VIABLE ARGUMENTS, CONCEPTUAL INSIGHTS, AND TECHNICAL HANDLES

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Findings from an empirical study on prospective elementary teachers' argument productions are reported. In order to analyse the data, a generative study was conducted to develop a framework for expressed actions that afforded the communication of viable arguments for generalizations. Identified are three types of technical handles that appear constructive in communicating viable arguments. Inappropriate and inadequate technical handles are also noted.

STATEMENT ABOUT THE FOCUS OF THIS PAPER

This research builds on and recasts previous work regarding conceptual insights (CI) and technical handles (TH) during argumentation. Raman, Sandefur, Birky, Campbell, and Somers (2009) identify *getting key ideas, discovering THs*, and *culminating the argument information into standard form* as significant "moments" important in creating a proof, in no particular order. Sandefur, Mason, Stylianides, and Watson (2013) recast the framework as "(1) finding a [CI], i.e., a sense of a structural relationship pertinent to the phenomenon of interest that indicates why the statement is likely to be true, and (2) finding some [THs], i.e., ways of manipulating or making use of the structural relations that support the conversion of the CI into acceptable proofs" (p. 328).

Based on empirical data generated by prospective elementary teachers (PSTs), who participated in a teaching experiment designed to improve argumentation skills, I found that I needed to again recast the TH framework. My objective was to describe PSTs' communicated argument as an expression of their "handling" of information, not necessarily to describe the processes PSTs go through or experience when addressing a prompt/task and developing an argument. While the concept of a CI remanded similar to that express in Sandefur et al, I found three related but distinct types of THs important to communicating a viable argument. While the offer of a new framework aligns with a theoretical essay, this article is a research report because it uses empirical data to establish that each of these handles is important to communicating a viable argument.

The presentation of these findings is intended to deepen our understanding of student actions that influence viable argument production and to promote further research in this area. My aim is to make international contacts with other researchers, who might benefit from these findings and who might wish to collaborate on future work.

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THEORETICAL FRAMEWORK FOR THE RESEARCH

Viable argument defined for this study

For this study, an argument is defined as viable if it (1) expresses a clear, explicit, and unambiguous claim with explicit conditions and a conclusion, (2) expresses support for that claim that involves acceptable data (foundation), (3) expresses acceptable warrants that link the data to the claim, and (4) identifies the mathematics on which the claim relies.

Criterion 2, acceptable data/foundations, is met by expressing/representing information or insights in a manner that (a) illustrates the conditions in the claim, (b) can be used to represent all cases in the domain of the claim, and (c) can be appealed to when connecting the data to the claim. Acceptable data can include examples, diagrams, prior results, axioms, definitions, narrative descriptions, stories, etc. provided that the three criteria are met.

The connection between the data and claim referred to in acceptable data criterion (c) is the warrant and is deemed "acceptable," viable argument criterion 3, when it expresses how the data is used to support the claim and takes into account all cases to which the claim applies. Using purely empirical warrants to support a generalization that applies to an infinite set is not acceptable. Using logical necessity, referencing prior results, and describing through narrative how the semantic meaning of mathematical objects support the claim can all be acceptable warrants.

Criterion 4 is defined as the meaning of the objects and operations involved in the claim and those meanings are determined by definitions, axioms, and theorems. Viable arguments must express these meanings, at least semantically, for the argument to be viable.

This framework for viable argument was developed by the author from existing frameworks found in Toulmin's (1958/2003) argument analysis scheme and the Common Core State Standards' (CCSS-M, 2010) description of the mathematical practice number 3, construct viable arguments and critique the arguments of others. Consistent with Toulmin, an argument is defined as a claim and its support. Other argument features in Toulmin's scheme include data that provide facts and other information that support the claim, warrants that link data to a claim, backings that give support for why a warrant should be accepted, qualifiers that give the strength of a claim, and rebuttals that give circumstances, cases, or facts under which the claim would not be true. For this study, I focus my attention on claims, data, and warrant, which Krummheuer (1995) calls the core of the argument, and I often combine warrant and backing into a single category labelled warrant, unless the arguer is clearly addressing the acceptability of an existing warrant.

Consistent with CCSS-M (2010), viable arguments can use referents such as objects, drawings, diagrams, actions, and in this study, examples, as the data/foundation of a viable argument. The term viable argument is not explicitly defined in CCSSM (2010),

but I make the assumption that this term is used instead of "proof" to note that there are plausible argument types other than formal, mathematically logical ones, and to draw distinctions between proofs and non-proofs. Consequently, my framework for viable argumentation leverages notions of less-than-formal mathematical arguments (e.g., Balacheff's, 1988, notion of generic example proofs), which some authors have called proofs, but I classify them as viable arguments in an effort to value them but not confuse them with mathematical rigorous arguments that are explicit about the logic and prior results used.

Conceptual insights and technical handles

For this study, the term conceptual insights (CI) can refer to any one of the following: developing a sense or belief based in pertinent mathematical structure that a claim is true or false, developing a sense or belief based in pertinent mathematical structure about what might be claimed (stated as true), or developing a sense or belief based in pertinent mathematical structure about why a claim is true or false or what causes the claim to be true or false.

As mentioned earlier, I found it necessary to recast earlier descriptions of THs (e.g., Raman et al, 2009 & Sandefur et al, 2013) to describe my data. My analysis only addresses the articulated argument, not what the arguer intended to write or say. Nor does my analysis attempt to document the processes through which the ideas expressed in the argument are generated. This focus helped me draw clearer distinctions between CI and THs and offered a purer description of what the arguer was able to express appropriately and viably when culminating ideas, findings, and insights into an argument product. From this narrowed focus, I was able to identify three distinct but related "handles." Technical handles of type 1 (TH1) describe the articulated claim in relation to the expressed data, CIs, or warrants. Technical handles of type 2 (TH2) describe how the data or CIs are expressed. Technical handles of type 3 (TH3) describe the expressed link between the data or CIs and the claim (i.e., warrant). THs are first described without any connotation of whether or not the handles are constructive. Adjectives (e.g., appropriate, inappropriate, adequate, inadequate) are applied to note a TH's subtype and potential for viable argumentation. This framework will be further exemplified in the results section.

DATA SOURCES AND JUSTIFICATION FOR ANALYSIS METHODS

Twenty-one PSTs enrolled in a undergraduate mathematics content course for elementary school teachers participated in the study. PSTs were given the definition of viable argument as presented in Section 2 and received two months of training and practice using the definition to construct and critique arguments. Five sources of data were collected and used in this study: (1) students' weekly posts in the online environment (2) teacher/researcher observations during inclass work (3) student writen responses to inclass tasks (4) task-based, clinical interviews, audio taped and transcribed, and (5) responses on paper-and-pencil assessments.

For the online posts, students were instructed that to receive full credit, they must initiate a discussion by making a claim or respond to another student's post. At minimum, a post needed to present at least one argument feature (a claim, data for a existing claim, a warrant, a backing for an existing warrant, a qualifier, or a rebuttal), but they were also told that this was a minimal criterion and that a complete argument was desired in each post. This concept of collective argumentation has been discussed in Krummheur (1995), Lampert (1992), and Yopp and Ely (under review), to name a few.

Data was analysed from the perspective of a *generative study* (Clements, 1999) to develop or recast an existing theoretical model to explain data. Qualitative analysis methods were akin to those described by Miles and Huberman (1994) in which the analyst begins with a theoretical coding framework that is constantly compared to the data until a model that fits the data emerges. A cyclic process of analysis, refinement, and reanalysis was used to test emerging the framework and outcome of the data analysis (similar to the methods described in Goldin, 1999, & Sandefur et al., 2013). As conceptual themes emerged, the themes were verified through triangulation with multiple data sources. For example, as student posts were memoed, emerging themes were triangulated with task-based interview data to confirm TH codes.

SAMPLE OF DATA AND RESULTS

A sample of results are reported for data collected from one of the online tasks:

Task 1: You are teaching a 6^{th} grade class. You ask the class to investigate the sums of consecutive numbers and develop some rules about the types of numbers that are sums of 2, 3, 4, and 5 numbers. After some set time, three students offer rules. Sally says that the sum of two consecutive numbers is odd. Sophia says that the sum of three consecutive numbers is divisible by 3. Isabella says, "I think that the sum of four consecutive numbers is divisible by 4". Write exemplary responses, which include viable arguments.

Adequate and inadequate THs

In this thread we find the same claim coded as an inappropriate TH1 in one post yet coded as appropriate TH1 in another post. We also find a follow-up post in which the arguer expresses appropriate or adequate THs for all three argument features and presents a viable argument.

Charli: Data for Claim #3: My example of the claim: 1+2+3+4=10 or 4+5+6+7=22 supports the claim that says: "The sum of four consecutive counting numbers is not divisible by four" because shown here proves that these consecutive counting numbers are not divisible by four.

Alex: Warrant for Claim 3: The sum of four consecutive counting numbers will not be divisible by four, because there is one multiple of four, but the other numbers do not add to equal a multiple of four. 1+2+3+4= 10[.] 4+6, only

the four is divisible by four, not the six[.] 5+6+7+8=26[.] 8+18, only the eight is divisible by 4[.]

Franni: I think that a better claim for this problem would be that for any 4 consecutive numbers added together, the sum will always be divisible by 2. The data supporting this would be: 2+3+4+5=14 which is divisible by 2, 10+11+12+13=46 which is divisible by 2, and 23+24+25+26=98 which is divisible by 2. The warrent [sic] would be: For any 4 consecutive numbers, N, N+1, N+2, N+3 [data] added together, the sum will always be divisible by 2 because when added you have N+(N+1)+(N+2)+(N+3) which can be written as 4N + 6. 4N is divisible by 2 and 6 is divisible by 2 so no matter what number N is, the answer will always be divisible by 2.

In the first post in this thread, Charli labels her post "data" but has presented an argument because both a claim and support are present. She presents empirical support and no CI about the generalization that no sum of four consecutives is divisible by 4. Although her claim is true, it is not appropriate with respect to the data. Charli's use of the word "proves" raises concerns of empiricism. However, follow-up cognitive interviews revealed that Charli wished to express "generality" in her warrant.

I mention "empiricism" and Charli's perspective to again to draw attention to the focus of this paper. The approach here is to analyse what students are able to articulate against a class standard for viable argument, not to analyse their views about adequate support. While it has been established in the literature that a PSTs' beliefs about what constitutes adequate support for a generalization can influence their argument production and their ratings of arguments (Martin & Harel, 1989; Sylianides, G.J. & Stylianides, A.J., 2009), other literature suggests that students who know the limitations of empirical evidence produce empirical evidence when they aren't able to produce better (e.g., more general) arguments (Sylianides, A.J. & Stylianides, G.J., 2009). There is a gap in the literature concerning the skill of argument production as separate from beliefs. The focus of this paper is to examine the argument product from a "handling" point of view as a technical skill, rather than an exploration of what a PST finds convincing. A PST with the appropriate TH training might express a more appropriate TH1 by stating "Based on the examples we can claim that at least some of the sums are not divisible by 4." With this change, Charli's argument would be viable.

This *skill* of making appropriately claims might be unique from the PST's views about sufficient support for claims. This makes this framework unique from the previous studies that gave their participants a claim, typically assumed true, and examined the participants' support for that claim. In this framework, the truth of the claim made by the PST is not the focus. Because PSTs must develop a claim, the focus is the claim's appropriateness relative the data presented.

In contrast to the inappropriate TH1 expressed by Charli, Alex, who responsed to Charli's post and wrote basically the same claim, expresses an appropriate TH1. This is because Alex expresses a CI that applies, at least in her mind, to all cases. Her claim is appropriate relative to the general nature of her insight, regardless of whether the

insight pans out as adequate support for the claim. Whether or not her insight has potential for an adequate representation (which it does) is not the point when assigning a TH1 code. This code refers only to whether or not the formation of the claim is prudent, and whether or not it is appropriately worded, expressing both the conditions/domain and the conclusion.

Yet, despite Alex's appropriately worded claim and interesting insight, Alex does not handle her insight adequately to communicate a viable argument or a viable argument features. It is noted that Alex labels her post as warrant, but it is also noted that warrants connect data to a claim. By default data must be present for a warrant to exist, either in the current post or a previous post. Thus, an argument is present in Alex's post.

There are at least two ways we might reconstruct the argument. One way is label the Alex's examples and observations about these examples as data and label Alex's statement "because there is one multiple of four, but the other numbers do not add to equal a multiple of four" as the warrant. Another way is to label the "because…" statement as both the data and implicit warrant and the examples as backing for the truth of the data and warrant. (A criticism of Toulmin's analysis is that multiple plausible arguments may be constructed, Aberdein, 2005.) Regardless of how we reconstruct the argument, the CI in the "because" statement is not expressed in a manner that can be appealed to generically. The expression or representation does not reveal how we know all sums of four consecutives have the two mentioned properties: that one summand is a multiple of 4 and the others do not sum to a multiple of 4. Consequently, Alex expresses an inadequate TH2.

Alex need not use a variable to represent these insights. Examples can be useful when crafting the arguments (Balacheff, 1988, Sandefur et al., 2013) and their concreteness makes them particularly accessible to PSTs. An example becomes a referent in an argument if the arguer uses the example to illustrate objects or relationships when supporting a claim. Despite their utility, examples are at times used in ways that do not lead themselves to viable argumentation (Balacheff, 1988; Healy & Hoyles, 2000).

Using examples to construct viable arguments requires that the pre-service teachers' develop CIs and are able to handle the data or insight in appropriate and useful ways (Raman et al., 2009; Sandefur et al., 2013). For a generic example argument, Alex might need several examples to argue all cases (e.g., the multiple of 4 as the first, second, third, or fourth summand). Alex might even develop "sub-arguments" to establishes each of the two properties separately. Never-the-less, the condition "sum of 4 consecutives" must be represented adequately for viable argumentation

In contrast, Franni communicates a viable argument in her response to Alex's post. Franni expresses an appropriate TH1 and adequate TH2 when she expresses a claim about divisibility by 2 and supports this claim using a variable to represent the claim's conditions as N+(N+1)+(N+2)+(N+3). The adequacy of this representation for viable argumentation is affirmed when Franni appeals to its equivalent form 4N+6 and notes

that both 4N and 6 are divisible by 2, expressing an adequate TH3. Admittedly, there are parts of the argument that could be improved: there are a few typos and the arguer should explicitly mention the "on the list" prior result that *if both summands are divisible by an integer, then sum is as well*, but the former issue is easily fixed through editing and the later is an issue of community norms about sufficient detail.

Franni offers another opportunity to reflect on the utility of the focus on THs independently from a PST's views about types of argument structure and sufficient support for generalizations. Franni presents several examples that she calls data prior to presenting her generic representation, which she calls the warrant. A cognitive interview revealed that Franni, like her group members, viewed the term "data" as numerical information as in a scientific experiment. This view was different from the way "data" was defined and used in class. Thus, despite her non-canonical expression of the argument structure, globally her data and CI are handled sufficiently well for viable argumentation.

CONCLUSION

The purpose of this work is to establish three types of THs as important for communicating a viable argument in response to a claim. In order to communicate a viable argument, as defined in this study and for this particular community, PSTs need to (1) communicate a claim that is appropriately worded, expressing the conditions and conclusion, and is prudent based on their data or CI; (2) express their data or CI in a manner that can be appealed to appropriate in a warrant that connects the data to the claim; and (3) express a warrant (and possibly backing) that appeals to the data, indicates how all cases in the domain of the claim are considered/expressed, and identifies the mathematics on which the claim relies.

This work is unique from previous work because it focuses solely on what PSTs are able to communicate, not on what PSTs might intend to communicate, and because it demonstrates three distinct, although related, THs. An arguer may adequately or appropriate express any one of the handles without doing so with respect to the other two. This separation allows researchers and educators to focus on students' abilities and deficits during argumentation as technical skills (e.g., given this type of data, examples, or counterexamples, what types of claims are appropriate?).

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