

Not Just IQ: Patterning Predicts Preschoolers' Math Knowledge Beyond Fluid Reasoning

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Abstract

Preschoolers' patterning skills are predictive of their concurrent and later math knowledge; however, it is unclear if patterning is only a proxy for general intelligence, or how it might support specific math skills. The current study examined the relation between 66 preschool children's patterning skills and their general cognitive abilities, including fluid reasoning, working memory, and spatial skill. Further, the link between patterning and general math knowledge and specific math skills (i.e., numeracy and shape knowledge) was examined. Children's patterning skills were significantly but only moderately linked to their performance on general cognitive ability measures (e.g., fluid reasoning). Further, beyond the effects of these general cognitive abilities, patterning significantly predicted both general math and numeracy knowledge, as well as verbal calculation and magnitude comparison. Thus, although patterning skills related to measures of general cognitive ability, patterning was a unique contributor to children's general math and numeracy knowledge and specific math skills. Theories of math development and early math standards should thus be modified to emphasize the role of children's patterning skills to their math development.

Keywords: Pattern skills, spatial skills, mathematical development, cognitive ability

Not Just IQ: Patterning Predicts Preschoolers' Math Knowledge Beyond Fluid Reasoning

Children's mathematics knowledge prior to formal schooling is important to their future achievement. Specifically, math knowledge at school entry varies substantially (Starkey, Klein, & Wakeley, 2004) and strongly predicts later math and reading skills (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Nguyen et al., 2016; Watts, Duncan, Siegler, & Davis-Kean, 2014). Further, better math knowledge is associated with higher incomes, attainment of more prestigious careers, and better healthcare decisions (Lipkus & Peters, 2009; Ritchie & Bates, 2013; Shapka, Domene, & Keating, 2006). While most research has explored the contributions of early numeracy skills to math development, some work has highlighted that patterning skills (the ability to notice and use predictable sequences) also predict children's math development (Rittle-Johnson, Fyfe, Hofer, & Farran, 2017; Rittle-Johnson, Zippert, & Boice, 2019). There is debate as to whether patterning skills are simply an indicator of general cognitive ability (i.e., fluid reasoning) or serves as a unique predictor of math ability (Burgoyne, Witteveen, Tolan, Malone, & Hulme, 2017; Papic, Mulligan, & Mitchelmore, 2011; Psnak et al., 2016), especially because patterning skills are predictive of and causally related to children's development in both math *and* reading knowledge in a few studies (Burgoyne et al., 2017). Further, little is known about what specific math skills might be supported by patterning skills.

The current study had three aims. The first was to examine the relation between children's general cognitive skills and their patterning skills. This was important to begin to distinguish patterning from general cognitive ability (e.g., fluid reasoning). Second, we examined whether patterning skills related to children's general math skills beyond general cognitive ability. Finally, we examined the relation between children's patterning skills and specific math skills (i.e., numeracy and shape knowledge and specific numeracy skills). In the

following sections, we describe patterning in preschoolers, how this skill is related to fluid reasoning and other general cognitive abilities, and how it might relate to math knowledge.

Patterning Skills

Patterning skill refers to the ability to notice and use predictable sequences, which include sequences of shapes, sounds, or numbers. One type of pattern, repeating patterns, refer to linear patterns with a repeating unit, such as triangle-triangle-circle-triangle-triangle-circle. These types of patterns are most developmentally appropriate for preschoolers because they do not require explicit content knowledge (e.g., number knowledge), as with some types of growing patterns (e.g., 2, 4, 6, 8). Instead, repeating patterns can often be identified by focusing on a single physical dimension, such as shape or color, and noticing that there is a part that repeats. Typical patterning assessment tasks, presented in increased difficulty, include duplicating and extending a visible model pattern with the same materials, and duplicating patterns with different materials (i.e., abstract patterning; Papic et al., 2011; Rittle-Johnson, Fyfe, Loehr, & Miller, 2015; Sarama & Clements, 2004; Starkey et al., 2004).

Repeating patterns are emphasized regularly by young children, parents, and teachers. Previous research has found that patterning and shape activities were the most common mathematical activities observed during free play of 4- and 5-year-olds (Ginsburg, Lin, Ness, & Seo, 2003). Parents report engaging their children in a range of pattern activities on a monthly basis (Zippert & Rittle-Johnson, 2018). Additionally, preschool teachers view patterning activities as important (Clarke, Cheeseman, & Clarke, 2006; Economopoulos, 1998) and report doing frequent pattern activities in the classroom (Rittle-Johnson et al., 2015). These classroom pattern activities focused on creating, duplicating, extending, and naming repeating patterns.

Relation Between Patterning and General Cognitive Abilities

Theory and emerging empirical findings have suggested that children's patterning skill is linked to a range of general cognitive abilities, including fluid reasoning, working memory, and spatial skills (Burgoyne et al., 2017; Wijns, Torbeyns, De Smedt, & Verschaffel, in press). We present evidence in support of these associations in turn.

Patterning and fluid reasoning. Patterning may be especially tied to fluid reasoning (Burgoyne et al., 2017), which is the general cognitive ability to think logically and problem solve in novel contexts without reliance on specific content knowledge (Carroll, 1993; Cattell, 1987). Part of this inductive type of thinking involves the ability to notice and perceive patterns, and use such evidence to predict future events (e.g., what comes next). In fact, tasks involving pattern detection and completion, such as the Raven's Progressive Matrices, are commonly included on fluid intelligence measures (Wechsler, 2003). Further, researchers who have successfully trained patterning skills to improve math knowledge in first graders have theorized that by training patterning skills, they were actually promoting fluid reasoning, which in turn improved children's mathematics knowledge (Kidd et al., 2014; Pasnak et al., 2016). The direct relations between patterning and fluid intelligence measures, however, have yet to be empirically assessed.

Patterning and working memory. Working memory, defined as a short-term mental system that manipulates temporarily stored information related to a task (Baddeley & Logie, 1999), may be especially important for thinking about patterns. For example, when abstracting repeating patterns (i.e., recreating the underlying structure of a model pattern using different materials), children may simultaneously mentally store and transform items from the model pattern to create an abstracted version (Collins & Laski, 2015). Further, repeating patterning

tasks likely become more taxing on working memory as the number of items in a pattern's structure increase (Miller, Rittle-Johnson, Loehr, & Fyfe, 2016).

Indeed, research has found support for this link. In two studies, preschoolers' repeating patterning knowledge correlated moderately with their concurrent verbal and visuospatial working memory skills (Rittle-Johnson, Fyfe, McLean, & McEldoon, 2013; Rittle-Johnson et al., 2019). In another study, visuospatial and verbal working memory explained almost half of the variance in repeating patterning task performance, but short-term memory and inhibitory control were not linked to patterning task performance (Collins & Laski, 2015). Further, preschoolers' working memory, but not their inhibitory control or set shifting (moving back and forth between tasks, ideas, and dimensions), explained growth in repeated patterning skill resulting from patterning instruction (Miller et al., 2016). Thus, working memory may be even more relevant for repeating patterning than other executive functioning skills.

However, this past research has used working memory measures that had high incidences of floor effects and limited variability and reliability (Collins & Laski, 2015; Rittle-Johnson et al., 2019). There are documented limitations in common measures of children's working memory when used with preschoolers. For example, many tasks involve reciting strings of numbers and letters in backward order, requiring verbal and sequencing skills and content knowledge (e.g., understanding of the term "backward") that many young children have yet to fully develop (Ramirez, Chang, Maloney, Levine, & Beilock, 2016; Wechsler, 2003). In response to this, nonverbal measures of working memory have been developed that are more sensitive to the developmental needs of young children. These measures include meaningful pictures and use of proactive interference as opposed to backward sequencing to tax working memory (Picture memory; Wechsler, 2012). The current study used a more developmentally

appropriate measure of working memory, Picture Memory (Wechsler, 2012), for the preschool age to better measure this construct.

Patterning and spatial skills. Children may also rely on spatial skills to complete repeating patterning tasks, especially when the tasks include working with visual patterns constructed with objects, such as geometric forms. Specifically, they might use the spatial skill of form perception (i.e. the skill of duplicating and distinguishing different shapes from each other and from symbols) to identify, distinguish between, and match geometric figures that make up patterns (Rittle-Johnson et al., 2019). Research is beginning to explore the relation between repeating patterning and spatial skills. The spatial skills of form perception, visuospatial working memory, and spatial visualization (imagining and transforming spatial information) were all correlated with repeating patterning skills, and form perception was most strongly, albeit moderately, related to patterning skills (Rittle-Johnson et al., 2019). Overall, theorists and prior work have suggested that patterning is linked to the general cognitive abilities of fluid reasoning, working memory, and potentially form perception.

Relation Between Patterning, Math Knowledge, and Specific Math Skills

Patterning and math knowledge. Repeating patterning is thought to be important to mathematical thinking. First, like patterning, math inherently involves identifying, extending, and describing predictable sequences in objects and numbers (Charles, 2005; Sarama & Clements, 2004; Steen, 1988). Some theorists liken patterning to early algebraic thinking due to its emphasis on awareness of regularities and structural relationships (Carragher, Schliemann, Brizuela, & Earnest, 2006; Mason, Stephens, & Watson, 2009; Sarama & Clements, 2009). Further, patterning instruction is prominently situated in several research-based early childhood math curricula (Greenes, Ginsburg, & Balfanz, 2004; Sarama & Clements, 2004; Starkey et al.,

2004).

Emerging empirical evidence provides support for the link between patterning and math. Longitudinal studies indicate that repeating patterning skills in preschool are predictive of broad math knowledge concurrently as well as months and years later, even after controlling for general cognitive abilities (Fyfe, Rittle-Johnson, & Farran, 2019; Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2019). Further, research-based early childhood math curricula with patterning components have been proven effective in improving general math knowledge. Most importantly, experimental work with both preschoolers and first graders indicates that intensive year-long patterning instruction can improve end-of-the-year math knowledge (Kidd et al., 2013, 2014; Papic et al., 2011).

Patterning and numeracy. While much progress has been made in linking patterning to math more broadly, we must begin to identify what math skills are specifically related to patterning to better understand how patterning skills are connected to math knowledge. First, repeating patterning skills may be linked to numeracy knowledge specifically. The reason for the link to numeracy may be that patterning skills involve deducing underlying rules in the sequence of objects, and numeracy knowledge also requires deducing underlying rules from examples, such as the successor principle for symbol-quantity mappings (i.e., the understanding that adding one means the next number in the count sequence). Repeating patterning skills may also promote some counting skills, such as counting by 2's or 5's (Clements & Sarama, 2014). Because repeating patterning tasks do not require prior number knowledge, even preschool children can deduce underlying rules in patterns with objects. Developing such skills with repeating patterns at a young age may support their noticing and use of patterns in numbers as they acquire basic numeracy knowledge. A rule that children may learn to support their

numerical knowledge is that the next number name in the counting sequence means adding one more, ultimately helping children to connect their knowledge of counting string to the process of calculation. This same rule may also help children to appropriately compare numerical magnitudes if they use counting as a strategy.

Empirical evidence is mounting for the link between patterning, numeracy knowledge, and specific numeracy skills. First, preschoolers' beginning-of-the-year repeating patterning skills predicted their numeracy knowledge concurrently and 7-months later (Rittle-Johnson et al., 2019). Further, end of preschool repeating patterning skills predicted symbolic mapping (e.g., comparing the magnitude of symbolic numbers) and calculation knowledge in early elementary school (Rittle-Johnson et al., 2017). A correlation between patterning and calculation skills has also been found in early elementary school (Fyfe, Evans, Matz, Hunt, & Alibali, 2017; MacKay & De Smedt, 2019). The pattern and math-specific skill link may even be causal. For example, repeating patterning training in preschool promoted counting, symbolic mapping, and calculation knowledge by the following school year, compared to children who had not received a patterning intervention in preschool (Papic et al., 2011). However, a number of methodological issues (i.e., no random assignment to condition, initial condition differences were not statistically confirmed, and selective reporting of study results) limit the generalizability of this finding. Thus, the current study aimed to further examine the relation between patterning and specific numeracy skills of calculation and magnitude comparison skills. We chose not to consider counting as a correlate of patterning because of the limited counting range of children at this age. The initial numbers in the count string require rote memorization to learn them, whereas larger numbers more explicitly showcase repeating patterns and structure.

Patterning and shape knowledge. Patterning may also be predictive of early geometric shape knowledge in preschoolers. The earliest evidence of shape knowledge in infancy involves categorizing dot patterns arranged as shapes (Quinn & Eimas, 1986) and detecting and distinguishing different shapes amongst patterns of dots (Quinn, 1987; Quinn, Brown, & Streppa, 1997). Further, children must recognize regularities in terms of what properties make up a given shape (e.g., all triangles have 3 connected straight sides), regardless of the variant (e.g., obtuse, scalene versus the more recognizable equilateral triangles) of that shape (Clements & Battista, 1992). By the preschool years, children can classify familiar geometric shapes by their names, though they have trouble doing so for more unusual variants. Further, patterns tasks are often created using shapes, requiring children to distinguish between different geometric forms. Overall, children's patterning skills could be related to their geometric shape knowledge, but prior research has not explored this connection.

In summary, patterning skills are linked to general math knowledge, but little is known about the reasoning behind its links to math knowledge. Specifically, we know little about the specific math skills to which patterning might relate. We sought to examine the specific link between patterning, numeracy, and shape knowledge, and two specific numeracy skills (i.e., calculation and magnitude comparison) in order to better understand why patterning relates to math knowledge.

Current Study

The current study thus sought to distinguish patterning from general cognitive ability (e.g., fluid reasoning) and better understand the relations between patterning and math in preschool-aged children. In order to further understand how patterning is related to specific early math skills, a battery of math assessments in addition to general cognitive assessments

were used. Guided by past work showing that patterning skills moderately correlated with other general cognitive abilities and significantly predicted math knowledge above these cognitive abilities (Rittle-Johnson et al., 2019), we tested the following hypotheses:

1. Hypothesis 1: Patterning skills and general cognitive ability, especially fluid reasoning, will be moderately correlated with one another.
2. Hypothesis 2: Patterning skills will concurrently predict broad math knowledge above the effects of age and general cognitive skills, including fluid reasoning.
3. Hypothesis 3: Patterning skills will be moderately correlated with general numeracy and shape knowledge, and specific numeracy skills of calculation and magnitude comparison, even after controlling for age and general cognitive skills.

Theoretically, this research contributes to a more comprehensive theory of early academic development in math by considering early evidence of how patterning skills are distinct from general cognitive ability and how they are linked to specific math skills. Practically, the timing of this research is critical to inform efforts to revise and implement the Common Core State Standards (*Common Core State Standards*, 2010), which currently give little attention to patterning skills in early math education.

Method

Participants

Participants were 4- and 5-year old children recruited from two public and three private preschool programs located in a metropolitan area in the U.S. Of the 66 children, 61% were female with an average age of 4.5 (SD = .36; range = 4.0 to 5.3 years). Half of the participating children were non-Hispanic white, 37.9% were African American, 7.6% were Asian, and 4.5% were Hispanic.

Measures

Patterning skills. We administered two measures to assess children's patterning skills. The first measured preschool children's ability to find the missing item, duplicate, extend and abstract visual repeating patterns using shapes and to identify the pattern unit (i.e., the part that repeats; see Figure 1 for sample item types). This measure consisted of nine items at varying levels of difficulty, as described and validated previously, plus 3 new items using easier AB patterns added to the beginning of the assessment (Miller et al., 2016; Rittle-Johnson et al., 2015, 2013). The order of items was: 1) missing item in AB pattern, 2) duplicate AB pattern, 3) extend AB pattern, 4) duplicate AABB pattern, 5) duplicate AAB pattern, 6) extend ABB pattern, 7) extend AABB pattern, 8) abstract AABB pattern, 9) abstract ABB pattern 10) abstract AABB pattern, 11) duplicate ABB pattern from memory, and 12) identify pattern unit in AAB pattern. Stop criteria was implemented to reduce testing time and child frustration, and the assessment ended if children answered all extend or all abstract pattern items incorrectly. This assessment took approximately eight minutes to administer and internal consistency was .84 in our sample.

Our second patterning measure was a recently developed teacher-based patterning assessment adapted from previous research (Rittle-Johnson et al., 2019; Zippert, Douglas, & Rittle-Johnson, 2019; Zippert, Loehr, & Rittle-Johnson, 2018). This 10-item measure was created using resources openly available to teachers online with the intention of measuring patterning ability in the context of patterns that might be used in a classroom setting (see Figure 2 for sample items). Children were presented with colorful pictures of model patterns that were missing pattern pieces. Children were asked to complete patterns or find missing pattern pieces using laminated pattern pieces. This measure included three types of tasks and four different pattern units: (a) what comes next in an AB and ABC pattern, (b) missing item in an AB, ABC

and ABB pattern and (c) extend pattern in an AB, AABB, and ABC pattern. Extend items required children to complete a pattern by adding four pieces to the end. Children received one point for each correct answer, and the assessment took less than five minutes. Internal consistency in our sample was good (.79).

Ability estimates for each child were generated for each patterning measure using a Rasch model with a Laplace approximation and empirical Bayesian prediction method, which is stable for sample sizes around 50 (Cho & Rabe-Hesketh, 2011). Laplace approximation was implemented in R (<http://www.r-project.org>), using the `glmer` function of the `lme4` package (Bates, Maechler, & Dai, 2008). The ability estimates for the two scales were averaged to create a patterning composite measure.

Math Measures

General math, numeracy, and shape knowledge. A short-form version of the Research-Based Early Mathematics Assessment (Weiland et al., 2012) was used to assess math knowledge. This 19-item version assesses numeracy knowledge in section one and primarily shape knowledge in section two. Numeracy items included non-symbolic magnitude comparison, rote counting to 5, 3 subitizing items with small and large numbers, object counting, enumerating, and production (e.g., give n when shown a model set of objects), matching numerals to their respective non-symbolic quantities 1 through 5, 4 non-symbolic addition and subtraction problems, and an item determining the smaller of two multidigit numbers. Shape knowledge items included identifying triangles and rhombuses amongst distractors (e.g., broken shapes) and unusual variants (e.g., isosceles triangles), creating shapes, determining the number of sides on a shape, and determining the result of cutting a shape in two pieces. Items for each section were ordered by IRT difficulty estimates from the norming sample. For each of the two

sections, stop criteria was met when a child answered three consecutive questions incorrectly. Children received one point for each correct answer. This assessment took less than 15 minutes to administer. Internal consistency previously reported by Weiland et al. (2012) was acceptable ($\alpha = .71$ and $.79$ in two samples).

IRT ability estimates were generated using a partial credit model. To improve the precision of ability estimates for our sample size below 100, we used Empirical Bayes estimation to constrain the item parameters (Baker & Kim, 2004), using WinBUGS 1.4.3 (Spiegelhalter, Thomas, Best, & Lunn, 2003). The informative prior distribution on the item difficulty parameters and the sum-to-zero constraints on the item location and threshold parameters were chosen based on results reported in Weiland et al. (2012). Internal consistency using IRT scores for the REMA Short-form total was $.77$ in our sample. Internal consistency was also good for the numeracy subscale ($\alpha = .79$), but was weak for the shape knowledge subscale ($\alpha = .58$).

Specific numeracy skills. Two subscales of the Preschool Early Numeracy Scales (Purpura & Lonigan, 2015) were used to assess number knowledge. Children received one point for each correct answer, and each subscale took approximately two minutes to administer.

Magnitude comparison. The first subscale, magnitude comparison, involved visually or verbally presenting the child with a set of four numbers. Four number sets were presented visually and two number sets were presented verbally. For half of the sets, children were asked which number meant the most. For the other half, children were asked which number meant the least. Purpura and Lonigan (2015) previously reported acceptable internal consistency of this subscale ($\alpha = .74$) and internal consistency was $.64$ in our sample.

Verbal calculation. The second subscale, story problems, involved seven verbally presented story problems involving addition or subtraction. An example included, "If Hugh does

not have any cookies, and his mom gives him two cookies, how many cookies does Hugh have now?”. Purpura and Lonigan (2015) previously reported acceptable internal consistency for this subscale ($\alpha = .71$) and was similar ($\alpha = .67$) in our sample.

Place value. Numerical knowledge was further tested using two place value tasks that have not been used to assess individual differences in the past, but which seemed promising: *Which is X?* and *Which is More?* (Mix, Prather, Smith, & Stockton, 2014). For the four *Which is X?* questions, children were shown two multi-digit numbers and asked to point to one. For example, children were visually shown the numbers 206 and 260 and asked “Which number is 206?”. For the four *Which is More?* questions, children were shown two multi-digit numbers and asked to point to the one that was more. For example, children were visually shown the numbers 101 and 99 and asked “Which is more?”. However, 56% of children scored at or below chance across the 8 items. Internal consistency in our sample was unacceptable ($\alpha < .30$) and was therefore excluded from analyses.

General Cognitive Skills

Fluid reasoning. The Matrix Reasoning subscale from the Wechsler Preschool and Primary Scale of Intelligence-Fourth Edition (WPPSI-IV) measures fluid reasoning skills (Wechsler, 2012). Each child was asked to select one of four visually presented response items to complete a two by two matrix shown in the middle of the page. The items get progressively more difficult, ranging from filling in the fourth box with the same image as appears in the other three, to understanding more complex relationships, such as “soccer ball is to goal as basketball is to basketball hoop.” The assessment was administered according to standardized instructions, and took approximately three minutes to administer. Children received one point for each

correct answer, and the total score was used in analyses. Internal consistency was previously reported as good for this subscale (Fisher's $z = .89$) and was good in our sample ($\alpha = .85$).

Working memory. The Picture Memory subscale, also from the WPPSI-IV, measures working memory in children aged 2-7 years (Wechsler, 2012). For this task, children see multiple pictures of everyday items such as a flower or a sock, for either three seconds or five seconds, depending on the difficulty of the item. They are then shown a series of pictures on the following page. These pictures include the pictures they have just seen along with multiple distractors, and the children are asked to point to the pictures they were just shown. On more difficult items, distractors include the target item from the previous trial. The assessment was administered according to standardized instructions and took approximately five minutes to administer. Children received one point for each correct answer, and the total score was used in analyses. Wechsler (2012) previously reported a good internal consistency (Fisher's $z = .89$). Internal consistency in our sample was .90.

Spatial skill. The Position in Space subtest of Developmental Test of Visual Perception–Second Edition (DTVP-II; Hammill, Pearson, & Voress, 1993) was used to measure the spatial skill of form perception. For each of the 25-items, children were presented with four or five similar pictures in slightly different positions. Participants were asked to find the picture that matched a target picture in orientation. The assessment was administered according to standardized instructions, including stop criteria. Children earned one point for each correct answer, and administration took less than five minutes. Total number correct was used in analyses. Internal consistency is reported as high (Cronbach's $\alpha > .80$) for children ages 4 to 10 (Hammill et al., 1993), and internal consistency in our sample was .86.

Procedure

Children were assessed individually by one of three assessors near the beginning of the school year. Each of the two 30-minute assessments, on average 4 days apart, took place in a quiet area chosen by an administrator in the child's preschool. Sessions were video or audio recorded. In the first session, assent was obtained and children were assessed on the research-based patterning task, numeracy scales, spatial skill, and working memory. In the second session, children were administered the math assessment, the teacher-based patterning task, and fluid reasoning. Children received a sticker at the end of each session for participating.

Two children (3.0%) had incomplete data because they asked to return to the classroom early. Both children were missing data for teacher-based patterning and fluid reasoning, and one of the children was also missing data for working memory. Missing data were imputed via multiple imputations. Data and study measure materials are available at osf.io/hetd5 (Authors, 2019).

Results

Relations Between Patterning and General Cognitive Abilities, Including Fluid Reasoning

For our first aim, we explored raw correlations between patterning and measures of general cognitive abilities (see Table 1). In line with our hypothesis, there was a moderate correlation between patterning skills, fluid reasoning, working memory, and spatial skill (r 's = .34 to .54). Overall, this suggests that patterning is distinct from these other cognitive abilities.

Relation Between Patterning and General Math Knowledge

In line with our second aim, we examined how patterning related to general math knowledge, over and above other cognitive abilities, including fluid reasoning. As shown in Table 1, patterning knowledge was related to most math outcomes, even after controlling for age and other cognitive abilities. Thus, we conducted hierarchical linear regression models predicting

children's general math knowledge, adding age and general cognitive measures in the first step and the patterning composite in the second step (see Table 2 for results). Patterning was significantly and positively predictive of general math knowledge above all cognitive controls, explaining an additional 22% of the variance once added to the model. Fluid reasoning, working memory, and spatial skills initially significantly predicted general math knowledge, but after patterning was included in the model, none were significant, though fluid reasoning predicted math knowledge marginally.

Relations Between Patterning and Specific Math Skills

In line with our third aim, similar hierarchical linear regression models were run predicting children's specific math skills. Results of these models are presented in turn.

Numeracy knowledge. Regarding specific math skills, patterning significantly predicted the numeracy knowledge subtest of the REMA-brief, explaining an additional 17% of the variance once added to the model (see Table 2). While less predictive overall, spatial skills were marginally predictive of numeracy knowledge after including patterning in the model.

Shape knowledge. Patterning skills did not significantly predict shape knowledge (the effect was only marginal), and fluid reasoning was a marginal predictor only before patterning was added to the model (see Table 2). However, internal consistency of the shape knowledge measure was low, making potential relations more difficult to detect.

Verbal calculation. Patterning skills significantly predicted verbal calculation controlling for other general cognitive skills, explaining an additional 11% of the variance once added to the model (see Table 2). Working memory was a significant predictor only in Step 1.

Magnitude comparison. Finally, patterning was the only significant predictor of magnitude comparison once it was included in the model, explaining an additional 17% of the variance (see Table 2). Spatial skill was a significant predictor only in Step 1.

Discussion

The current study examined relations between preschoolers' patterning skills and general cognitive abilities (e.g., fluid reasoning), as well as broad and specific math skills after controlling for general cognitive abilities. Patterning skills were moderately related but not redundant with general cognitive abilities. Further, patterning skills predicted broad math knowledge and specific math skills (i.e., general numeracy, magnitude comparison and verbal calculation), even when we controlled for general cognitive abilities. We discuss the potential role of patterning in mathematical thinking more broadly, as well as children's general reasoning about numbers and individual numeracy skills. We end with implications of findings for math theory, research, and early childhood curricula, as well as study limitations, and suggestions for future research.

Patterning is Unique from Fluid Reasoning and other General Cognitive Abilities

In line with our first hypothesis, repeating patterning related significantly but only moderately to fluid reasoning, working memory, and the spatial skill of form perception. Our study was the first to determine the association between patterning and fluid reasoning, and it aligns with other studies that have found moderate relations between patterning, working memory, and a range of spatial skills (Collins & Laski, 2015; Miller et al., 2016; Rittle-Johnson et al., 2019). Patterning is likely linked to fluid reasoning because both involve logical thinking in novel contexts and predicting what is next given limited evidence, as has been suggested by others (Lee et al., 2012). Further, neither relies on specific content knowledge, and may require

identifying and duplicating relationships between objects and information. Patterning is additionally linked to working memory because it requires children to remember, reproduce, and mentally manipulate objects in a model pattern, and this becomes more difficult as the repeating unit increases in the number of items it contains (Miller et al., 2016). We propose that patterning and spatial skills also may be connected given that children must differentiate between and match different geometric figures in model patterns in order to determine and recreate their underlying structures. At the same time, repeating patterning was not associated with each of these skills to such a large extent that too much variance overlapped, which would suggest that they were redundant skills. This suggests that while patterning draws on fluid reasoning, working memory and spatial skills, it is a unique cognitive skill meriting its own targeted empirical and theoretical attention.

Patterning Contributes to General Math Knowledge

To further demonstrate that patterning is an important skill, we tested the concurrent associations between repeating patterning and general math knowledge, over and above children's general cognitive abilities. In line with our second hypothesis, while all general cognitive abilities were initially significant predictors, considering the additional predictive role of patterning resulted in it being the sole unique contributor to general math knowledge. These results reflect past work which has also shown that preschoolers' patterning skills predict concurrent and later general math knowledge beyond general cognitive skills, including age, a range of spatial skills, working memory, and language skills (Nguyen et al., 2016; Rittle-Johnson et al., 2017, 2019). However, we extend this work by also considering children's fluid reasoning skills and measuring working memory skills in a more sensitive way. This further highlighted the unique and important role that patterning plays in children's general mathematical thinking.

Linking Patterning, General Number and Shape Knowledge and Specific Number Skills

To better understand why patterning is linked to mathematics knowledge, we explored the link between patterning and specific math skills, namely general number and shape knowledge and specific numeracy skills. In line with our third hypothesis, we found significant positive associations between patterning and many of children's specific math skills, including general numeracy and the specific numeracy skills of verbal calculation and magnitude comparison, even after including our cognitive controls. Past research has also shown correlational and even causal links between patterning knowledge and specific aspects of math knowledge, including general numeracy knowledge in preschoolers and calculation ability in elementary school-aged children (Fyfe et al., 2017; MacKay & De Smedt, 2019; Papic et al., 2011; Rittle-Johnson et al., 2017, 2019). These associations may exist because children rely on similar mental processes when comparing and combining numerical magnitudes and when solving patterning problems. These shared processes might include considering and applying rules and regularities in numbers and sequences of objects. We expand upon this idea below.

First consider how this might proceed for numerical cognition. The numerical skills of object counting and reciting the count sequence are thought to involve consideration of several rules or counting principles, including that the order of the number words must remain stable (although the order in which the objects are counted does not), that each object should be assigned one count word only, and that the last word in the count sequence determines the set size (Gelman & Gallistel, 1978). In addition, in order to solve magnitude comparison problems, children must consider the rule that numbers appearing later or earlier in the count sequence are larger and smaller in magnitude, respectively. In engaging in calculation, children must understand the rule that adding to or decreasing an existing quantity will ultimately result in a

larger sum or smaller difference, respectively, and that the next or previous number in the count sequence is exactly one more or one less.

In each of these instances of problem solving with numbers, children can rely on the regularities of the number sequence to help them find the answer. For example, children may rely on the regularities of the number sequence to engage in counting to 100, namely that numbers 1 through 9 all repeat for each decade. Further, they must recognize the regularity that the aforementioned rules of counting (e.g., counting principles) apply to all different types of objects, including sounds (Gelman & Gallistel, 1978). Additionally, children can rely on the regularity of the number sequence to remind them that 9 comes after 8 when counting, which should help them reason that 9 is the larger number. Similarly, for calculation, children can rely on counting strategies to reach the correct sum when adding or subtracting two numbers (Briars & Siegler, 1984).

Next, consider how rules and regularities may also be integral for patterning. An important rule for repeating patterning is recognizing that there is a unit of repeat, and that this determines the structure for the entire pattern. Thus, when children are solving pattern extension problems, they must continue the pattern beginning with the right shape (e.g., an ABABA sequence must be followed by a “B” rather than another “A” to extend the pattern correctly). Children can also always rely on the regularity of reciting the items in order to determine the correct next items in a pattern problem.

Finally, although we hypothesized that we would find relations between children's patterning skills and their shape knowledge, our results did not support this prediction. We had anticipated that this link might exist because identifying and composing shapes may involve making sense of patterns of groups of objects (Quinn, 1987; Quinn et al., 1997; Quinn & Eimas,

1986) and understanding regularities in what is classified as a particular shape (e.g., triangles all have 3 sides, the 3 sides much touch; Clements & Battista, 1992). At the same time, repeating patterning tasks often involve a series of geometric shapes, so having knowledge of shape names, or even familiarity with their forms could aid children in processing pattern items more efficiently by reducing cognitive load. This could be especially helpful when abstracting patterns since children must use a different set of shapes to duplicate the structure of a model pattern. Despite the lack of a relation in the current study, we are hesitant to rule out the possibility that a link exists between repeating patterning and shape knowledge, especially given the weak internal consistency of the shape knowledge measure in the current study, which limited our ability to detect a relation. In addition, the relation may appear later in development, after children develop more advanced repeating patterning and shape knowledge. For example, few children reached the pattern abstracting items in the current study, so if shape knowledge is especially important at this level of patterning, we may not have been able to uncover this link in the current sample.

Limitations, Implications and Future Directions for Math Theory, Research, and Curricula

A number of limitations of the current study should be noted. First, our study was conducted in a single timepoint, and was correlational in nature. Thus, we cannot infer causality or directionality from our conclusions. In addition, the findings may be limited to our particular age range of 4- to 5-years or context (U.S. preschools). Additionally, we faced measurement issues with our shape knowledge subtest and place value measures, preventing us from making substantive claims about how they each are associated with patterning.

However, given the consistent correlational links found between patterning and general and specific math and numeracy skills in the current study and past research (Rittle-Johnson et

al., 2017, 2019), we provide recommendations for modifying math theory, empirical research, as well as educational standards. First, building evidence indicates an important place should be made in theories for the role of children's repeating patterning skills in supporting their mathematical thought. This is especially important as many theories of math development put sole emphasis on the contributions of numeracy skills to math development, and sometimes treat math as if it is synonymous with numeracy (Dehaene, 1997; Siegler & Lortie-Forgues, 2014).

Second, future empirical studies should aim to explore the causal nature and directionality of the associations between patterning and math, and determine the mechanisms by which patterning might support mathematical thinking, especially specific math skills (e.g., general numeracy knowledge and specific numeracy skills). Past researchers have theorized that the positive effect of patterning instruction on math knowledge may have been explained by improvements in fluid reasoning (Kidd et al., 2013, 2014; Papic et al., 2011; Parniak et al., 2016); however, our current evidence suggests that this is a less likely explanation, given that patterning and fluid reasoning were not strongly related. Future pattern training studies should move beyond basic practice with patterning activities to explicitly emphasize finding rules and regularities in numbers and patterns to improve patterning, general math knowledge, and specific math skills (e.g., numeracy). Such future training studies should additionally take appropriate methodological precautions (e.g., conduct random assignment, statistically check initial condition differences), and provide a complete report of differences in all math outcomes. Such studies could additionally control for fluid reasoning when assessing math and patterning learning gains.

Third, more work is needed to develop a more reliable measure of children's shape knowledge. It was unclear if we found no relation between children's shape knowledge and

patterning skill because of reliability issues with the shape knowledge measure. Thus, it is important for future work to both better measure children's shape knowledge and determine if its association to patterning can be ascertained. Additional work could also explore if shape knowledge might be related to specific patterning skills of differing difficulties. For example, it could be that knowing shape names may be more important for completing and duplicating patterns than abstracting them.

Fourth, in terms of Common Core curricula, explicit mention of patterning instruction does not currently appear until the fifth grade, although making use of and looking for structure is a process standard throughout (*Common Core State Standards*, 2010). Thus, we argue for emphasis in existing standards that structure can be used and found in repeating pattern tasks specifically. Finally, in line with other math consensus documents and many research-based math curricula (Greenes et al., 2004; National Association for the Education of Young Children, 2014; National Council of Teachers of Mathematics, 2006; Sarama & Clements, 2004; Starkey et al., 2004), we urge policymakers to consider the reintroduction of patterning into instructional standards as early as school entry. We especially encourage a particular focus on children finding rules and regularities in repeating patterns, and helping them link this skill to number patterns.

In conclusion, patterning skills in preschoolers were found to be independent of other general cognitive abilities including fluid reasoning, and were uniquely predictive of general math and numeracy knowledge and specific numeracy skills of verbal calculation and magnitude comparison. Our correlational findings corroborate evidence from experimental studies that have found causal links between patterning and math and numeracy knowledge (Kidd et al., 2013, 2014; Papic et al., 2011). Thus, theory and educational standards should be modified to emphasize the contributions of children's patterning skills for their math development.

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Table 1

Correlations Among Key Variables

Variables	<i>M</i>	<i>SD</i>	1	2	3	4	5	6	7	8	9	10
Control Variables												
1. Age	4.54	0.36	---	.39**	-.06	.18	.33**	.28**	.21 [†]	.28*	.34*	.29*
2. Fluid Reasoning	8.40	3.76		---	.23 [†]	.53**	.47**	.51**	.50**	.35**	.32**	.25*
3. Working Memory	11.29	5.00			---	.21 [†]	.34**	.33**	.33**	.13	.12	.27*
4. Spatial Skills	6.27	3.45				---	.54**	.47**	.58**	.17	.43**	.31*
Target Variables												
5. Pattern Composite	.02	1.40					---	.74**	.72**	.37**	.63**	.53**
6. General Math	-1.41	1.17						.58**	---	.89**	.58**	.43**
7. Numeracy	-1.02	1.55						.54**	.84**	---	.39*	.50**
8. Shape	-1.80	0.89						.24 [†]	.52**	.29*	---	.23 [†]
9. Magnitude Comparison	1.80	1.62						.48**	.23 [†]	.30*	.01	---
10. Verbal Calculation	2.88	1.94						.37**	.27*	.36**	.12	.42**

Notes. Values above the diagonal are raw correlations ($df = 65$). Values below the diagonal are partial correlations after controlling for age, fluid reasoning, working memory, and spatial skills ($df = 60$). Pattern composite, General Math, Numeracy, and Shape used IRT ability estimates.

[†] $p \leq .1$. * $p < .05$. ** $p < .01$.

Table 2

Hierarchical Multiple Regression Models Predicting Math, Numeracy, and Shape Knowledge, and Calculation and Magnitude Comparison Skills

Variable	General Math Knowledge			Numeracy Knowledge			Shape Knowledge			Verbal Calculation			Magnitude Comparison		
	<i>B</i>	β	ΔR^2	<i>B</i>	β	ΔR^2	<i>B</i>	β	ΔR^2	<i>B</i>	β	ΔR^2	<i>B</i>	β	ΔR^2
Step 1 ^a			.37**			.42**			.15*			.20**			.26**
Age	.45	.14		.26	.06		.45	.18		1.49	.28*		1.25	.28*	
Fluid reasoning	.08	.27*		.09	.22 [†]		.06	.27 [†]		-.02	-.04		.00	.01	
Working memory	.05	.22*		.06	.20 [†]		.01	.08		.10	.25*		.02	.06	
Spatial skills	.09	.26*		.18	.41**		-.01	-.02		.13	.23		.17	.37**	
Step 2 ^b			.22**			.17**			.05 [†]			.11**			.17**
Age	.00	.00		-.27	-.67		.28	.13		.97	.18		.69	.15	
Fluid reasoning	.06	.19 [†]		.06	.15		.06	.23		-.05	-.09		-.03	-.06	
Working memory	.02	.07		.02	.07		.00	.01		.06	.14		-.03	-.08	
Spatial skills	.01	.02		.09	.20 [†]		-.03	-.13		.04	.06		.08	.16	
Pattern skills	.51	.61**		.60	.54**		.19	.29 [†]		.59	.43**		.63	.54**	

Note. ^a $df = (4, 61)$. ^b $df = (1, 60)$. [†] $p < .1$. * $p < .05$. ** $p < .01$.

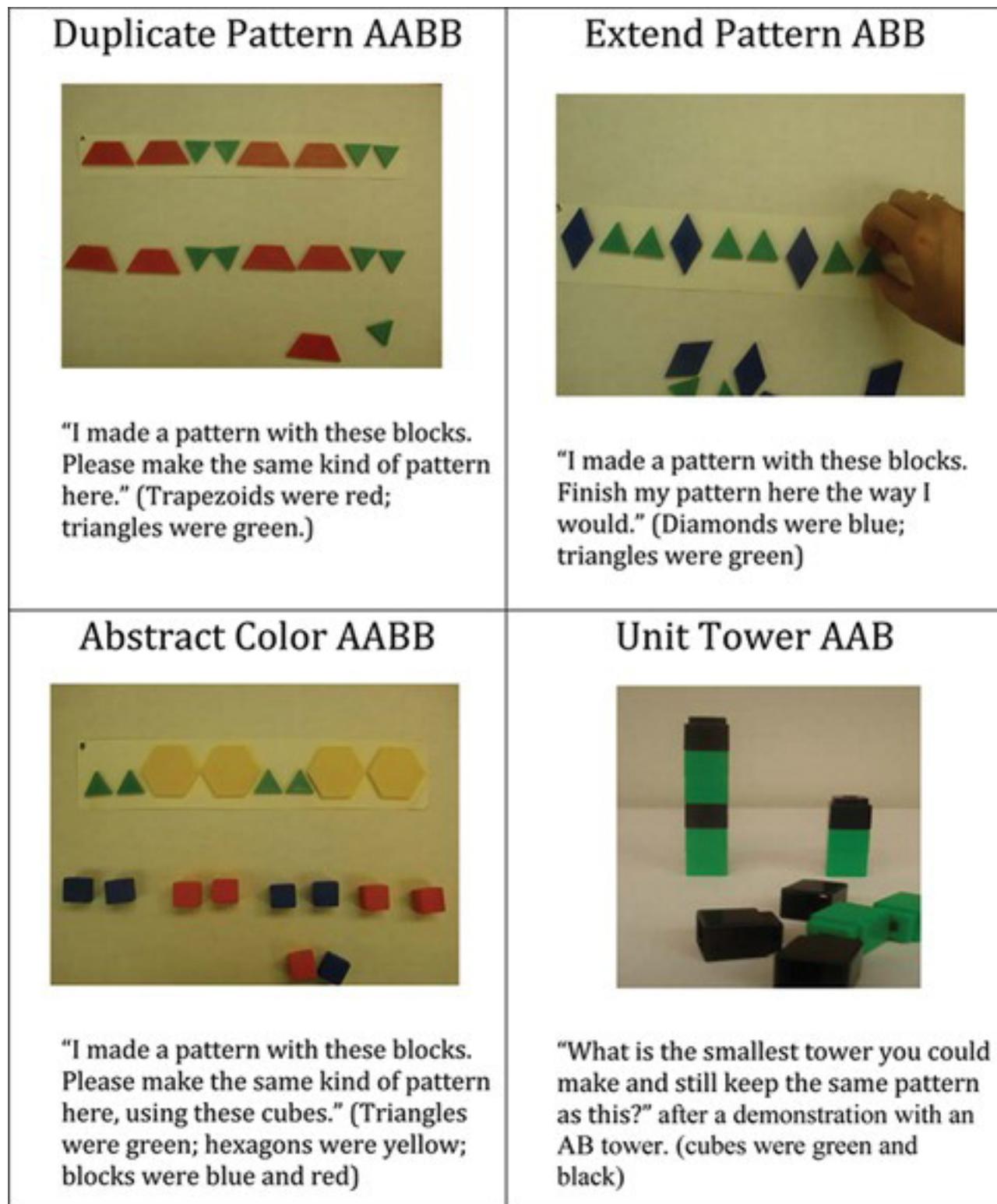


Figure 1. Sample item types from each level, including a sample correct response, from the research-based patterning assessment. From "Emerging Understandings of Patterning in 4-Year-

Olds,” by B. Rittle-Johnson, E. R. Fyfe, L. E. McLean, and K. L. McEldoon, 2013, *Journal of Cognition and Development*, 14, p. 382. Copyright 2013 by Taylor & Francis Group, LLC.

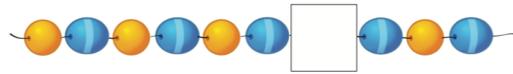
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What Comes Next Pattern ABC



“What comes next in the pattern? Use one of these.” [Experimenter gestures to picture cutout response options below.]

Missing Item Pattern AB



“Find the missing bead [experimenter gestures to picture cutout response options below] to complete the pattern [experimenter gestures across pattern].”

Extend Pattern ABC



“Can you complete the pattern?” [Experimenter gestures to circles on the right of the pattern.]

Figure 2. Sample items from the teacher-based patterning assessment.