ARGUMENTATION IN UNDERGRADUATE MATH COURSES: A STUDY ON PROBLEM SOLVING

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The purpose of this study is to analyze the complex argumentative structure in undergraduate mathematics classroom conversations during problem solving by taking into consideration students' and teacher' utterances in the classroom using field-independent Toulmin's theory of argumentation. Analyzing students' and teacher' utterances in the class allowed us to reconstruct argumentations evolving in the classroom talk as argumentations in classrooms are generally teacher guided. The analyses contributed to an emerging body of research on classroom conversations.

INTRODUCTION

Problem solving requires argumentation (Cerbin, 1988). Argumentation is a process of making claims and providing justification for the claims using evidence (Toulmin 2003; Mejia-Ramos & Inglis, 2009; Knipping, 2008). On the other hand, argumentation is a verbal and social activity of reason aimed at increasing (or decreasing) the acceptability of a controversial standpoint for the listener or reader, by putting forward a constellation of propositions intended to justify (or refute) the standpoint before a rational judge' (van Eemeren et al., 1996). Argumentation requires problem solvers to identify various alternative perspectives, views, and opinions; develop and select a preferred, reasonable solution; and support the solution with data and evidence (Voss, Lawrence, & Engle, 1991).

Toulmin's model has provided researchers in mathematics education with a useful tool for research, including formal and informal arguments in classrooms (Knipping, 2008) as it is intended to be applicable to arguments in any field. Studies using Toulmin model focused on analyzing students' arguments and argumentations in proving processes in a classroom (Knipping, 2002, 2008; Krummheuer, 1995) and, individual students' arguments in proving processes (Pedemonte, 2007). Toulmin himself noted that his ideas has no finality. Indeed his model has been reshaped in various ways, his claims have been contested by some and in response reformulated by others, and some but not all aspects of his approach have been incorporated in applications in different domains (Hitchcock & Verheij, 2006).

Having established these facts, the goal of our research is to study the argumentation in undergraduate mathematics classrooms during problem solving using Toulmin's theory of argumentation. Specifically, the aim is to analyze the structure of the arguments accomplished in the course of interaction where the teacher and students involvement in this accomplishment. This study is part of a wider study investigating

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the argumentation generated in undergraduate mathematics classes while proof generation (see Ubuz, et al., 2012), definition construction (see Ubuz et al., 2013), and problem solving. This paper suggests a method by which complex argumentation in problem solving can be reconstructed and analyzed. Analyzing students' and teacher' utterances in the classroom according to Toulmin model allows us to reconstruct argumentations evolving in the classroom talk since arguments are produced by several students together with the guidance of the teacher.

THEORETICAL FRAMEWORK

In the following sections we will expose some theoretical considerations on the Toulmin model, and the problem solving process.

The Toulmin Model

According to Toulmin, an argument is like an organism. It has both a gross, anatomical structure and a finer, as-it-were physiological one (Toulmin, 2003). He is interested in the finer structure. The Toulmin model is differed from analysis of Arisitotle's logic from premises to conclusion. First, we make a claim(C) by asserting something. For the challenger who asks "What have you got to go on?" the facts we appeal to as foundation for our claim is called data (D) by Toulmin. After producing our data, we may being asked another question like "How do you get there?" He notes, at this point we have to show that the step from our data to our conclusion is appropriate one by giving different kind of propositions like rules, principals, inference – licenses or what you will, instead of additional items of information (Toulmin, 2003). A proposition of this form Toulmin calls a warrant (W). He notes that warrants are of different kinds and may confer different degrees of force on the conclusions they justify. We may have to put in a qualifier (Q) such as "necessarily", "probably" or "presumably" to the degree of force which our data confer on our claim in virtue of our warrant. However there may be cases such that the exceptional conditions which might be capable of defeating or rebutting the warranted conclusion. These exceptional conditions Toulmin calls as rebuttal (R). For our challenger may question the general acceptability of our warrant: "Why do you think that?" Toulmin calls our answer to this question our backing (B) (Hitchcock & Verheij, 2006). The diagram of the Toulmin model is as follows:

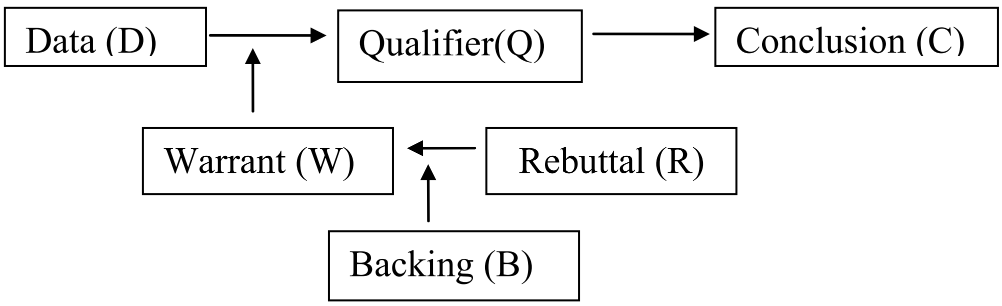

Figure 1: The Toulmin Model

Reconstructing and analyzing the complex argumentative structure in classroom conversations follow their own structure. For example, careful analyses of the types of warrants (and backings) that students and teachers employ in classroom situations allowed two distinctions in the justifications: visual and conceptual (Knipping, 2008). The warrants and backings based on conceptual aspect or deductive are mathematical concepts or mathematical relations between concepts, and make reference to theorems, definitions, axioms and rules of logic. The warrants and backings based on visual or figural aspect make reference to figures as part of the argumentation.

Problem Solving Process

Problems are identified as such if the participant sees a quandary or feels a difficulty or doubt that needs to be resolved (Hiebert et al., 1996). Once a problem has been identified, the participant actively pursues a solution by calling up and searching out related information, formulating hypotheses, interacting with the problem, and observing the results (Hiebert et al., 1996). Eventually some conclusion is reached, some resolution is achieved, some hypotheses are refined. The outcome of the process is a new situation, and perhaps a new problem, showing new relationships that are now understood (Hiebert et al, 1996). So, problem solving has two aspects: (a) the process, or set of behaviors or activities that direct the search for the solution, and (b) the product, or the actual solution. Both the process and the product are essential components of the problem-solving experience (Kantowski, 1977). The teacher bears the responsibility for developing a social community of students that shares in searching for solutions. Analyzing the adequacy of methods and searching for better ones are the activities around which teachers build the social and intellectual community of the classroom (Hiebert et al, 1996).

METHODOLOGY

Data were collected through nonparticipant observations that were videotaped. Observation was conducted 2009-2010 spring semesters in real analysis course for eight weeks, and 2010-2011 spring semesters in advanced calculus course for six weeks, offered to mathematics education student at the third and second years, respectively. These courses were selected as both formal and informal argumentations were at the focus of these courses. In these courses the number of students were 45 and 40, respectively. Formal proof approaches are given to the students at the "Abstract Mathematics I - II" courses provided in the first year. In these courses, students learn what a proof is and how to prove theorems. That is, they learn how to argue mathematically, justify their claims and encounter the cases named "counter example" for the first time which rebuttals their claims.

The analysis of the observations is based on the transcripts. As Toulmin (2003) noted, "an argument is like an organism. When set out explicitly in all its detail, it may occupy a number of printed pages or take perhaps a quarter of an hour to deliver; and within this time or space one can distinguish the main phases marking the progress of the argument from the initial statement of an unsettled problem to the final presentation of a conclusion" (p. 87). Based on this explanation, eleven argumentations were determined and five of them were on problem solving. These five argumentations were observed in real analysis course.

Observations were conducted by the second author. He analyzed the transcripts by marking the progress of the argument from the initial statement to the final conclusion through using Toulmin model components. He noticed that some aspects of observed argumentations were overlooked. He modified the Toulmin model by integrating *guide – backing* and *guide – redirecting* additional components which were observed in almost all argumentations. We called an approval given by teacher to the warrants, backings or intermediate conclusion as *guide – backing*. When the argumentation does not start from a right point or students get stuck on an argument point, teacher intervenes with an example, a question or a suggestion to arrange the argument. We called such intervenes as *guide – redirecting*.

Having discussed with the first author who is a mathematics educator and doing research on proof, it was decided that observed argumentations could be considered into three classes: proof generation, definition construction, and problem solving. She also noted that some components could be classified in itself. After re-analyzing observed argumentations, *warrant* component were divided in two categories: *deductive warrant* and *reference warrant*. Students appeal reasoning like numerical computing, applying a rule to an inequality, creating new ideas from a definition, a theorem or a rule in producing their warrants. We called this kind of warrants as *deductive warrants* as Inglis et al. (2007) did. When a warrant referred to a theorem, a definition, a rule or a problem, we called such a warrant as *reference warrant. Guide – backing* was divided into three categories: *approval, reference* and *terminator*. When teacher just approve the students' warrant, backing or conclusion by saying "good, fine, great, well done" and does not use any mathematical phrase, we called this kind of guide backing as *approval guide backing*. When teacher approve the students' warrant, backing or conclusion by referring a definition, a theorem or a problem recently solved, we called this kind of guide backing as *reference guide backing*. Argumentations come to an end when teacher or students reach the final conclusion to be achieved. In case, teacher reaches the final conclusion, students convince that the conclusion is legitimate. In case, students reach the final conclusion, teacher serves a backing. This backing shows the final conclusion and we called it as *terminator guide backing*. One important point that must be noted here is that argumentations were not analyzed according to their mathematically correctness.

Finally, full transcriptions together with analysis model components explanation are provided to an external auditor who is a researcher in mathematics education field. After a week, the auditor completed her analysis and a complete consensus was reached on analysis of argumentations.

RESULT

Five open-ended problems requiring the search of counter-examples and/or application of the definitions, rules, theorems for the solution constituted five different problem solving argumentation context. In this paper only one of these problems is considered as example because of page restrictions. Here we analyze a transcript of a short argumentation in which *deductive warrant, guide - redirecting and terminator guide – backing* appear. The following argumentation occurred when teacher asked if a boundary point of a set is an accumulation point of that set in R^n.

1 Deniz: The boundary of a set A is defined as the intersection of its closure with its complement.

2 Teac: Correct.

3 Stu: And the closure of A is the union of A with the set of its accumulation points. Eeehm… "Or" operator… I got a mistake! I mean x(a boundary point of A) is in A or in the set of its accumulation points, so x does not have to be in the set of accumulation points of A.

4 Teac: Well, you are right but it is not the way what is supposed to be. Instead, you should have a set, say A. Then x would be a boundary point of A but would not be a accumulation point of A. I mean you should give a counter example. Got it? Do you have a such example?

5 Alpaslan : Would it be one-point set?

6 Teac : Well done! Is it one-point set? Yes, it is.

7 Alpaslan : The set of its accumulation points is empty set.

8 Teac : Which means that a boundary point does not have to be an accumulation point.

In line 1, the student defined the boundary of a set. He considered it as a data. In line 3, he realized that he needs to use "or" conjunction. He used this reasoning as a deductive warrant to conclude that a boundary point of A shouldn't be in the set of accumulation points of A. In line 3, teacher intervened with a suggestion to arrange the argument. He clearly stated that he needs to have a counter example. So teacher gave a guide – redirecting. Hereon, Alpaslan suggested one-point set as his data in line 5. In line 6, teacher gave an approval guide backing by using phrases "Well done! Is it one point set? Yes, it is.". Therewith, Alpaslan could produce easily the final conclusion at the end of line 7. In line 8, teacher gave a terminator guide – backing by confirming the final conclusion. Therefore, his conclusion is valid. We observed that student producing deductive and/or reference warrant get easily the final conclusion after getting terminator backing guide. We think that if students have an ability to produce deductive and/or reference warrants and get any kind of guide – backing, then he/she could get easily the final conclusion. The diagram corresponding to the argumentation above is as follows:

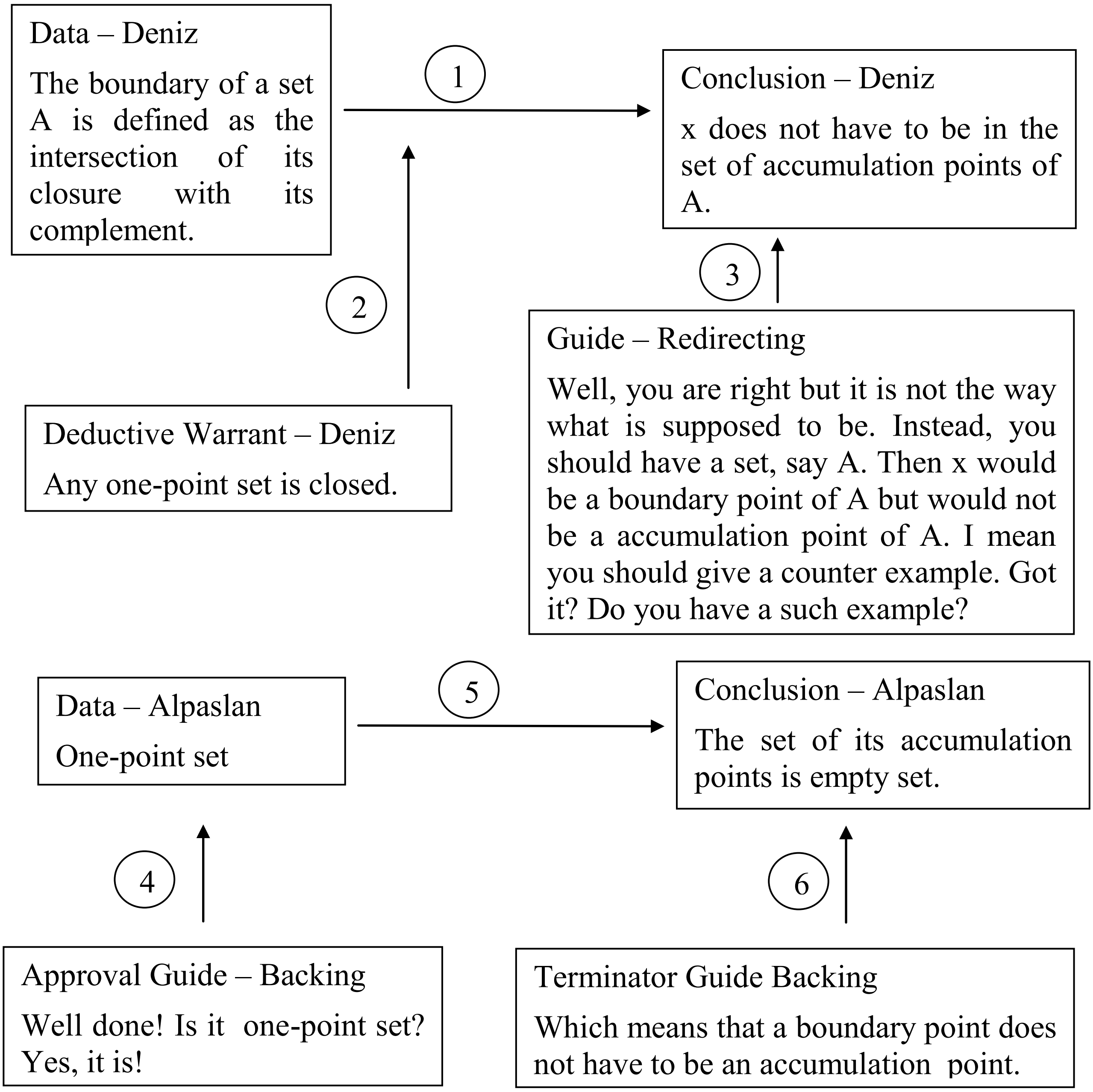

Figure 2

CONCLUSIONS

The model of Toulmin, which is helpful for reconstructing argumentation steps and streams, is not adequate for more complex argumentation structures. Argumentations in classrooms requires a different model for capturing the global structure of the argumentations developed there. Analyzing students' and teacher' utterances in the class according to the Toulmin model allowed us to reconstruct argumentations evolving in the classroom talk. Argumentations in classrooms are generally teacher guided. Teacher acts as a guide who exactly knows the path to follow i.e. where to start and to end the argumentation. Therefore argumentation guided by the teacher in the classroom comes to an end. During the argumentation if students follow the wrong path, get a false intermediate conclusion or get stuck in a point, teacher intervene the students to put them on the path in which they have to follow. If students on their own can manage to get the conclusion of argumentation, then they are sure about the conclusion when they get the terminator guide backing. According to this and based on our observations, teacher played a role in argumentation like guide – backing and guide – redirecting. According to our view, guide – redirecting is an important component for searching out related information, formulating hypotheses, interacting with the problem, and observing the conclusions which are essentials of problem solving process. We also think, guide – backing and guide – redirecting components prevent emergence of qualifier component in argumentations in proof generation, in definition construction and in problem solving process. There are two reasons for that. Firstly, the conclusion needs to be reached is absolute. Secondly, guide – backing and guide – redirecting are components which leads the way to the absolute conclusion.

In sum, new components were identified to be added to the Toulmin argumentation model as well as interactions between them. They were named *guide-backing* and *guide-redirecting*. *Guide backing* was divided into three classes: *approval, reference* and *terminator*. *Approval guide-backing* and *terminator guide-backing* occurred in almost all argumentations related to proof generation, definition construction, and problem solving but *reference guide-backing* occurred in almost all except argumentation on constructing a definition (see also Ubuz, et al, 2012, 2013). Furthermore, *warrants* were divided into two classes: *deductive* and *reference*. *Deductive warrant* occurred in any type of an argumentation but *reference warrant* occurred in any type of an argumentation except in an argumentation for constructing a definition (see also Ubuz, et al, 2012, 2013).

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