understanding. Controlling for cohort, there was a significant omnibus effect of whole-number difficulty on difficulty fractions understanding, $F(2, 1015) = 43.18$, $p < 0.001$, indicating that the proportion of students who experienced difficulty in fractions understanding at the end of fourth grade was related to start-of-year whole-number difficulty. Pairwise comparisons between whole-number difficulty groups revealed significant differences between (a) students with adequate

Table 3
Results for the logistic regression analysis examining the effects of whole-number difficulty on difficulty in fractions understanding.

	Estimate (SE)	p
Fixed effects		
Intercept	− 1.30 (0.28)	< 0.001
Cohort	− 0.48 (0.21)	0.027
Whole-number difficulty[a]		
Adequate (reference group)	–	–
Less severe	1.52 (0.25)	< 0.001
Severe	3.48 (0.38)	< 0.001
Variance components		
Classroom-level random intercept	0.12 (0.11)	
School-level random intercept	0.30 (0.18)	

Note. Adequate = Students with adequate whole-number competence; Less Severe = Students with less severe whole-number difficulty; Severe = Students with severe whole-number difficulty.

[a] $F(2, 1015) = 43.18$, $p < 0.001$.

whole-number competence (18% [95% CI: 12.04, 25.06] predicted to experience difficulty in fractions understanding, after controlling for cohort) versus those with less severe whole-number difficulty (49% [95% CI: 36.62, 62.28] predicted to experience difficulty in fractions understanding, after controlling for cohort), $t(1015) = -5.96$, $p < 0.001$; (b) students with adequate whole-number competence versus those with severe whole-number difficulty (87% [95% CI: 77.56, 93.28] predicted to experience difficulty in fractions understanding, after controlling for cohort), $t(1015) = -9.06$, $p < 0.001$; and (c) students with more versus less severe whole-number difficulty, $t(1015) = 5.57$, $p < 0.001$.

Compared to students with adequate whole-number competence, those with less severe whole-number difficulty were 4.57 (95% CI: 2.77, 7.52) times as likely to experience difficulty in fractions understanding, and those with severe whole-number difficulty were 32.37 (95% CI: 15.24, 68.76) times as likely to experience difficulty in fractions understanding. Students with severe whole-number difficulty were 7.09 (95% CI: 3.56, 14.14) times as likely to experience difficulty in fractions understanding compared to those with less severe whole-number difficulty. To examine the robustness of results, analyses were also conducted with two contrasting cutoffs for difficulty in fractions understanding (a more lenient criterion used in the literature to define mathematics learning disabilities [i.e., 15th] and low achievement [i.e., 35th]). Across the analyses, the omnibus effect of whole-number difficulty on difficulty in fractions understanding was significant, and all pairwise differences between whole-number difficulty groups were significant with the 95% confidence intervals for the odds ratios overlapping across analyses, thus supporting the robustness of findings.

Table 4 shows results for the logistic regression analysis examining the effect of individual whole-number calculation items on difficulty in fractions understanding among students with adequate whole-number competence. Significant unique effects were found for item 10 (i.e., 2-digit dividend ÷ 1-digit divisor), $t(667) = -2.78$, $p = 0.006$, and item 12 (i.e., 3-digit × 1-digit without regrouping), $t(667) = -2.88$, $p = 0.004$. Controlling for cohort and responses to the other whole-number calculation items, students with adequate whole-number competence who missed item 10 were 2.27 (95% CI: 1.27, 4.07) times as likely to experience difficulty in fractions understanding than students with adequate whole-number competency who answered the item correctly. Likewise, controlling for cohort and other responses, students with adequate whole-number competence who missed item 12 were 1.99 (95% CI: 1.24, 3.17) times as likely to experience difficulty in fractions understanding compared to those who answered the item correctly.

Table 4
Results for the logistic regression analysis examining the effects of individual whole-number calculation items on fractions difficulty among students with adequate whole-number competence.

	Estimate (SE)	*p*
Fixed effects		
Intercept	2.20 (2.61)	0.411
Cohort	− 0.67 (0.30)	0.025
WRAT-4 item 1	− 0.33 (1.11)	0.763
WRAT-4 item 2	0.87 (1.34)	0.514
WRAT-4 item 3	− 1.25 (0.73)	0.087
WRAT-4 item 5	− 1.24 (0.68)	0.066
WRAT-4 item 6	0.53 (0.65)	0.415
WRAT-4 item 7	− 0.54 (0.85)	0.525
WRAT-4 item 8	0.24 (0.48)	0.609
WRAT-4 item 9	− 0.41 (0.40)	0.310
WRAT-4 item 10	− 0.82 (0.30)	0.006
WRAT-4 item 11	− 0.33 (0.43)	0.437
WRAT-4 item 12	− 0.69 (0.24)	0.004
WRAT-4 item 13	− 0.10 (0.32)	0.757
WRAT-4 item 16	− 0.45 (0.34)	0.184
WRAT-4 item 17	− 0.63 (0.98)	0.521
WRAT-4 item 22	− 2.28 (1.87)	0.222
WRAT-4 item 25	1.67 (1.25)	0.184
WRAT-4 item 29[a]	–	–
Variance components		
Classroom-level random intercept	0.29 (0.24)	
School-level random intercept	0.40 (0.34)	

Note. WRAT-4 = Wide Range Achievement Test–4 Math Computation.
[a] No students got this item correct so the item was not included in the analysis.

4. Discussion

The purposes of this study were to explore whether early fractions understanding is differentially challenging for students with versus without adequate whole-number skill and to identify specific whole-number skill associated with difficulty in early fractions understanding. Toward that end, we assessed (a) whether the proportion of students who experience difficulty in fractions understanding differs as a function of students' whole-number competence; and (b) among students with adequate whole-number competence, which specific whole-number skills discriminate between students who do versus do not experience difficulty in fractions understanding.

We found that the proportion of students who experienced difficulty in fractions understanding at the end of fourth grade differed significantly as a function of students' whole-number competence at the beginning of fourth grade. Approximately 18% of students with adequate whole-number skill experienced difficulty in fractions understanding after controlling for cohort. This compared to 49% of students with less severe and 87% of students with severe whole-number difficulty after controlling for cohort. Compared to students with adequate whole-number competence, those with less severe whole-number difficulty were almost five times as likely to experience difficulty in fractions understanding whereas those with severe whole-number difficulty were about 32 times as likely to experience difficulty in fractions understanding. Students with severe whole-number difficulty were about seven times as likely to experience difficulty in fractions understanding than those with less severe whole-number difficulty. At the same time, among students with adequate whole-number competence, the start-of-year whole-number skills that distinguished students completing the year with versus without difficulty in fractions understanding were basic division facts (2-digit dividend ÷ 1-digit divisor, such as 10 ÷ 2) and simple multiplication (3-digit × 1-digit without regrouping, such as 421 × 2).

As discussed in the introduction to this paper, many students initially rely on their whole-number knowledge to interpret the meaning of fractions, which creates conflict between students' prior knowledge about whole numbers and unfamiliar ideas about fractions (e.g., Cramer et al., 2002; Cramer & Wyberg, 2009; Siegler et al., 2011). Consequently, whole-number principles and operations (e.g., a bigger number indicates a larger amount) is thought to interfere with learning fractions, resulting in widespread difficulty with initial fractions concepts. This is in part supported by our finding that a considerable percentage of students (i.e., 18%) did develop fractions difficulty despite having adequate whole-number competence.

However, our findings do not support the widespread assumption that initial learning about fractions concepts present similar challenges for students regardless of their whole-number competence. Instead, they suggest that initial fractions understanding develops in accord with whole-number competence: Relatively few fourth graders with initially adequate whole-number knowledge experience difficulty in fractions understanding, and fourth graders with weak whole-number knowledge experience such difficulty at a dramatically greater rate. This is consistent with and extends the individual differences literature, which found that whole-number skill concurrently and longitudinally predicts fractions calculations (Bailey et al., 2014; Hansen et al., 2015; Hecht et al., 2003; Jordan et al., 2013; Vukovic et al., 2014).

Our findings further indicate that whole-number skill is a key predictor of fractions understanding and that a strong foundation in whole numbers facilitate, rather than hinder, initial fractions understanding. Our findings, combined with the individual difference literature, suggest that mastery of whole-number skill is foundational to fractions understanding in addition to fractions calculations. Results also lend further support to the integrated theory of numerical development proposed by Siegler et al. (2011), who argue that fractions knowledge develops as students broaden their understanding of whole numbers to accommodate the meaning of fractions magnitudes. That is, whole-number and fractions understanding are understood within an integrated number system that requires magnitude understanding, not as separate number systems.

Of course, the individual differences literature is not entirely consistent on whole-number capacity as a predictor of fractions understanding (e.g., Hecht et al., 2003; Hecht & Vagi, 2010; Namkung & Fuchs, 2016). Moreover, another potential explanation for the link between whole-number and fractions understanding is that they may be associated via a third variable, such as underlying cognitive processes responsible for both forms of learning. In line with this idea, Namkung and Fuchs (2016) found that distinctive cognitive processes underlying whole-number versus fractions competence (e.g., language for fractions and working memory for whole-number competence), even as they documented that whole-number and fractions magnitude understanding rely on nonverbal reasoning. So, it is possible that difficulty with whole-numbers and fractions understanding are connected via limitations in the cognitive resources or reflect other cognitive processes that commonly underlie many forms of mathematics competence. A third possibility is that competence with whole numbers frees up other resources, such as working memory and attention, to be devoted to fraction tasks that are more complex and require more steps. In a related way, one limitation of the present study is that we did not have data on other relevant factors such as general intelligence and competence in other foundational mathematics skills (e.g., number sense, magnitude understanding), which may explain the link between students' whole-number and fractions understanding. Further studies are needed on identifying potential mediators and moderators of the relation between whole-number and fractions understanding.

With respect to the specific whole-number calculation skills that were most directly related to fractions understanding, our findings suggest two items suggest connections between whole-number and fractions understanding: basic division facts (2-digit dividend ÷ 1-digit divisor, such as 10 ÷ 2) and simple multiplication (3-digit × 1-digit without regrouping, such as 421 × 2). Even after controlling for responses to other items, students who missed these items were about twice as likely to experience difficulty in fractions understanding than students with correct responses. As we had hypothesized in addition to the prior findings indicating that division skill plays an important role in fractions learning, it is not surprising that competence with whole-number division predicts difficulty in fractions understanding. The close relation between whole-number division and fractions learning may be explained by the fundamental equivalence of whole-number division and fractions. Any fraction can be defined as a quotient, representing a numerical value obtained by dividing one whole number by another (Siegler & Pyke, 2012). Besides, whole-number division is the only operation that yields fractions (e.g., 12 ÷ 5 = 2 2/5).

Moreover, when students initially form ideas about fractions, it is often based on equal-sharing, in which students divide whole amounts equally into groups. For example, 12 cookies divided equally among 4 friends is represented as 12 ÷ 4, which is also equivalent to 12/4. This equal-sharing of whole amounts provides the basis for initial understanding of part-whole relations, which are further developed via common unit fractions encountered when first learning fractions (e.g., 1 divided equally into 3 parts represents 1/3). Therefore, skill with whole-number division may provide a basis for and reflect understanding of fractions.

It is, however, interesting that simple multiplication skills (multi-digit by single-digit multiplication without regrouping) may be related to fractions difficulty. This runs contrary to our hypothesis, and it is less obvious how simple multiplication skill (compared to division skill) may be related to fractions difficulty. One potential link between whole-number multiplication and fractions may be the multiplicative nature of creating units (Behr, Harel, Post, & Lesh, 1994; Thompson & Saldanha, 2003), in which 3/7 is understood as "three 1/7." This supports more complex interpretation and understanding of fractions, such as fractions exceeding 1 (e.g., 7/3 represents "seven 1/3"), in which a part-whole and repeated addition interpretation do not easily make sense. Multiplicative reasoning has been found to play a key role with more advanced mathematics competence, such as fractions, proportional reasoning, and algebra (NMAP, 2008; Van Dooren, De Bock, & Verschaffel, 2010).

One might wonder why other whole-number division items on WRAT-4 were not predictive of fractions competence. We remind readers that the unique effects of 18 whole-number calculation items were assessed simultaneously. So although other items did not uniquely predict fractions difficulty after controlling for the other response, they may still have a bivariate relation to fractions difficulty. Additionally, we remind readers that due to the exploratory nature of our analysis, additional studies are warranted to confirm that basic division facts and simple multiplication uniquely predict fractions difficulty. With that in mind, item-level descriptive analysis suggests that the predictive utility may be related to the range of skills students have or have not mastered at the start of fourth grade. Other WRAT-4 division and multiplication items represent more complex division skills beyond the scope of skills mastered at the beginning of fourth grade (e.g., multi-digit division with a remainder, multi-digit multiplication with regrouping). The percentages of students in our sample who correctly answered such items on the WRAT-4 ranged from 0 to 3.6%.

In sum, our findings provide evidence that initial fractions learning is less challenging for students with strong whole-number competence. That is, students with poor whole-number skill are far more likely to develop fractions difficulty compared to those with strong whole-number skill, which suggests that whole-number competence plays a foundational role in initial learning of fractions. More specifically, basic division facts and simple multiplication skill may be important predictors of successful fractions understanding. This provides some insight on the nature of interventions that may help students learn fractions. First, given the importance of the whole-number competence in subsequent learning of fractions, early intervention to ensure strong whole-number competence before fractions are introduced seems warranted. Also, because basic whole-number division facts and simple multi-digit multiplication may reflect important connections with fractions understanding, interventions should focus on strategies to improve students' understanding and operational skill with these forms of whole-number skill and to explicitly help students make connections between these whole-number operations and fractions understanding. Further research is warranted on the effects of explicitly connecting whole-number division and multiplication, as they are introduced in the primary grades, to key fraction ideas.

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