A COMPARATIVE STUDY ON BRAIN ACTIVITY ASSOCIATED WITH SOLVING SHORT PROBLEMS IN ALGEBRA AND GEOMETRY

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This paper presents study that investigates brain activity (using ERP methodology) of male adolescents when solving short problems in algebra and geometry. The study design links mathematics education research with neuro-cognitive studies. We performed a comparative analysis of brain activity associated with the translation from visual to symbolic representations of mathematical objects in algebra and geometry. The findings demonstrate that electrical activity associated with the performance of geometrical tasks is stronger than that associated with solving algebraic tasks. Additionally, we found different scalp topography of the brain activity associated with algebraic and geometric tasks. Based on these results, we argue that problem solving in algebra and geometry are related to different patterns of brain activity.

INTRODUCTION

This paper presents a small segment of a large scale project that analyses components of mathematical abilities in three dimensions: basic cognitive traits, brain activity associated with solving mathematical problems and mathematical creativity (Leikin, Leikin, Lev, Paz, & Waisman, 2014). In this paper we choose information about neuro-cognitive activity related to solving short problems in algebra and geometry, and perform a comparative analysis of brain activity associated with translation from visual to symbolic representations of mathematical objects in algebra and geometry. Algebraic tasks required translation from graphical to symbolic representation of a function, whereas tasks in geometry required translation from a drawing of a geometric figure to a symbolic representation of its property. In this paper we do not analyse relationship between students' mathematical performance and their mathematical abilities since no interaction between students' abilities and test effect was identified.

BACKGROUND

Neuro-cognitive research in mathematics education

Neural basis of the use of mathematical cognition has been investigated in several directions mostly focusing on brain location of cognitive functions related to mathematical processing. Here we provide several examples. Research on number processing and simple arithmetic (Dehaene, Piazza, Pinel, & Cohen, 2003) emphasized the role of the parietal cortex to number processing and arithmetic calculations. The horizontal intraparietal sulcus has been found to be involved in calculations; the

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posterior superior parietal lobule has been linked with the visuo-spatial and attention aspects of number processing while the angular gyrus has been found to be associated with the verbal processing of numbers and involved in fact retrieval (Grabner, Ansari, Koschutnig, Reishofer, Ebner, & Neuper, 2009). The parietal cortex has been found to be involved, too, in more complex mathematical processing such as word problem solving (Newman, Willoughby & Pruce, 2011). The posterior superior parietal cortex has been found to be involved in visuo-spatial processing including the mental representations of objects and mental rotations (Zacks, 2008). Some studies demonstrated that when complexity of the problems rises, more brain areas simultaneously support the solving process (Zamarian, Ischebeck, Delazer, 2009). Note, however, that the neural mechanisms involved in complex mathematics have not been studied sufficiently, and our study enters this lacuna.

Studying functions and geometry in high school

Function is one of the fundamental concepts in mathematics in general and in school algebra and calculus in particular (Da Ponte, 1992). Kaput (1989) argued that the sources of mathematical meaning-building are found in translations between representation systems. The ability to translate from one representation of the concept of function to another highly correlates with success in problem solving (Yerushalmy, 2006) while flexible use of representations is part of cognitive variability, which enables individuals to solve problems quickly and accurately (Heinze, Star, & Veschaffel, 2009).

Geometry in school mathematics is considered an important source for development of students' reasoning and justification skills (Lehrer & Chazan, 1998; Hanna, 2007; Herbst & Brach, 2006). Learning geometry in high school involves analyzing geometric objects, their properties and the relationships between them. Mental images of geometrical figures represent mental constructs possessing simultaneously conceptual and figural properties (Fischbein, 1993). Geometrical reasoning combines visual and logical components which are mutually related (Mariotti, 1995), while perceptual recognition of geometrical properties must remain under the control of theoretical statements and definitions (Duval, 1995).

This paper focuses on brain activity associated with solving short geometry problems that require translation between visual representation of a geometric object and the symbolic representation of its property.

METHODS

The study goals

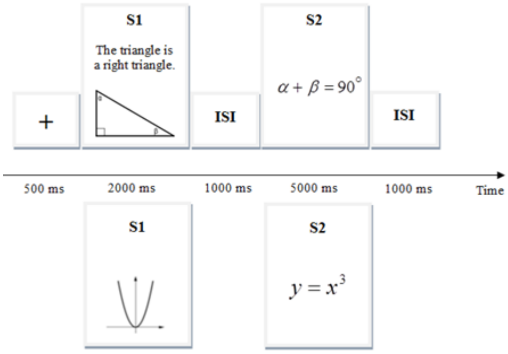
This study examined behavioural measures, i.e., Accuracy of responses (Acc) and Reaction time for correct responses (RTc), and electrophysiological measure, i.e., amplitudes, latencies, and scalp topographies, related to solving short problems that require translation between symbolic and graphical representations in algebra and geometry. We asked: How do the examined measures revealed when solving algebraic problems differ from those revealed when solving geometry problems?

Participants

We report herein our findings on 71 right-handed male adolescents. All participants were paid volunteers, native speakers of Hebrew, right-handed, with no history of learning disabilities and/or neurological disorders. All participants and their parents signed an informed consent form. The study received the approval of the Helsinki Committee, the Israel Ministry of Education, and the Ethics Committee of the University of Haifa.

Materials and procedure

A computerized test that required students to perform a translation between symbolic and graphical representations of function was designed with 60 tasks (trials) using E-Prime software (Schneider, Eschman, & Zuccolotto, 2002).



S1 – Introducing a situation; S2 – Question presentation;
+ – Fixation cross; ISI – Inter Stimulus Interval

Figure 1: The sequence of events and task examples

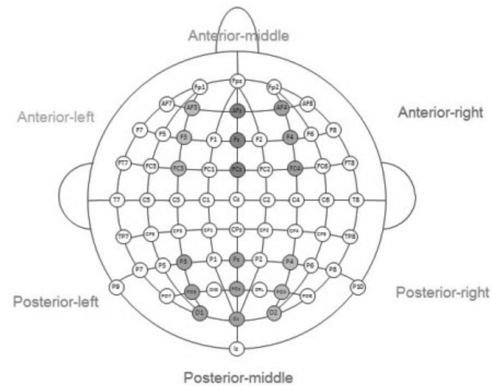


Figure 2: Location of the electrodes and selected electrode sites

Each task on each test was presented in two windows with different stimuli (S1 – Task condition; and S2 – Suggested answer) that appeared consecutively. The sequence of events and examples of the tasks are presented in Figure 1. At S2 each subject had to decide whether the suggested answer was correct or not by pressing an appropriate button on the keyboard. Alpha-Chronbach was determined by accuracy criteria and found to be sufficiently high (α_C = .859 and α_C = .760 for algebraic and geometric tasks, respectively).

Scalp voltages were continuously recorded using a 64-channel BioSemi ActiveTwo system (BioSemi, Amsterdam, The Netherlands) and ActiveView recording software. Two flat electrodes are placed on the sides of the eyes in order to monitor horizontal eye movement. A third flat electrode is placed underneath the left eye to monitor vertical eye movement and blinks. During the session electrode offset is kept below 50 µV. Figure 2 depicts location of the electrodes and the selected electrode sites.

Data analysis and statistics

Trials with correct responses were used for both ERP and behavioural analysis. Behavioural data of trials excluded from electrophysiological analysis by artifact rejection were excluded from the behavioural analysis.

Behavioural measures: We examined Accuracy (Acc) and Reaction time for correct responses (RTc) for each participant. Acc was determined by the mean percentage of correct responses to 60 tasks on the test. Reaction time for correct responses (RTc) was calculated as the mean time spent for verification of an answer (stage S2) and for correct responses only. We performed between the tests comparisons using repeated measures MANOVA.

Electrophysiological measures: Event related potentials (ERPs) were analysed offline using the Brain Vision Analyzer software (Brain-products). Ocular artifacts were corrected using the Gratton, Coles and Donchin (1983) method. The ERP waveforms were time-locked to the onset of S1 and to the onset of S2. Due to the space contrarians we do not report analysis of ERP early components.

Following visual inspection of grand average waveforms and appropriate scalp topographies we divided the late potential wave into three time frames: 300-500, 500-700 and 700-900 ms. We used repeated measures ANOVA tests on the ERP mean amplitude, to examine effects of Tests, Laterality (Left, Mid-line and Right) and Caudality (Anterior and posterior). Analysis was done for each of the two stages of a task (S1 and S2). Table 1 depicts Electrophysiological data analysis performed for Late Potentials.

ERP component	Stage	Time frame (ms)	Factors	Measures
Late potentials	S1 S2	300-500 500-700 700-900	*Laterality:* 3 levels: Left, Middle and Right *Caudality:* 2 levels: Anterior, Posterior *Test:* 2 levels: Algebra, Geometry	Mean amplitude

Table 1: Electrophysiological data analysis of Late Potential

RESULTS

We report on the significant effects and interactions only. If a particular effect (or interaction) is not reported, this indicates that it was not significant.

Differences in Acc and RTc

Measure	Algebra Mean (SD)	Geometry Mean (SD)	F (1, 67)	η_p^2
Acc	78.2 (9.7)	83.2 (7.5)	18.877^{***}	.220
RTc	1686.4 (398.6)	1593.7 (382.5)	5.513^{*}	.076

$p<.05$, $^{**}p<.01$, $^{***}p<.001$; Acc – Accuracy, RTc – Reaction time for correct responses

Table 2: Acc and RTc in the different groups of participants

Repeated measures MANOVA demonstrated significant effects of the Test factor [F (2, 66) = 10.082^{***}, *Wilks* $\wedge$ = .766] both on the Acc and on the RTc (Table 2). On algebraic test we found significantly lower Acc along with significantly higher RTc as compared to Acc and RTc on the geometry test (Table 2).

Electrophysiological findings reflected in late potential

Figure 3 depicts examples of the grand average waveforms and topographical maps of the late potentials for Algebra and Geometry tests.

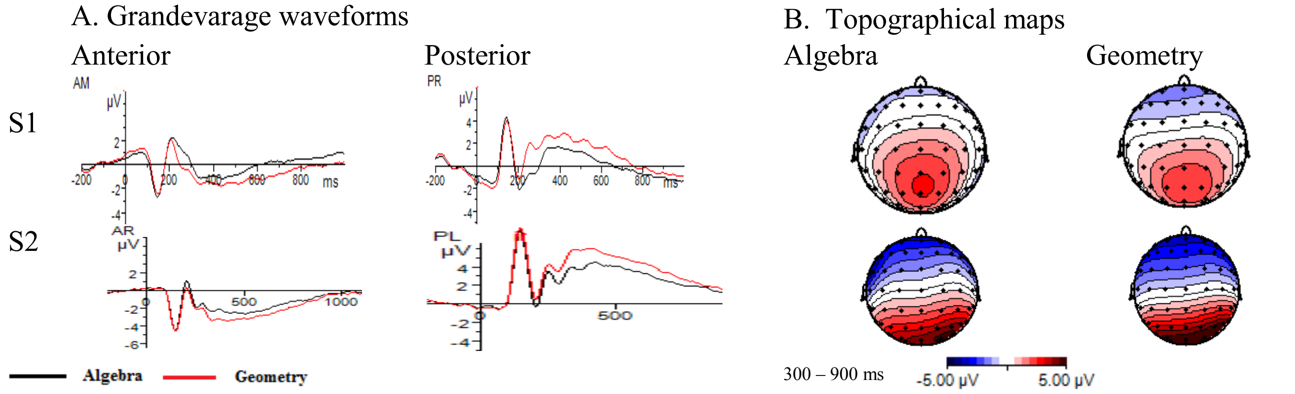


Figure 3: (A)Examples of the grandevarage waveforms associated with Algebra and Geometry tests in the selected electrode sites.; (B) Topographical maps of voltage amplitudes for Algebra and Geometry tests

	Significant factors and interactions	Time	F	η_p^2
S1	**Test** Amp(Algebra) < Amp(Geometry)	300-500 ms	7.687**	.103
	Test × Caudality	300-500 ms	6.763*	.092
		500-700 ms	21.417***	.242
		700-900 ms	13.211***	.165
	Test × Laterality Amp (Algebra) < Amp(Geometry) in **RH** Amp(Algebra) > Amp(Geometry) in **ML**	300-500 ms	$F(1.657, 111.033) = 10.480^{***}$	.135
		500-700 ms	$F(1.749, 117.165) = 8.561^{***}$	.113
	Test × Laterality × Caudality	300-500 ms	$F(1.692, 113.371) = 7.709^{***}$	.103
S2	**Test** Amp(Algebra) < Amp(Geometry)	300-500 ms	46.560***	.410
		500-700 ms	25.155***	.273
		700-900 ms	5.615*	.077
	Test × Caudality	300-500 ms	41.390***	.382
		500-700 ms	10.248**	.133
	Test × Laterality Amp (Algebra) < Amp(Geometry) in **LH** and **ML** Amp(Algebra) ~ Amp(Geometry) in **RH**	500-700 ms	3.650*	.052

$p \le .05$, $^{**} p \le .01$, $^{***} p \le .001$, when not mentioned *d.f.* (1, 67); RH –Right hemisphere; ML – Mid-line; LH – Left hemisphere

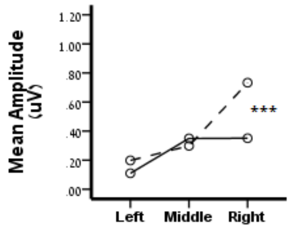
Table 3: Significant results in mean amplitude in the selected electrode sites associated with the Algebra and Geometry tests

Significant differences associated with the Test were found at both S1 and S2 stages. Table 3 demonstrates time frames at which Test, Laterality and Caudality factors had significant effects and interactions. We found that the *mean amplitude* was significantly higher for Geometry test than for Algebra test. Significant *interaction between Test and Caudality* revealed at posterior and anterior regions while absolute values of voltages for the Geometry test were larger than for Algebra test with negative voltages in anterior regions and positive voltages in posterior regions.

A significant *interaction of Test with Laterality* was found at S1 and at S2: At S1, the mean amplitude reveal for the Geometry test was larger than for the Algebra test in the right and left hemisphere, whereas in the mid-line the mean amplitude for Algebra test was higher as compared to mean amplitude for the Geometry test (Figure 4). At S2, the mean amplitude for the Geometry test was larger in the left hemisphere and mid-line as compared to that for the Algebra test, while the difference between the tests in the left hemisphere and the mid-line was significant. A significant *interaction between Test, Caudality and Laterality* was found at S1 (Figure 4).

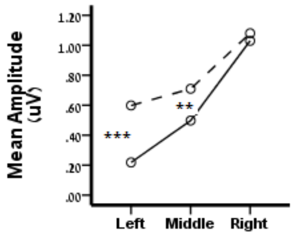
Test × Laterality

S1: 300-500 ms



**right:*
95% *CI* [.243, .521]

S2: 500-700 ms

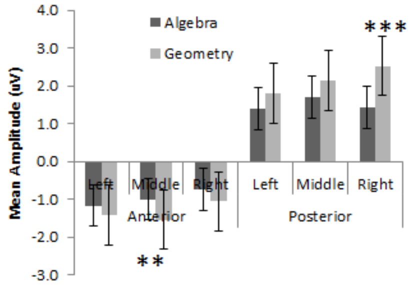


****Left:* 95% *CI* [.215, .546]
***Middle:* 95% *CI* [.072, .351]

—— Algebra
– - Geometry

Test × Laterality × Caudality

S1: 300-500 ms



*** *posterior right*: 95% *CI* [.597, 1.583]
** *anterior middle*: *p* = .007, 95 % *CI* [.158, .936]

$^{*}p \leq .05$, $^{**}p \leq .01$, $^{***}p \leq .001$

Figure 4: Mean amplitude for Algebra as compared to Geometry test in the Left, Middle and Right electrode sites in the 300-500 ms at S1 and in the 500-700 ms at S2.

DISCUSSION

Strength of electrical potentials: The first major finding of our study shows that solving geometry tasks emerged higher electrical potentials that solving algebraic tasks both at S1 and S2 stages at both the anterior and posterior parts of the scalp. Following previous research that connects cognitive functions of different types with activation of different brain regions, we hypothesise that the greater electrical activation in the anterior parts of the scalp demonstrated that geometry test requires greater cognitive control and activation working memory (Arsalidou & Taylor, 2011; Newman et al., 2011). The enhanced voltages in the posterior (especially right) parts of the scalp during geometry test may be connected to the greater demands in visuo-spatial processing, including manipulation of internal representations (Zacks, 2008) in geometry problem solving.

Brain topography: The second major result of our study revealed a significant interaction of hemispheric laterality with Test. At S1, the mean amplitudes associated with the geometry test were higher as compared to the algebra test in the right hemisphere, whereas the mean amplitudes associated with the algebra tests were higher in the mid-line. Moreover, in the right hemisphere the difference between the amplitudes elicited by the two tests was significant. In contrast, at S2 the mean amplitudes associated with the geometry test were significantly higher in the left hemisphere and the mid-line. Previous studies demonstrated that the left hemisphere is thought to be more involved in processing verbal and symbolic information and is also shown to be more analytic from the processing viewpoint, while the right hemisphere seems to deal more with the processing of visuo-spatial information (e.g., Dien, 2008). Thus we hypothesise that differences in activation patterns between the Algebraic and Geometry tests in our study may be explained by the differences between the processing strategies used by the participants in algebra and geometry. We also suggest that when solving geometry tests, at the visual and symbolic stages, participants activate different hemispheres.

Students' problem solving performance on geometry task as compared to their performance on algebra tasks revealed higher electrical potentials along with higher accuracy and shorter reaction times for correct responses. In contrast to the previous findings that task complexity lead to higher electrical potentials, by combining our research findings related to Acc and RTc with findings related to ERP measures, we speculate here that strength of electrical potentials when solving mathematical tasks are not necessarily connected to the task difficulty but subject-matter-dependent.

This study emphasizes the contribution of neuropsychological research, which adds important information to previous findings of cognitive studies in the field of the psychology of mathematics education. Based on our findings we argue that problem solving in algebra and geometry is associated with different patterns of brain activity and, thus, we hypothesize that teaching algebra and geometry may require different didactical approaches. We also assume that our findings on the differences in scalp topology associated with solving algebra and geometry tasks may explain why different students are not equally good in geometry and algebra.

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