AN INFERENTIALIST ALTERNATIVE TO CONSTRUCTIVISM IN MATHEMATICS EDUCATION

Ruben Noorloos[1], Sam Taylor[1], Arthur Bakker[2], Jan Derry[2]

[1]Utrecht University, the Netherlands, [2]University of London, UK

The purpose of this paper is to draw attention to a relatively new semantic theory called inferentialism as developed by the philosopher Robert Brandom. We argue that it offers a better alternative to the still present representational view of mind than does (socio)constructivism. After a discussion of the shortcomings of (socio)constructivism, we summarize the key features of inferentialism that make it worth thinking through more carefully in mathematics education research.

REPRESENTATIONALISM AND THE MYTH OF THE GIVEN

This paper invites mathematics educators to study a semantic theory that Brandom (1994, 2000) has elaborated in recent years. He critiques a representational view of mind that has long been criticized in several disciplines but is still common among cognitive scientists and, in our experience, in how mathematics educators and teachers talk about teaching and curriculum (Bakker & Derry, 2011). Rather than seeing representation to be the basis for reasoning, Brandom explains the meaning of representations through their origin in reasoning practices.

Representationalist theories describe the activity of learning as that of the modification or construction of internal representations. The student is assumed to possess a pre-existing internal faculty of representing external phenomena which the teacher is supposed to expand in a given, pre-determined direction. The goal of teaching is accordingly characterized as bringing about the correct representations in the student's mind, while its success is determined by the measure of correspondence which exists between the internal representations and external reality. In recent decades, representationalism has come under attack. Cobb, Yackel, and Wood (1992) argue that the central problem of representationalist theories is their implicit appeal to a *dualism* between individual representation and external reality. They write:

> At the outset, mathematics in students' heads (internal representations) is separated from mathematics in their environment (external representations that are transparent for the expert). The basic problem is then to find ways of bringing the two back into contact.(p.14)

This dualism is endemic to representationalism: Once representations are cut off from the world and the practices that contribute to their constitution, they cannot be brought together again except by the assumption that mind and world do stand in some sort of primitive relationship, for example through a fundamental form of immediate sense experience. Sellars (1956/1997), one of Brandom's inspirations, attacked this assumption of immediate sense experience, which he called the "Myth of the Given." He argued that the normativity inherent in conceptual content – the ways in which

2014. In Liljedahl, P., Oesterle, S., Nicol, C., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 4, pp. 321-328. Vancouver, Canada: PME.

concepts and speech, in contrast to objects and causes, can be correct and incorrect – cannot in any way be derived from a nonconceptual, nonnormative reality which is simply "given" to the mind. His argument was amplified by Brandom's colleague McDowell (1994/1996), who pointed to a complementary and, he claimed, equally pernicious assumption. Once the failure of appeals to the given is recognized, it is natural to reject any form of foundationalism. But this response would go too far and may reject the need for any relation between mind and world at all, because it is thought that a form of foundationalism is the only way to make internal representations square up with the world. In McDowell's term, one becomes a *coherentist*, rejecting any form of constraint on one's thinking imposed by external reality and preferring instead pragmatic, deflationary notions such as the internal consistency of one's ideas.Such an approach risks being like a "frictionless spinning in a void" (McDowell, 1996, p. 11).

One way to escape from the "oscillation" (p. 17) in the history philosophy between the given and coherentism is to accept Sellars' view that, though humans have access to external objects, these objects or representations of them can only play a normative role (be used in assertions which can be correct or incorrect) by being taken up into the language games of people: They must become subject to norms which were already in place in human practices. In his influential phrase, they must be placed in the "space of reasons" (Sellars, 1997, §36). Brandom systematically elaborates this idea (Bransen, 2002). He pictures humans' interactions as engaging in a the essentially social "game of giving and asking for reasons"; objectivity and representation are not basic but must be mediated through this game. This also necessarily involves an emphasis on the *holistic* and *social* nature of concepts.

Cobb et al. (1992) presented constructivism as an alternative to representationalism. We summarise some of the problems faced by constructivism and suggest inferentialism as an alternative that is more convincingly rooted in philosophical traditions. In brief, inferentialism has been argued to be compatible with Vygotsky's ideas (Derry, 2013). It also connects to a "domesticated" Hegelianism (e.g., the aforementioned notions of mediation, holism and social nature of concepts) which maintains links with analytic philosophy (Bernstein, 2002). Inferentialism is more explicitly concerned with concept use and reasoning practices than much published work arising from sociocultural and activity-theoretical perspectives, which makes it especially interesting to mathematics education research. However, due to space limitations we concentrate on why we think it is a better alternative to variants of constructivism.

CONSTRUCTIVISM

A basic constructivist presupposition is that learners "create their own understandings" through the process by which they are immersed in meaning making (Rogers, 2011, p. 178). In this way, the constructivist approach to knowledge overcomes the problematic representational tie between mind and world by downplaying the world element. As

such, the constructivist hopes to reject the correspondence relation that sits at the heart of representationalist theories by undermining the implicit *dualism* of the representational doctrine. Consequently, constructivism is able to make room for two desiderata which representationalism was shown to lack: It can offer accounts of individual learning and the social embedding of learning processes.

The central problem the constructivist alternative faces, however, is how to come to terms with accommodating these central desiderata without postulating that the learner is in direct contact with the world. This section aims to show that constructivism has been unable to offer a satisfactory account of the learning process because it adheres to a thinly veiled neo-Kantian pre-supposition, which dictates that on some level mind and world must be kept apart if we are to avoid the problematic representationalist idea that mind and world stand in some sort of primitive relationship to one another. We hold that this neo-Kantian dogma undermines the constructivist's position because, in trying to avoid falling into the Myth of the Given, it embraces a dualism of its own.

There are many variants of constructivism, but for our argument it is sufficient to recognise the general distinction between the cognitive constructivist and socioconstructivist approaches, because most individual manifestations of constructivist theory can broadly be categorized under one of these constructivist positions (Mason, 2007). Cobb and Bowers (1999) delineate this general distinction by considering two metaphors:

> In the case of the cognitive perspective, a central organising metaphor is that of knowledge as an entity that is acquired in one task and conveyed to other task settings. In contrast, a primary metaphor of situated learning perspective is that of knowledge as an activity that is situated in regard to an individual's position in a world of social affairs. (p. 2)

This conceptual difference affects the methodology of both approaches, because, "If from the cognitive point of view, knowing means possessing, from the sociocultural perspective it means belonging, participating, and communicating" (Mason, 2007, p. 2). In this sense the focus is *either* on the "construction of internal knowledge or meaning" *or* the "construction – if it can even be called that – of new communities of discourse and social practice" (Kaartinen & Kumpulainen, 2002).

Thus there is a tension between cognitive and socioconstructivism resulting from their contrasting attempts to satisfy the desiderata lacking in representationalist theory, whilst simultaneously adhering to implicit and problematic neo-Kantian presuppositions, which dictate that on some level mind and world are disconnected. Consider cognitive constructivism (e.g., Von Glasersfeld, 1980). By focusing upon the *individual's* internal cognitive mechanisms it explains the process of learning in terms of the individual's construction of internal knowledge or meaning. The problem is that in attempting to circumvent the issue of a nonconceptual reality which the mind is primitively able to represent, cognitive constructivist's take a coherentist approach, which tries to conceptualise knowledge claims and articulate standards of objectivity on the basis of the internal coherence of beliefs. In this way, the individual's loses

touch with the world. So, even if cognitive constructivism satisfies one desideratum – that of individual learning – it can only do this by neglecting objective external constraint and falling prey to relativism. Learners, therefore, are left spinning in McDowell's frictionless void.

Socioconstructivism, on the other hand, appreciates that learning cannot be segregated from social practices and must take place within certain communities of discourse, but typically offers no account of how individual's go about constructing internal knowledge claims. If we cast learning as that mental phenomena that take place only inside of the learner's head, then we ignore the now well established idea that a theory of learning should include an account of the reflexive relation between an individual student's reasoning and the evolution of the classroom practices that constitute the immediate social situation of their mathematical development (e.g., Bowers, 1996). On the socioconstructivist view, however, whilst we sidestep the coherentist pitfall, we have no account of how differing contexts and discourse practices provide the sufficient basis for *individual* understanding. We have no explanation of how the same knowledge – of simple mathematical truths, for example – can be internalised by differing individuals with often markedly different learning processes. Subsequently, if learning concerns only the construction of social practice, then we lack a convincing account of how individuals come to know anything. How, for example, can I know that my knowledge of the mathematical truth, $1 + 1 = 2$, squares with yours? In Cobb et al.'s words, students "have no way of knowing whether their individual interpretations of a situation actually correspond to those of others" (1992, p. 17-18). For the socioconstructivist, therefore, learning refers solely to an individual's integration into a particular environment to such an extent that the role of the mind – in forming and modifying knowledge claims and meaning designation – plays almost no part.

To attend to this problem and so illuminate why there exists a tension between cognitive and socioconstructivism, we must look to constructivism's neo-Kantian presuppositions about the mind-world relation. How can individual learning be any more than the coherence of my own beliefs and so satisfy the notion of objectivity? And how can the constructivist make an appeal to sociocultural practices whilst still accommodating individual learning? We claim that these two desiderata cannot be *jointly* satisfied within a rigidly neo-Kantian constructivist theoretical framework, because the constructivist maintains that the best way to deal with representationalism's Myth of the Given inspired deficiencies is to adopt the neo-Kantian doctrine that says, on some level, mind and world must be kept apart. But this will not do, because it functions as an incomplete renunciation of the dualism at the heart of representationalism and causes problems for constructivism's own account of learning processes. The constructivist wants to renounce the representationalist assumption that the world is simply given to us, but to do so he endorses the neo-Kantian notion that the world is not entirely open to the mind. But then, instead of *overcoming* dualism, the constructivist merely has a choice between deciding in favour of the mind and discounting the world (cognitive), or in favour of the environment

(socioconstructivist) and losing his way back to the mind. Just as in Kant's transcendental story, the constructivist has – perhaps implicitly – decided that there exists a gap between mind and world, so that we can have a theory of learning centered on mind *or* environment, but not both.

It is clear from the preceding discussion that if the constructivist is to offer a theory of learning that conceives of the learning process as one in which individual learners have "taken-as-shared mathematical interpretations, meanings, and practices institutionalized by wider society" (Cobb et al., 1992, p. 16), he needs additional theoretical tools to overcome the neo-Kantian gap described above. He requires an *evolution* of constructivism's theoretical commitments.

In our view, one interesting evolution of constructivism, which can be found in the ideas of Cobb et al., is the introduction of normativity. For Cobb et al., mathematical learning is multidimensional; it contains individual and sociocultural elements, because individual learners are subject to intersubjective sociomathematical norms (Cobb & Yackel, 1996). These norms, as instantiated via social practice, bind individual learning to the collective in a way inconceivable from within the traditional constructivist picture by regulating "what counts as an acceptable mathematical explanation and justification" (p. 461). Sociomathematical norms thus provide the basis by which constructivism can answer to both desiderata lacking in representationalist theory without succumbing to the Myth of the Given or coherentist tendencies. But Cobb's account has not provided an explication of the functionality of the normative process that underpins the learning process; the *how* and *why* concerning normativity. As such, Cobb's evolution only goes so far. Moreover, we wonder whether a perspicuous and systematic description of this process could be given by making further use of the metaphor of construction. As an alternative or supplement to this evolution in Cobb's work, we argue that Brandom's inferentialism offers an approach that permits of a clearer articulation and expansion of what Cobb is trying to achieve.

INFERENTIALISM

Inferentialism differs from representationalism and constructivism in privileging the metaphor of inference over those of representation and construction. Following Sellars, Brandom espouses an inferentialist semantics, which sees the meaning of a word as determined by the inferences in which the word plays a role – a Hegelian idea also found in Vygotsky (see Bakker & Derry, 2011). This distinguishes him from typically representationalist theories, which understand meaning in terms of reference to objects. For example, the meaning of "red," for Brandom, is defined by that one can, *inter alia*, derive "p is colored" or "p is not blue" from "p is red." From this it follows that inferentialism must be *holistic* in nature: To understand one concept, one must understand many. So, to

> grasp or understand […] a concept is to have practical mastery over the inferences it is involved in – to know, in the practical sense of being able to distinguish (a kind of

> know-*how*), what follows from the applicability of a concept, and what it follows from. (Brandom, 2000, p. 48, his italics; cf. Sellars, 1997, §36)

This is a *practical* mastery because the assessment of one's concept use is not up to oneself (or to the way one's concepts mirror reality). Rather, it is up to the people one engages with in the aforementioned social game of giving and asking for reasons. Brandom (1994, Part I) develops a social scorekeeping account of language, in which one keeps track of the claims one takes oneself or other persons to be committed to on the basis of their utterances, their actions, or what one takes to follow from those utterances or actions. These practices are constrained by norms, as in Cobb; but we wish to emphasize that, contrary to Cobb, Brandom gives a perspicuous account of their provenance from within scorekeeping practice. Brandom's account is essentially socially perspectival (1994, Section 8.6). It is primarily important what one *takes* people to be committed to; only later does the question arise whether they are or should *really* be committed to those inferences. One's commitments are, however, genuinely subject to external constraint, in that they may be challenged or endorsed on the basis of facts about reality; Brandom allows for noninferential access to the external world through perception. This access is not immediate, however, because it is filtered through the game of giving and asking for reasons.

In sum, Brandom understands concepts in terms of the set of inferences in which they play a role. Concepts are not "in the head," nor are they completely "out there" – rather, they reside in the game of giving and asking for reasons. Though concepts do refer to the external world and may be said to be "constructed," in a loose sense, by the discovery and articulation of new inferences which involve them – a process which may occur both on an individual and a supra-individual level – both representation and construction can be explained in terms of inferences. The individual develops new concepts by becoming aware of new possible inferences. It is the task of the teacher to support and guide this process, while being aware of the individual learning which manifests itself in the situative teacher-student webs of reasons, and without being able to retreat to an external vantage point outside of the social game.

The upshot is that Brandom gives a *dynamic*, *holistic*, *nondualistic*, and *social* picture of human rationality. Reasoning is not primarily an internal phenomenon which depends on essentially private cognitive structures, but takes place in the social game of giving and asking for reasons. This game is dynamic in that the inferences a given word is engaged are not fixed, and in that Brandom sees the status of utterances as depending on how they are recognized (in the Hegelian sense of *anerkennen*) by one's conversational partners. The way they understand your utterance informs the content it is eventually seen to possess. This is one of the ways in which Brandom's account of linguistic activity is *relational*. Moreover, it is inherently practical, stressing the concrete contexts in which learning can only take place, and allowing us to see, for example, how the teaching of concrete, meaningful examples may aid teaching more than the abstract transfer of structural knowledge that does not help students to make inferences. Finally, it is well suited to being applied at different levels of grain. By

introducing the notion of a web of reasons, which is the manifestation of the game of giving and asking for reasons in a particular situation (Brandom, 1994, p. 5), it is possible to accommodate both individual learning as well as more general features of classroom learning.

IMPLICATIONS AND CONCLUSION

It is worth emphasizing recommendations that inferentialism offers mathematics educators (Bakker & Derry, 2011). Firstly, it asks them to understand concepts primarily in *inferential* rather than representational terms, as the set of moves available in the social game of giving and asking for reasons. Secondly, it privileges holism over atomism, emphasizing the interrelations between concepts. This has implications for curriculum and teaching, not only in statistics education but also in, for example, vocational education, Bakker and Akkerman (in press) suggest that inferentialism has something important to offer when we intend to overcome common dichotomies such as between school-based and work-based knowledge, and between mathematical and contextual knowledge. In particular the concept of webs of reasons can help to do more fine-grained analyses of the many types of reasons in making claims or decisions. We assume that more areas within mathematics education can benefit from an inferentialist perspective, for example research into reasoning, proof and argumentation, but also research into the role of signs and representations. One key area for further research may be to study the relation between reasoning and representing. Brandom's reversal of philosophical methodology of explaining the meaning of representations in terms of inference may be too extreme for explaining learning. It may turn out that a co-evolvement of reasoning and representing, as in Peirce's theory of diagrammatic reasoning (Bakker, 2007), is more convincing.

Inferentialism has been applied in educational settings and found useful, also for the analysis of mathematical learning (e.g., Huβmann & Schacht, 2009). This paper has sought to urge its theoretical virtues. We do not claim that it surpasses all its competitors, but we have given some reasons for studying if the metaphor of inference is preferable to those of construction or representation – both of which can be explained in terms of it. We also think that an inferentialist theory of education provides the systematic resources needed to solve some of the problems which have plagued educational researchers. Because of its systematicity and roots in both analytic and continental philosophy, it may prove a more perspicuous alternative to many proposals currently on the market. Inferentialism escapes the oscillation between the given and coherentism we discussed and avoids the dichotomies between, for example, mind and world, and the individual and the social. It promises a systematic and coherent way of dealing with these dichotomies without reifying them as is often done. Though promising *evolutionary* developments of the constructivist paradigm are found in Cobb's work, we propose that a *revolutionary* account – a replacement by an inferentialist framework – may ultimately be more useful. We hope that mathematics educators feel invited to study inferentialism and related philosophical perspectives.

References

Bakker, A. (2007). Diagrammatic reasoning and hypostatic abstraction in statistics education. *Semiotica*, *164*, 9-29.

Bakker, A., & Akkerman, S. F. (2013). A boundary-crossing approach to support students' integration of statistical and work-related knowledge. *Educational Studies in Mathematics*. doi: 10.1007%2Fs10649-013-9517-z

Bakker, A., & Derry, J. (2011). Lessons from inferentialism for statistics education. *Mathematical Thinking and Learning*, *13*, 5-26.

Bernstein, R. J. (2002). McDowell's domesticated Hegelianism. In N. H. Smith (Ed.), *Reading McDowell: On mind and world* (pp. 9-24). Abingdon, UK: Routledge.

Brandom, R. (1994). *Making it explicit: Reasoning, representing, and discursive commitment*. Cambridge, MA: Harvard University Press.

Brandom, R. (2000). *Articulating reasons: An introduction to inferentialism*. Cambridge, MA: Harvard University Press.

Bransen, J. (2002). Normativity as the key to objectivity: An exploration of Robert Brandom's Articulating Reasons. *Inquiry*, *45*(3), 373-391.

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher*, *28*(4), 1-15.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, *31*(3/4), 175-190.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education*, *23*(1), 2-33.

Derry, J. (2013). *Vygotsky: Philosophy and education*. Hoboken, NJ: Wiley Blackwell.

Hußmann, S., & Schacht, F. (2009). Toward an inferential approach analyzing concept formation and language processes. In V. Durand-Guerrier (Ed.), *Proceedings of CERME 6* (pp. 842-851). Lyon, France: CERME.

Kaartinen, S., & Kumpulainen, K. (2002). Collaborative inquiry and the construction of explanations in the learning science. *Learning and Instruction*, *12*, 189-212.

Mason, L. (2007). Introduction: Bridging the cognitive and sociocultural approaches in research on conceptual change: Is it feasible? *Educational Psychologist*, *42*(1), 1-7.

McDowell, J. (1994/1996). *Mind and world*. Cambridge, MA: Harvard University Press.

Rogers, G. (2011). Constructivism and the pedagogy of teacher education: Reflections on practice. In J. Adams, M. Cochrane, & L. Dunne (Eds.), *Applying theory to educational research: An introductory approach with case studies* (Ch. 11). Chichester: John Wiley.

Sellars, W. (1956/1997). *Empiricism and the philosophy of mind*. Cambridge, MA: Harvard University Press.

Von Glasersfeld, E. (1980). Radical constructivism. *Revue Internationale de Philosophie*, *36*(4), 612-635.