USING PRACTICAL WORKSHEET TO RECORD AND EXAMINE METACOGNITVE STRATEGIES IN PROBLEM SOLVING

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We adopted Brown's (1987) conceptualisation of metacognition to examine how student teachers can be taught metacognitive control while solving mathematical problems. In addition, tasks given to these student teachers were in-built with opportunities for them to be aware of the need for metacognition. We describe the use of the Practical Worksheet as a way to make visible their metacognition, within the context of solving mathematics problems. Findings suggest that the greater awareness of control due to the use of the Practical Worksheet contributed to the greater employment of control in subsequent problem solving.

INTRODUCTION AND LITERATURE REVIEW

Metacognition is an important idea in the teaching of problem solving. However, its definition has remained elusive. Some have interpreted it as "thinking about thinking" (Lai, 2011; Larson, 2007). Others have attributed this difficulty to the fact that several terms such as self-regulated learning, reflective learning, executive control, meta-memory and monitoring are used interchangeably with metacognition; for example, Holton and Thomas (2001) view students' ability to carry out self-interrogating and using self-scaffolding in problem-solving as metacognition.

Brown (1987) conceptualised metacognition as having two components: the knowledge (what one knows about one's cognition) and the control (what one does to regulate one's cognition). Metacognitive knowledge refers to three different types of knowing: declarative knowledge (about one's skills and intellectual resources); procedural knowledge (about how to execute procedural skills and apply strategies) and conditional knowledge (about when and why to use declarative and procedural knowledge). The control component refers to the actual metacognitive control actions applied to cognitive processes – they are planning (such as goal setting and allocation of resources), monitoring (assessing one's strategy used) and evaluating (appraising the products and efficiency of learning). Metacognitive knowledge and control do not function separately or independently; rather, they complement each other for a person to achieve optimal performance.

According to Desolete (2007), we cannot assume that metacognitive skills will develop in the mathematics classrooms. Veenman, Van Hout-Wolters and Afflerbach (2006) argued that in order for students to develop metacognitive skills, it is crucial that teachers model metacognitive skills since learners acquire metacognitive skills through implicit socialization with experts.

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In the study reported here, we adopted Brown's (1987) conceptualisation of metacognition to examine how student teachers can be taught metacognitive control while solving mathematical problems. In addition, tasks given to these student teachers were in-built with opportunities for them to be aware of the need for metacognition. In the next section, we describe the use of the *Practical Worksheet* as a way to make visible their thinking, including metacognition, within the context of solving mathematics problems.

THE PRACTICAL WORKSHEET

In Schoenfeld's (1985) framework of mathematical problem solving, control is listed as one of four components essential for success. This idea of the importance of control, together with the well-known Pólya's (1954) model of problem solving, formed the theoretical basis for our design of the *Practical Worksheet* (PW).

The PW consists of four pages exactly corresponding to the four stages of Pólya: (1) Understand the Problem; (2) Devise a Plan; (3) Carry out the Plan; and (4) Look Back. Through the Look Back (which we renamed "Check and Expand") stage, the problem solver may revisit the solution, check the reasonableness of the answer/solution, look for alternative solutions to the problem, and make changes/extensions to the solution; in the process, this looping back is a location where metacognition can be identified. In addition, a "Control" column was added in the Stage 3 so that any conscious metacognitive acts that are utilised can be recorded.

THE PARTICIPANT AND METHOD

One of the authors (hereafter referred to as the "tutor") taught the mathematics methods module in the Postgraduate Diploma in Education (PGDE) programme. The PGDE is a pre-service teacher education programme. This module is taken by university graduates in Mathematics or in a Mathematics-related discipline such as Engineering who are seeking certification to become a secondary school mathematics teacher. Six hours of this 24-hour module are devoted to the idea of teaching of problem-solving and the teaching of mathematics *through* problem solving.

Twenty-two pre-service teachers (PT) participated in this study. The tutor started by explaining what a mathematical problem is, emphasizing that it is different from a routine exercise and that it requires time and effort to solve. The tutor modelled the processes of problem solving before he discussed in detail Pólya's model and Schoenfeld's framework for problem solving. He then demonstrated how the PW should be used by working it through with a specific problem. During the first problem solving lesson, Example 1 was used to explain how the practical worksheet could be used to guide one towards problem solving.

> <u>*Example 1*</u> *: ABC is an equilateral triangle. P is a point inside the triangle such that the distances from its three sides are 4, 5 and 6 cm. Find the length of one side of the triangle.*

Before concluding the problem solving part of the course module (which formed the first three lessons), the tutor presented two other problems to be solved in the next two tutorial sessions.

> *Problem 1 : The coordinates of a given point A are (6, 2). Find a point B on the line y = x and another point C on the x-axis such that the perimeter of the triangle ABC is minimum. Find the coordinates of point B and C.*

At this point, it was found that almost all students had not paid much attention to noticing the thought processes involved in solving the problem. The tutor discussed Problem 1 again in class and attempted to get the student teachers to be more aware of their own thinking and to focus more on the metacognitive control action for the next problem. This was followed by another tutor demonstration on how the PW could be used to guide their thinking. However, one of the PTs mentioned that he needed time to examine and write down what was actually happening in his mind while solving a problem. In response to this request, the tutor gave 10 more minutes (making it a total of 30 minutes) for them to solve Problem 2 in their last problem solving lesson.

> *Problem 2 : An equilateral triangle ABC with sides 4 cm is inscribed in a circle. If a point P lies on the minor arc BC, find the value of* $PA^2 + PB^2 + PC^2$.

For both problems, the PTs had to solve the problem using the PW in class within the stipulated time. They knew that their performance in the two problem-solving sessions would not be graded.

METACOGNITIVE STRATEGIES DEMONSTRATED IN THE PROSPECTIVE TEACHERS' WORK

In this section, we shall present using some PT's solution of Problem 2 and examine them for evidence of the use of metacognitive strategies in their solution.

For Metacognitive *planning* activity, we looked for evidence from the PW on statements about possible cognitive resources and heuristics that may be involved in solving the problem. For example, drawing a diagram is a common problem solving heuristic. While it is usually considered a cognitive rather than metacognitive behavior, we think that a diagram can also become a vehicle for stimulating metacognitive strategies, leading to a useful insight of the problem. In the case of Problem 2, a diagram can be seen as a tool for metacognitive planning in this way: when point P is moved along the arc BC to coincide with either points B or C, this special case reveals a way forward in the solution strategy. Using the PW, all except one PT indicated in writing what they planned to do for Stage 1 when they attempted to solve Problem 2. Figure 1 shows one of the PT's metacognitive planning strategies.

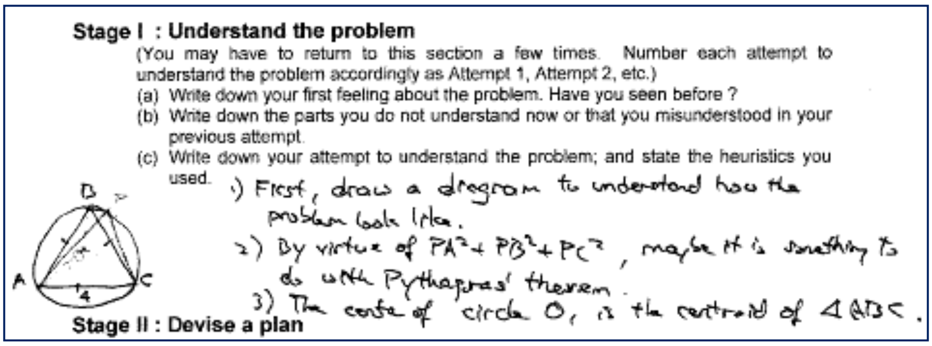
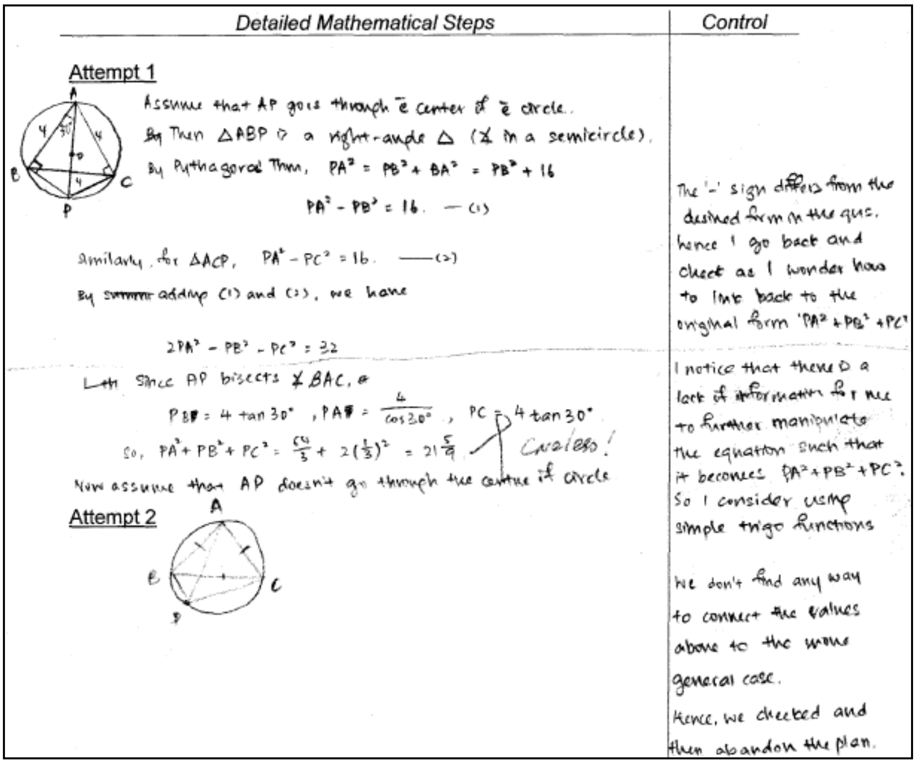

Stage I : Understand the problem
(You may have to return to this section a few times. Number each attempt to understand the problem accordingly as Attempt 1, Attempt 2, etc.)
(a) Write down your first feeling about the problem. Have you seen before ?
(b) Write down the parts you do not understand now or that you misunderstood in your previous attempt.
(c) Write down your attempt to understand the problem; and state the heuristics you used.
1) First, draw a diagram to understand how the problem looks like.
2) By virtue of $PA^2 + PB^2 + PC^2$, maybe it is something to do with Pythagoras' theorem.
3) The centre of circle O, is the centroid of $\triangle ABC$.
Stage II : Devise a plan

Figure 1: PT4's metacognitive planning strategies.

There is also evidence for metacognitive *monitoring* activity. This is illustrated in PT11's work as shown in Figure 2. She used Pythagoras' Theorem and did not manage to obtain the form required by the question. She switched her solution path to using trigonometric ratios to find *PA*, *PB* and *PC* correctly (though she made a slight computational mistake). This is a demonstration of the exercise of metacognitive monitoring strategy, in which the solver constantly monitors his or her solution plan and is ready for error-detection and correction, and to self-question if the current approach is on the correct path leading to the correct solution.



Detailed Mathematical Steps	*Control*
Attempt 1 Assume that AP goes through the center of the circle. Then $\triangle ABP$ is a right-angle $\triangle$ ($\angle$ in a semicircle). By Pythagoras Thm, $PA^2 = PB^2 + BA^2 = PB^2 + 16$ $PA^2 - PB^2 = 16.$ — (1) Similarly, for $\triangle ACP$, $PA^2 - PC^2 = 16.$ — (2) By adding (1) and (2), we have $2PA^2 - PB^2 - PC^2 = 32$	The '–' sign differs from the desired form in the qns. hence I go back and check as I wonder how to link back to the original form '$PA^2 + PB^2 + PC^2$'
Since AP bisects $\angle BAC$, $PB = 4\tan 30°$, $PA = \frac{4}{\cos 30°}$, $PC = 4\tan 30°$ So, $PA^2 + PB^2 + PC^2 = \frac{64}{3} + 2(\frac{1}{3})^2 = 21\frac{5}{9}$ Careless! Now assume that AP doesn't go through the centre of circle Attempt 2	I notice that there is a lack of information for me to further manipulate the equation such that it becomes $PA^2 + PB^2 + PC^2$. So I consider using simple trigo functions We don't find any way to connect the values above to the more general case. Hence, we checked and then abandon the plan.

Figure 2: PT11's metacognitive strategies in the "control" column.

Figure 3 shows PT8's careless mistake as part of the solution to Problem 2. As he did not proceed further on the PW, the tutor spoke with him after the activity about how he would have proceeded if he were given the additional time. He mentioned that he would have checked his working again before abandoning his solution. We think that this is an example of the use of metacognitive *evaluating*.

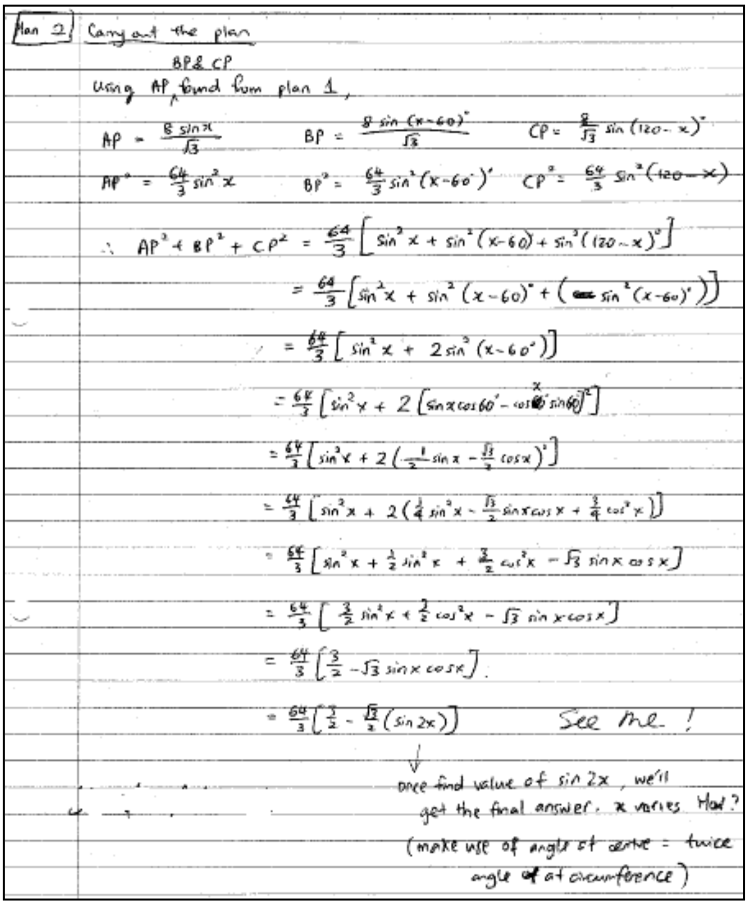


Figure 3: Metacognitive action by PT8. Note that $x = \angle ACP$

The PWs were examined and marked. Metacognitive strategies written on the PWs were coded and classified into metacognitive planning, metacognitive monitoring and metacognitive evaluating. Table 1 shows the rules for classification.

Metacognitive activity or strategy	Prescriptions	Sample written responses on the Practical Worksheet for Problem 2
Planning	Using heuristics to make sense of the problem.	PT1: trying to find the distance using sine or cosine rules.
	Stating goal and sub-goal of Indicating of possible cognitive resources that may solve the problem	PT5: Draw and label the diagram to translate the problem into a pictorial form.
Monitoring	Indicating the need for answering to the question Indicating of the solution steps make sense	PT11 : … I wonder how to link back to the original form $PA^2 + PB^2 + PC^2$
	Indicating of the 2nd approach	PT19: Stuck at this point, go back to Stage II.

Evaluating	Indication of a problem or error encountered Indication of possible short-coming of the method	PT7. I do not know how to extend – my method does not seem to be able to work for other regular polygons. PT8: Once (I) find the value of sin2x, we'll get the answer. x varies. How ?

Table 1: Summary of the coding scheme for metacognitive activity during mathematics problem solving.

Summary of Data

Table 2 shows the percentages of the three different metacognitive strategies used in solving Problems 1 and 2.

	Metacognitive planning	Metacognitive monitoring	Metacognitive evaluating	Correct answer (out of 22)
% of PT (Problem 1)	90.1%	27.3%	13.6%	9.1%
% of PT (Problem 2)	95.5%	72.8%	50.0%	22.7%

Table 2: Percentage of PTs demonstrating different metacognitive strategies outlined in Table 1.

Stage 1 of the PW provided the PTs an avenue to express in writing how they understand the problem and the plan to solve it—this explains the very high percentage of them (90% for Problem 1 and about 96% for Problem 2) demonstrating the metacognitive planning strategy. Although the percentage of PTs demonstrating metacognitive monitoring and metacognitive evaluating strategies were not high for both problems, especially for Problem 1, we are glad to see that there was an increase in the number of PTs displaying their metacognitive monitoring and evaluating strategies in solving Problem 2 (see Table 2). While this may be attributable in part to the PTs being more familiar with the use of PW to record their metacognitive strategies during problem solving, we think that the additional 10 minutes that were given to the PTs (for Problem 2) also allowed them to record their metacognitive processes on the PW. When we examined the PWs of those who obtained the correct answer for Problem 2, we noticed how PT18—who didn't write about her metacognition at all for Problem 1 (she could not solve Problem 1)—were able to solve Problem 2 correctly. In addition, in the process of solving it, she revealed the metacognitive traces that could have helped her: realizing that her initial conjecture of "$PB + PC - PA = 0$" in her working may be correct but was unable to prove it, she wrote that there "may be another way of calculating area of triangle ABC to develop the relationship" (the relationship she meant is $PB + PC - PA = 0$). As it turned out, that metacognitive

evaluating act directed her to more productive ways of exploring the relationship among the three sides and solve the problem successfully.

DISCUSSION AND CONCLUSION

Research has shown that if students have gone through some metacognitive training, they could improve their ability in mathematics problem solving (Jacobse and Harskamp 2009). But the issue of measuring thinking-related variables such as metacognition accurately and effectively is still an issue of concern and it remains an ongoing area of research. In addition, there are debates and discussions about the suitability of instruments. Think-aloud protocols have been commonly used in measuring metacognition (Ku & Ho, 2010; Veenam et al, 2006) but the actual implementation of the measuring process is time-consuming and complex and thus less practical. The search for alternative instruments that are more classroom-friendly and that helps students more directly continues.

From the results of this exploratory study, we think that the PW has the potential to aid learners keep track of their ongoing metacognitive behaviour and strategies used during the whole problem solving process. The PW ‘forces’ them to ‘think-aloud’ in the written form and teases out quite a good variation of metacognitive activities that were going on in problem solving process. Thus the PW can make metacognitive behaviour more visible, allowing the learners as well as the teachers to have information about the thinking processes. This information can, in turn, feedback to the problem solver and the teacher of problem solving.

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