THE DEVELOPMENT OF THE VISUAL PERCEPTION OF THE CARTESIAN COORDINATE SYSTEM: AN EYE TRACKING STUDY

Dmitry Chumachemko[1], Anna Shvarts[1], Aleksandr Budanov[2]

[1]Lomonosov Moscow State University, [2] Russian State University for the Humanities

The aim of the research is to investigate the transformation of the perception process through mathematics education, by an example of scanning the Cartesian coordinate system in order to locate a target point. We compared participants with different competence in mathematics. Historically, motion along axes appeared as a specific "theoretical" action that constituted the Cartesian coordinate system. We detected this specificity as a dominance of vertical and horizontal saccades of eye movements in perception processes of all groups of participants. Experts-novices differences dealt with an ability of experts to use additional essential information and to discard unnecessary data. Furthermore, a lot of evidences of shortening of the perception actions from novices to experts are presented.

THEORETICAL FRAMEWORK

The theoretical background of this work is based on the culture-historical tradition and an activity approach in its application to the development of perception, which are based on the dialectical-materialistic philosophy. From this point of view the process of perception should be constituted in accordance with scientific theoretical understanding of a represented object. Davydov supposes that the correct perception of visual models should appear by developing "special *object-related actions* by which they [students] can disclose in the instructional material and reproduce in models the essential connection in an entity" (Davydov, 1972/1990, p. 174). These special actions have been elaborated through cultural-historical development of the represented object and they, according to Davydov, need to be approached by a child through specially constructed educational activity.

Radford, following Marx, also assumes that an eye as a receptive organ should be converted to a "theoretician" and then it would be able to perceive scientifically essential features of figures. Radford (2010) supposes that this transformation occurs due to participation of a student in spontaneous but cultural forms of activity in classroom, which includes gestures, voice intonations and other embodied aspects of learning: "the senses … *become* shaped in certain historically formed ways as we engage in sociocultural practices" (Radford, 2010, p.2). So, by one or another way, education transforms the perception process of a student into historically elaborated system of actions, which allows detecting essential features of a visual model.

Let us now turn to some historical information in order to trace the main steps of the formation of the Cartesian coordinate system and to reveal the transformations of

2014. In Nicol, C., Liljedahl, P., Oesterle, S., & Allan, D. (Eds.) Proceedings of the Joint Meeting of PME 38 and PME-NA 36,Vol. 2, pp. 313-320. Vancouver, Canada: PME.

perception that are needed to approach this mathematical visual model. The history of the Cartesian coordinate system had started long before R. Descartes in ancient time from the practical usage of a rectangular grid on the plane in astronomy and in geography independently (Jushkevich, 1970, p. 98). The Cartesian system of coordinates appeared in mathematics due to attempts to find analytical descriptions of geometrical curves by Fermat and Descartes (Jushkevich, 1970). An idea was that the distance from one given line to a point of a curve could be counted by measuring the distance *along* another selected direction. Both Fermat and Descartes drew only one axis and the direction of another one, which usually wasn't perpendicular to the first one. Gradually, a rectangular system of coordinates became more popular.

Another important difference from the modern Cartesian coordinate system was that Descartes used only positive numbers; directions of axes could differ from one illustration to another. Fluent usage of both axes and negative coordinates appeared in 18 century (Jushkevich, 1970; Burton, 2011).

So, we suppose that the specific action, which is needed to perceive the Cartesian coordinate system, is a motion along one of the axis in order to find a distance from zero-point to a projection of a target point. Another important step towards correct perception of coordinate system is an ability to find a correspondence between a positive or negative value of a coordinate and an axis orientation.

This study claims that a child needs to acquire the specific ways of perception that were elaborated in the history of mathematics, during his educational practice. There are *three stages* of perception development distinguished by investigations framed in activity theory (e.g. Zaporozhets, 1986/2002). The first stage includes external, material actions with objects; for example a child could run along the axes by an index finger. The second stage reflects deployed sensory processes in which perceptual actions "are performed with the aid of motions of receptor apparatuses and anticipate subsequent practical actions" (Zaporozhets, p. 41). At this stage we should find a movement of eyes by the same route as fingers run at first stage. The third stage is a stage of most mature perception, the stage of shortening and automation. "Orienting-research action transforms into ideal action, into the movement of attention across the perception field" (p. 42), writes Zaporozhets.

Our *research question* dealt with the transformation of the perception process by mathematics education: whether indeed matured perception of Cartesian plane includes specific "theoretical" actions, revealed in this research in our historical analysis, and whether these kinds of actions become dominant in perception of highly mathematically educated respondents. We also investigated a shortening of perception actions proposed by psychologists of activity approach, as a way of transformation of external perceptual actions into mental ideal actions.

METHODOLOGY OF RESEARCH

Eye-tracking methodology appears more and more often in educational researches especially in area of multimedia learning, including multi-representational materials (van Gog & Scheiter, 2010). Only a few papers are devoted to mathematics education. Most of these papers are framed by a semiotics paradigm and analyze perception of representations as signs of mathematical objects. They calculate numbers and describe kinds of saccades between representations and compare number and duration of fixations in different area by participants with different level of mathematics competence. As Andra et al. write (2013), all kinds of analyses could be distinguished to macrolevel (analysis of shifting between representations, e.g. Andra et al., 2009; Andra et al., 2013), mesolevel (analysis of attendance of each representation by a gaze) and microlevel (analysis of activity inside of one representation, e.g. Epelboim and Suppes, 2001; Peter, 2010).

Our research is focused on microlevel analysis since a development of perception, which doesn't supposed by classical semiotics perspective, could be understood only through deep analysis of interaction of a subject with a visual representation. The most well founded fact is that experts are able to detect significant parts of representations (thus more fixations by experts were found in these areas in comparison with novices' fixations). It was shown for such knowledge domains as sports, medicine, transport (Gegenfurtner, 2012), zoology (Jarodzka, Scheiter, Gerjets & van Gog, 2010), meteorology (Canham & Hegarty, 2010) and others. In mathematics there are evidences that experts are able to focus on a blank area, which is essential for additional constructions in geometry (Epelboim & Suppes, 2001).

From cultural-historical point of view, all evidences of magnetism by essential parts of a picture or a text for experts could be interpreted as a reorganization of the perception process in accordance with their deeper theoretical knowledge. In our research we investigated if experts are able to choose an appropriate quadrant of the Cartesian plane faster than novices. We supposed that theoretical knowledge about negative or positive coordinate of a target point would influence on the perception process of experts to create an ability to use this information in a search for a point.

Applying ideas of the activity approach we were focused on a procedural aspect of perception trying to understand perceptual actions. We supposed that directions of saccades reflect cultural way of approaching the Cartesian coordinates system: saccades should be performed along axes. So vertical and horizontal saccades should prevail on any other directions if perception is reorganized in a theoretical way.

Participants

In our research we compared eye movements of participants of 3 levels of mathematics competence. There were 11 participants with higher mathematics education, 23 students of a first year of non-mathematical departments at University (they have passed school mathematics exam), 10 students of 9-11 grade of high school (14-16 years old). We will refer to these groups as experts, intermediates and novices.

Apparatus

For data collection we used SMI RED eye tracker with a sampling frequency of 120 Hz, participants seated at approximately 40-50 cm distance from a monitor. IViewX was used for tracking eyes' activity. Stimuli were presented by Experiment Center 3.0. Data analysis was conducted by Begaze 3.1 and SPSS 20.0. Before the main experimental procedure, eye tracker was calibrated for an each participant by 9-points procedure with validation. Only those participants who showed better than 0,5 degree accuracy at the calibration stage were accepted for main experiment.

Materials and procedure

Each participant had to solve 9 tasks on detection a point on the Cartesian plane with determined coordinates. There was an instruction at the beginning of experiment: "Now you will receive tasks on the Cartesian coordinate system. Try to solve these tasks as accurate and as fast as you can." Each task consisted of three slides: 1) a task with the coordinates of a point, 2) the Cartesian plane with two axis and four labeled points, 3) labels of the points to choose a correct answer. All tasks had an equivalent wording, for instance: "Choose a point with coordinates (3, -4)". There were one or two points (of four) in target quadrant of the Cartesian plane. Participants switched from one slide to the next by pressing Space bar. There were no time limits either for reading of a task or for searching for a point.

Hypotheses

1. Vertically and horizontally directed saccades are prevailed on saccades with other directions. This ratio is more pronounced for experts than for novices.
2. A number of fixations in irrelevant quadrants of Cartesian plane decrease with growing of participant competence.
3. Perceptual actions lessen with growing of competence: the better participants are educated, the shorter are their gaze paths, and the more the number of their fixations is reduced, and the durations of their tasks solving become shorter.

DATA ANALYSIS AND RESULTS

First part of analysis dealt with directions of saccades. The problem is that the standard algorithm of saccades detection implemented in Begaze 3.0 defines a saccade as a vector from the center of an initial fixation to the center of an ending fixation. Thus our observations of raw eye-movements showed that directions of many saccades were calculated incorrectly due to significant drifts during fixations (Figure 1 gives an example of raw data). We elaborated our own software that detected saccades by simple Velocity Threshold algorithm (Salvucci & Goldberg, 2000). Eye movements were considered

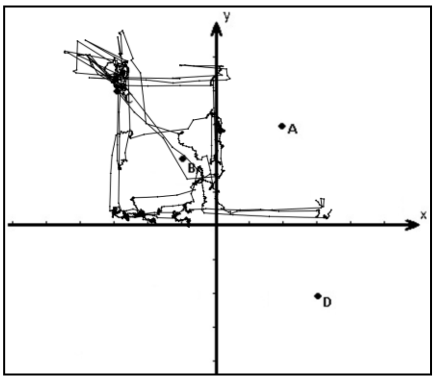

Figure 1: An example of raw eye movements by experts

saccades when velocity exceeded 120°/sec. Therefore, in respect to fixations, a saccade appeared as a vector from the point where the previous fixation is completed to the first point of the next fixation (instead of fixation centers). Saccade direction was computed as an angle from 0° to 90°. All saccade directions were divided into 6 sectors by 15 degrees: from a sector 0°-15° to a sector 75°-90°. Saccades of the first and the last sector were considered as horizontal and vertical respectively. The mean numbers of saccades of different directions were compared by repeated measures ANOVA with mathematics competence as a between group factor and saccade direction sector as within subject factor.

Saccades with vertical or horizontal orientation appeared approximately 4 times more often than those with directions from other sectors ($F=31.554$, $p<0.001$), see Figure 2. This ratio is stable across groups. In spite of this fact we found a significant interaction between factors ($F=4.225$, $p=0.021$): a dominance of vertical and horizontal saccades is most noticeable for novices, and intermediates also use vertical and horizontal saccades 4 times more often than all other saccades. But experts use only vertical, but not horizontal saccades as often as intermediates. Also it was shown that a total number of saccades decreases with mathematics competence ($F=5.446$, $p=0.008$); the result confirmed our third hypothesis about the shortening of the perception process.

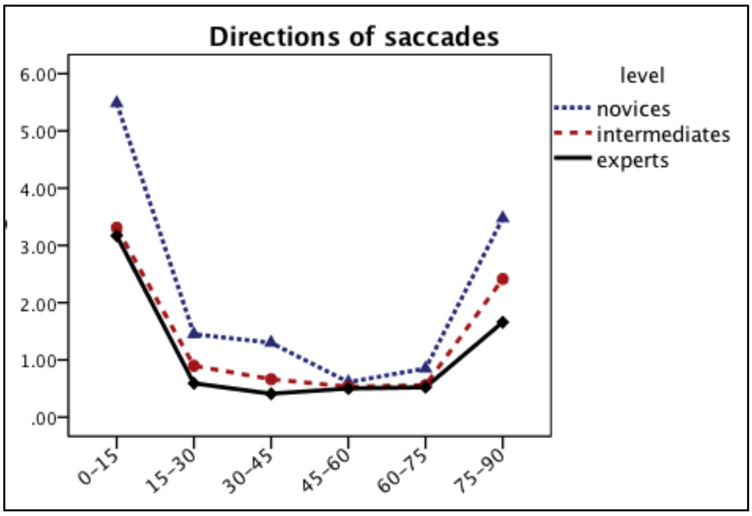

Figure 2: Directions of saccades in groups with difference competence

Next part of analysis was dedicated to participants' ability to be focused on essential parts of a diagram. We defined six AOI (Aries of Interest): four quadrants of the plane and two axes. A target point belonged to one of the quadrants and this target quadrant could be figured out only by taking into account the sign (positive or negative) of the both coordinates. Three other (non-target) quadrants are irrelevant to the task. Number of fixations in irrelevant AOIs decreases with mathematics competence (Kruskal Wallis Test, $\chi^2=11.065$, $p= 0.004$). There were large individual differences but at the average expert did only 3.5 fixations (6% of all fixations) in irrelevant AOIs for the whole session of 9 tasks, intermediates and novices did 10.2 (14.8%) and 14.1 (15.3%) irrelevant fixations correspondently.

To investigate shortening of perceptual actions we compared 1) number of fixations, 2) length of the gaze paths, 3) total time to solve each task in different groups using repeated measures ANOVA with mathematics competence as between group factor and task as within subject factor. Means and statistics for all parameters for each group are presented in Table 1. All parameters significantly indicated the reduction of explicit perceptual actions from novices to experts. Also specific tasks significantly influenced the process of problem solving (p<0.001). Significant interactions between tasks and competence were found for all parameters (p<0.03). Below we'll consider only the number of fixations.

	Level of competence			Results of ANOVA	
Parameter	novices	intermediates	experts	F	p (sig.)
Time of solution (sec)	4.638	3.285	2.681	4.916	0.013
Number of fixation	14.02	9.8	7.54	5.794	0.006
Gaze path (px)	1810.2	1250.1	814.5	5.744	0.007

Table 1: Parameters of the shortening of perceptual actions (between group analysis)

Figure 3 represents mean number of fixations for each task in different groups. First task provoked the most explicit search by novices and intermediates, which was reduced in next tree tasks. Experts solved first task with the number of fixations comparable to all other tasks. In contrast to the previous tasks, tasks 5, 6, 7, 8 had two points in the target quadrant (one correct and one wrong). Figure 3 shows that the presence of an additional point influenced the search process in novices, while perception of intermediates and experts is kept as short as it was in tasks with the only one point in the target quadrant. Altogether, the results provide evidence for shortening of perception from one group to another and from the first task to the following tasks.

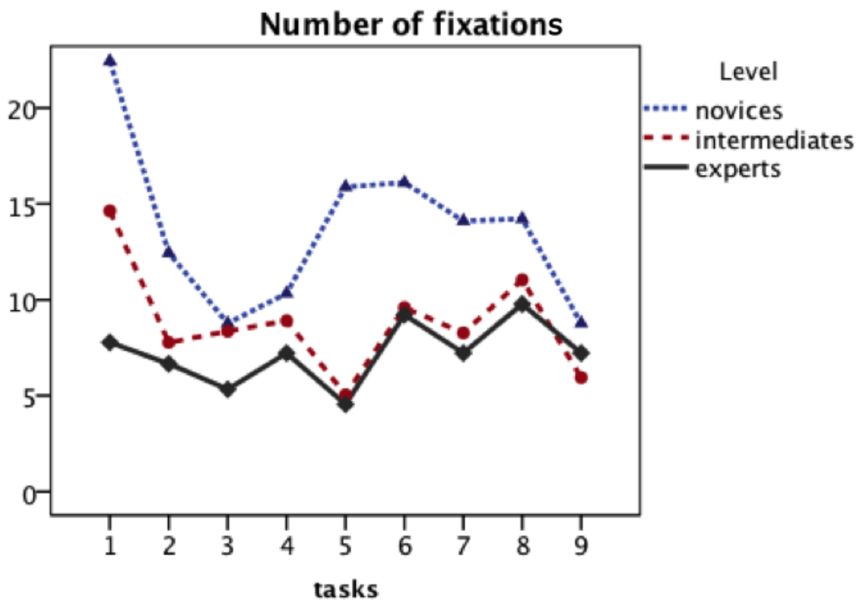

Figure 3: Reduction of fixations from group to group

DISCUSSION AND SOME CONCLUSIONS

The main result is that we have found an evidence of "theoretical" perceptual actions: vertical and horizontal saccades appeared much more often than saccades of other directions. We observed these specific actions in perception of participants with all levels of mathematics competence. It means that their perception had been transformed in a cultural way already. And that special system of tasks (as Davydov (1972/1990) claims) is not needed for it; but it could be acquired through classroom practices as Radford (2010) supposed.

Another possible interpretation is that this way of perception is a natural one for these tasks. In order to trace “non-theoretical” perception, where the main principle of motion along the axes is not approached yet, we intend to collect data from less experienced participants in our future work, and check if their perception is not vertically and horizontally organized. Only then will we be able to claim that perception was transformed to be “theoretical” as Radford anticipates (2010).

Davydov supposed that the transformation of the perception process is a result of theoretical understanding. Our results show that perception, which is structured by special actions (vertical and horizontal motions), could still be enriched by additional knowledge about negative or positive values of coordinates. Our Hypothesis 2 was confirmed: experts were almost never focused in irrelevant parts in comparison with other groups (6% for experts vs. about 15 % for other groups). Empirically the result repeats the evidences that experts are able to distinguish essential parts of visual representations (e.g. Gegenfurtner, 2012; Jarodzka et. al, 2010; Canham & Hegarty, 2010). But what is more interesting is that this result is similar to observations by Andra et. al (2009), that novices more often revisit different alternatives of answer than experts. From the activity theoretical point of view it means that experts conduct only executive actions, which lead to almost algorithmic solution, while novices need to perform orient-research activity to construct an image of a representation of the task and an algorithm how to perceive it (e.g. Zaporozhets, 1986/2002).

We also observed that the experts were able to solve the tasks using only necessary information, while missing additional data: there was a reduction of vertical saccades from intermediates to experts when horizontal saccades were performed with the same frequency (Figure 2). Indeed, the second coordinate wasn’t necessary to choose a correct answer.

As it was expected (Hypothesis 2) we have shown that orient-research parts of actions are reduced in perception of experts (they had the less amounts of fixations, the shorter gaze paths and the faster solutions (Table 1). It is interesting that this difference was especially strong for the first task (see Figure 3). We can explain it as follows: in the first task the orient-research activity of novices and intermediates was unfolded and it allowed them to construct an appropriate algorithm of perceptual actions. This algorithm was applied in further tasks. But new elements in tasks 5-8 (see Figure 3) broke the perception process of novices and returned it to the stage two of orient-research actions (see above from Zaporozhets, 1986/2002), while perception of experts and intermediates kept its maturity.

In summary, an inclusion of special “theoretical” actions in perceptual process appears as only a first stage in the transformation of perception by education. The difference between experts and novices deals with the ability of experts to use additional essential information and to discard unnecessary data. Apart from this other evidences related to the shortening of perceptual actions from novices to experts was found. So, being culturally organised, perception continues its development in order to find the shortest and simplest way and at the same time to include theoretical information. Future investigation of less experienced participants is also necessary.

Acknowledgment

This research was supported by RFH research grant № 12-36-01408.

References

Andrà, C., Arzarello, F., Ferrara, F., Holmqvist, K., Lindström, P., Robutti, O., & Sabena, C. (2009). How students read mathematical representations: An eye tracking study. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proc. 33rd Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 2, pp. 49-56). Thessaloniki, Greece: PME.

Andrà, C., Ferrara, F., Arzarello, F., Holmqvist, K., Lindström, P., Robutti, O., & Sabena, C. (2013). Reading mathematics representations: An eye tracking study. *International Journal of Science and Mathematics Education*, 1-23. doi: 10.1007/s10763-013-9484-y

Canham, M., & Hegarty, M. (2010). Effects of knowledge and display design on comprehension of complex graphics. *Learning and Instruction*, *20*, 155-166.

Davydov, V. V. (1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricula*. (J. Teller, Trans.) Reston, VA: National Council of Teachers of Mathematics. (Original work published 1972)

Epelboim, J., & Suppes, P. (2001). A model of eye movements and visual working memory during problem solving in geometry. *Vision Research*, *41*, 1561-1574.

Gegenfurtner, A., Lehtinen, E., & Säljö, R. (2011). Expertise differences in the comprehension of visualizations: A meta-analysis of eye-tracking research in professional domains. *Educational Psychology Review*, *23*(4), 523-552.

Jarodzka, H., Scheiter, K., Gerjets, P., & van Gog, T. (2010). In the eyes of the beholder: How experts and novices interpret dynamic stimuli. *Learning and Instruction*, *20*, 146-154.

Jushkevich, A. P. (Ed.). (1970). *History of mathematics, volume 2: Mathematics of 17th century* [in Russian]. Moscow: Nauka.

Radford, L. (2010). The eye as a theoretician: Seeing structures in generalizing activities. *For the Learning of Mathematics*, *30*(2), 2-7.

Salvucci, D. D., & Goldberg, J. H. (2000). Identifying fixations and saccades in eye-tracking protocols. In S. Spencer (Ed.), *Proceedings of the eye tracking research and applications symposium* (pp. 71-78). New York: ACM Press.

Van Gog, T., & Scheiter, K. (2010). Eye tracking as a tool to study and enhance multimedia learning. *Learning and Instruction, 20*, 95-99.

Zaporozhets, A.V. (2002). The development of perception and activity. (V. Zaitseva, Trans.) *Journal of Russian and East European Psychology*, *40*(2), 35-44. (Original work published 1986)