CONNECTING A MATHEMATICS TEACHER'S CONCEPTIONS AND SPECIALISED KNOWLEDGE THROUGH HER PRACTICE

Eric Flores, José Carrillo

University of Huelva, Spain

The aim of this paper is to show connections between a teacher's conceptions about the teaching and learning mathematics reflected in the planning designed by a secondary level mathematics teacher, and the specialised knowledge deployed both at the design stage and in the teacher's reflections after the lesson. The research method followed was an instrumental case study via content analysis. The study contributes to the development of an analytical model for studying mathematics teachers' specialised knowledge.

INTRODUCTION

In Skott, Van-Zoest and Gellert (2013), there is a call for research into the connections between mathematics teachers' knowledge, conceptions, and identity. In this work, we focus on the two-way connections between a mathematics teacher's conceptions and her specialised knowledge in the context of several typical practices.

By considering the design of, and reflection on, various class activities, we study the knowledge brought into play by a mathematics teacher at the planning stage, and the connections between this knowledge and the teacher's conceptions about teaching and learning the subject.

Viewed from a cognitivist perspective (Ponte, Quaresma & Branco, 2012), we consider the design of learner tasks, their management and the teacher's subsequent reflection upon them, as something which embraces multiple professional practices. In this instance, we consider the teacher's intentions, management and reflections regarding the interaction of the activities with her pupils and with hypothetical situations arising from aspects of the plan.

In response to the teacher's plan, which takes an experimental approach with equally likely outcomes, we delve deeper into the Conceptions about Mathematics Teaching and Learning (CMTL) reflected in the design itself, and seek to locate the specialised knowledge brought into play via descriptors drawn from the corresponding subdomains of the Mathematics Teacher's Specialised Knowledge model [MTSK] (Carrillo, Climent, Contreras & Muñoz-Catalán, in press).

THEORETICAL FRAMEWORK

In this section, we situate the study within the ambit of professional practice, focusing discussion on the practices of anticipating and interpreting the pupils' modes of thinking, and on the teacher's classroom management and *post hoc* reflections. As

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regards specialised knowledge, we draw on the subdomains of MTSK and its corresponding theoretical underpinnings. Finally, we consider the notion of conception, and the position we take in this respect *vis-à-vis* the interpretation of the data extracted from the design of the activities.

Mathematics teachers' professional practices

We consider professional practice as anything which forms part of the teacher's workload which is closely related to the promotion of their pupils' learning (e.g. Branco & Ponte, 2012). Viewed thus, professional practices go beyond the teacher's role *at the front of the class* and include activities which are undertaken outside the classroom. Such scenarios of professional practice offer plentiful opportunities to deepen our understanding of specialised knowledge (Flores, Escudero, & Aguilar, 2013). Below we cite examples of professional practices noted by various researchers (including unintentional ones), and indicate those we analyse in this study.

Stein, Engle, Smith and Hughes (2008) propose a series of professional (interdependent) practices for orchestrating productive discussions about mathematics, which they sequence thus:

> (1) anticipating likely student responses to cognitively demanding mathematical tasks, (2) monitoring students' responses to the tasks during the explore phase, (3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase, (4) purposefully sequencing the student responses that will be displayed, and (5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas. (p. 312)

Most of these practices directly involve the teacher's interaction with their pupils. Nevertheless, behind each, especially that of anticipating, is the need for a practice undertaken outside the classroom in the form of planning and reflecting on the outcomes of the lesson.

Ponte *et al.* (2012) describe and discuss two common practices, the presentation of tasks to students and group discussions. They propose a framework for studying these practices, which is intended to be serviceable irrespective of whether such studies take a cognitivist or sociocultural approach. The framework considers:

> (1) the teacher's aims, the way in which these give rise to achievable objectives, and how they are given shape through various professional actions, […] (2) the social context and the educational context, […] (3) the classroom context, […] (4) the teacher's professional knowledge, […] (5) the teacher's know-how, [… and] (6) the teacher's capacity for reflection. (p. 84)

In our study we focus specifically on the facets numbered 1, 4 and 6 above.

In their model of Mathematical Knowledge for Teaching (MKT), Ball, Thames and Phelps (2008) include within the knowledge subdomain they dub Specialized Content Knowledge (defined as the mathematical knowledge and skill unique to teaching) elements such as:

> Teaching […,] requires understanding different interpretations of the operations in ways that students need not explicitly distinguish […, teachers] must be able to talk explicitly about how mathematical language is used […]; how to choose, make, and use mathematical representations effectively […]; and how to explain and justify one's mathematical ideas. (p. 400)

In Flores, Escudero and Carrillo (in press), the authors conclude that, more than identifying mathematics teachers' specialist knowledge, the examples describe tasks forming part of teachers' work, and that different kinds of knowledge (mathematical, syntactic, learning styles and others) are required for teachers to carry these out. In other words, although the authors talk about SCK in terms of knowledge, what is actually exemplified seems to relate closer to the idea of mathematics teachers' professional practice.

The professional activity on which we focus in this paper is the design of classroom tasks, and we explore aspects of conceptions and knowledge, looking at three professional practices: the prediction and the interpretation of the pupils' way of thinking, and the *post hoc* reflection by the teacher involved in the study.

Mathematics Teacher's Specialised Knowledge

Various models relating to mathematics teachers' professional knowledge are available (e.g. Usiskin, 2002; Bretscher, 2012). In particular, MTSK focuses on the study of the kind of knowledge which is relevant only to mathematics teachers (Escudero, Flores, & Carrillo, 2012). This model is based on consideration of two of the knowledge domains proposed by Shulman (1986), Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), and offers a refinement (e.g. Montes, Aguilar, Carrillo, & Muñoz-Catalán, in press) to the knowledge subdomains proposed in MKT by Ball et al (2008). It seeks to address, principally, two issues detected in MKT – the difficulty in demarking some subdomains from others, and the tendency of some descriptors not to be phrased purely in terms of elements of knowledge (Carrillo *et al.*, in press).

In MTSK, there are three subdomains in respect of MK: Knowledge of Topics, KoT (including phenomenological aspects, meanings, definitions, and examples characterising aspects of the topic of study), Knowledge of the Structure of Mathematics, KSM (including an integrated system of connections which enables advanced concepts to be understood and developed from an elementary perspective, and elementary concepts from an advanced one), and Knowledge of the Practice of Mathematics, KPM (knowledge of the forms of knowing, creating and producing in mathematics, knowledge of aspects of mathematical communication, reasoning and proof). Three other subdomains are considered in PCK: Knowledge of Mathematics Teaching, KMT (knowledge of different strategies enabling the teacher to develop procedural and conceptual mathematical abilities, knowledge of the potential of resources, examples and other means of representation for making a specific content more comprehensible, and knowledge of educational theory relating to mathematics), Knowledge of Features of Learning Mathematics, KFLM (knowledge of the characteristics of the pupils' learning process for different contents, the language associated with each concept, and potential errors, difficulties and obstacles, theoretical knowledge about learning mathematics) and Knowledge of Mathematics Learning Standards, KMLS (knowledge of what the pupils should/can achieve by the end of a particular school year, knowledge of the procedural and conceptual abilities and mathematical reasoning promoted in specific educational stages).

MTSK offers this study useful categories for exploring knowledge. We start with general questions arising from the nature of the two knowledge domains and analyse these with specific categories drawn from each subdomain.

Conceptions of teaching and learning mathematics

We understand a conception as the "conscious or unconscious [set of] beliefs, concepts, meanings, rules, mental images and preferences concerning mathematics" (Thompson, 1992, p. 132).

Leatham (2006) takes a position regarding the study of conceptions, with which we concur. Introducing the term Sensible System Framework, the paper suggests that rather than focusing on inconsistencies between declared conceptions, those inferred from classroom performance and those drawn from teacher reflections, all such aspects could be observed as a sensible system which accounts for itself. We also agree that conceptions represent a predisposition towards action and that they cannot be directly observed or measured, only inferred.

For data analysis, we used the categories and indicators put forward by Carrillo (1998), which, in terms of CMTL, distinguishes four kinds of conceptions (referred to as teaching tendencies in order to foreground the difficulty of ascribing an individual teacher to any single conception): the traditional, the technological, the spontaneous, and the investigative. Again, it should be stressed that these categories are not designed for placing teachers in particular boxes according to their conceptions, but it is the case that teachers tend to show predilections towards the indicators of one tendency or another.

METHOD

The research design follows that of an instrumental case study (Stake, 1994), and was carried out by means of content analysis (Bardin, 2002). The study itself is part of a wider study seeking to establish connections between varying elements of MTSK.

The work uses the indicators described by Carrillo (1998) to identify the CMTL reflected in the design of activities by a secondary level mathematics teacher (Carol), and we allowed the rationale underpinning this design to guide our analysis.

The identification of the specialised knowledge Carol brought into play was achieved through an open-ended interview in which she was presented with hypothetical situations. The interview was structured according to the tasks that Carol had used in class, and focused on the following aspects of MTSK:

With respect to knowledge of content: (a) the knowledge she expected her students to learn; (b) the knowledge she used, or could have used, in the design and execution of the tasks and in reflecting on the results; and (c) the knowledge which, as researchers, we anticipated could be appropriate to planning the tasks, carrying them out and reflecting on the results.

With respect to pedagogical content knowledge: (d) knowledge of the students' habitual ways of working; (e) knowledge of the ways in which the students' thinking develops; and (f) knowledge of teaching strategies which promote specific behaviour in the students.

RESULTS

This section is divided into three parts. The first talks about our findings regarding the CMTL reflected in Carol's design. The second part concerns the items of specialised knowledge we identify with the help of the design itself and Carol's responses in the open-ended interview and the hypothetical situations. Finally, we suggest connections between the CMTL and the items of knowledge identified.

The rationale of the design: inferred conceptions

Carol's design consisted in choosing which result would appear most frequently when an object is thrown, first a coin, and second a dice. The complete rationale of the design (with each object) is thus: (a) predicting which event will occur the most number of times on throwing an object n times; (b) experimenting, recording the results and comparing these with the prediction; (c) predicting which event will occur the most number of times on throwing an object m times ($m>n$); (d) experimenting, recording the results and comparing these with the prediction; (e) predicting which event will occur the most number of times on throwing an object s times ($s>m$); and (f) dividing the number of times the pre-selected result occurred in the experiment by the total number of throws. In each stage, the students compare their results with those of classmates.

The repetition of predicting and experimenting was intended to guide the students towards recognising a pattern of equal probabilities, and this, taken together with the increased number of throws and the calculation of the quotient, indicates a conception of the acquisition of mathematical knowledge as a reproduction of the logical processes of the construction of content. For Carol, the significance of including experimentation in the class was both as a source of motivation encouraging student participation, and as a means of informally assessing student knowledge of the sample space of the event.

Although it is not our intention to categorise Carol as pertaining to a particular teaching tendency, the association she establishes between her design and the students' learning, based on the construction of meaning through the application of logical procedures, is a characteristic feature of the technological tendency (Carrillo, 1998).

Knowledge based on MTSK

With respect to her knowledge of the theme of *equally probable events*, Carol demonstrates her understanding of the connections with this topic and that of fractions, percentages and sample spaces. Likewise, she distinguishes between those events which have equal probability and those in which certain outcomes are more likely to occur:

> Carol: There are fewer combinations to add up [the faces of two dices] to appear *one* and *one* [...] the ones with a greater probability are the ones in the middle [… there are events in which you can consider] previous results [so as to] predict, but it's by no means certain.

The knowledge represented here can be considered as pertaining to KoT (knowledge of connections between elements of a concept, definitions and properties). Nevertheless, although Carol recognises the importance of determining the sample space of the events, neither in her design, nor her subsequent reflections, does she include as part of the space the event of, at least, two outcomes occurring exactly the same number of times, although she does recognise that her students do not typically consider this event as part of the space.

> Carol: My students never say it will turn out a draw [that heads and tails will occur an equal number of times], they choose either more heads or more tails, but not an equal number… well, perhaps a few say so, but most of them don't.

One area which is considered part of KFLM is that of predicting how students will think and act, and in this respect the teacher mentions strategies her students employ in predicting results based on previous outcomes, such as looking for patterns:

> Carol: [My students] would have thought to themselves: "there's a pattern here, first I called heads and it came out tails, then I called tails and it came out heads, now I'll see if it comes out tails again [...] which gives them the same result" [out of 10 throws].

Carol regards experimentation as a learning strategy which, besides motivating the students, allows them to explore possible outcomes. Knowledge of such teaching strategies which directly bear on mathematical content is considered part of KMT.

As for mathematical knowledge recognised by the researchers as being necessary to Carol's design, this consists of fully determining the sample space (that is, the consideration that at least two outcomes might occur an equal number of times), and knowledge of Bernoulli's experiment for deliberately choosing the number of experimental repetitions.

Potential connections between items of MTSK and CMTL

The analysis has brought to the fore the appearance of features of the technological teaching tendency. Although all teachers need knowledge of the logical processes of constructing the mathematical knowledge to be learnt by the students, the use of this knowledge is especially relevant in relation to the aforementioned features. According to evidence drawn from Carol's lesson episodes, this has meant the incorporation of elements of distinct natures. On the one hand, the knowledge of definitions, connections within the concept and properties such as the law of large numbers, shows a deep knowledge of the topic which allows its reconstruction. On the other hand, with respect to knowledge of connections with more advanced topics, Carol considers it unnecessary for this lesson, although she admits to using more advanced knowledge than that actually deployed in class at other times in the planning phase. Carol's attested pedagogical content knowledge centres on the objectives of her plan and the impact this might have on her students, as a result of which there is an emphasis on being aware of the options facing the students when they come to do the activities, and the strategies they might employ, whether correct or incorrect, in carrying them out. Carol's knowledge in this respect bears features of a technological conception regarding the teacher's role, specifically, the transmission of knowledge through technological procedures and a presentation style in which she adopts the role of technician organising content and design.

CONCLUSIONS

Through the case study of Carol's teaching we aimed to understand the two-way connections between conceptions and mathematics teachers' specialised knowledge. The study focused on various practices typical of mathematics teachers, and explored the utility of an emergent model designed to study the knowledge involved, MTSK. The connections are consistent in that the knowledge deployed by Carol (and likewise that which the researchers detect as potentially necessary) emerges from her intentions for the lesson. Further studies are clearly necessary to explore the connections between the multiple elements of teachers' knowledge, and the ways these impact on their teaching and their students' learning.

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