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Enactment of Lessons from a Technology-Based Curriculum: The Role of Instructional Practices in Students' Opportunity to Learn

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Abstract

Digital tools and technology-based activities offer new and promising opportunities for students to actively explore mathematical concepts and ideas in ways supported by current reforms and visions of mathematics instruction. This report provides an in-depth look at the implementation of SunBay Digital Mathematics (SunBay Math) during the second year of an i3 validation project, in two large Florida districts. SunBay Math is a set of middle-school curriculum replacement units centered on the use of technology-based, dynamically linked representations to learn core mathematical concepts. We focus specifically on patterns and relationships between instructional practices and instructional quality in 26 videotaped lesson enactments that were purposefully collected to represent variation in implementation.

Disciplines

Education | Educational Technology

Comments

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Executive Summary

Digital tools and technology-based activities offer new and promising opportunities for students to actively explore mathematical concepts and ideas in ways supported by current reforms and visions of mathematics instruction. This report provides an in-depth look at the implementation of SunBay Digital Mathematics (SunBay Math) during the second year of an i3 validation project, in two large Florida districts. SunBay Math is a set of middle-school curriculum replacement units centered on the use of technology-based, dynamically linked representations to learn core mathematical concepts. We focus specifically on patterns and relationships between instructional practices and instructional quality in 26 videotaped lesson enactments that were purposefully collected to represent variation in implementation.

Chapter 1 describes the conceptual framework and context for the study. As part of the scale-up and evaluation of SunBay Math, two units were implemented in sixth-, seventh- and eighth-grade classrooms in 30 middle schools randomly selected from two large Florida school districts serving diverse, high-need student populations.

To study the implementation of SunBay Math lessons in this context, we draw on the constructs of *textbook integrity* (Chval, Chávez, Reys, & Tarr, 2009) and *fidelity to the author's intended lesson* (Brown, Pitvorec, Ditto, & Kelso, 2009) to consider two dimensions in the enactment of SunBay Math lessons in relation to the curricular design: *structural integrity* and *pedagogical integrity*. We define structural integrity as adherence to the curriculum in terms of the observable parts of a lesson and activity structures, while pedagogical integrity refers to the interactions and conditions put in place for students to engage with the content of the lesson. SunBay Math's dynamic technology-based tools feature multiple, linked mathematical representations designed to support student understanding of both underlying concepts and procedures. The design of the lessons in the modular two-week units revolves around teachers facilitating student reasoning through small group exploration with dynamically linked representations and whole group discussions to elicit, highlight and consolidate the important mathematical ideas.

In **Chapter 2**, we present the research methods and findings related to the structural and pedagogical integrity of the 26 videotaped lesson enactments. We developed a set of four rubrics to focus on specific features of SunBay Math and also used four rubrics from the validated Instructional Quality Assessment (IQA) toolkit (Boston, 2012) to focus on overall instructional quality. Applying these eight rubrics to every lesson allowed us to capture variation in different dimensions of both structural and pedagogical fidelity, along with students' overall opportunity to engage in high-level reasoning around the mathematics. The analysis shows that while SunBay Math lessons in the written curriculum materials reflect a high level of cognitive demand, many of the observed lessons were enacted at a low level, focusing primarily on procedures rather than reasoning and conceptual understanding. Patterns across both high- and low-level implementations highlight how pedagogical integrity to the lesson influences the opportunity for students to engage in reasoning and make conceptual connections to the mathematics. Through both quantitative and qualitative analysis, we demonstrate that pedagogical integrity is more consequential than structural integrity for students' overall opportunity to learn.

To further explore and contextualize pedagogical integrity and the role of teacher instructional practices, four case studies of lesson enactments from the same unit are presented in **Chapter 3**. While these lesson enactments focus on the same mathematical content, they reflect differences in activity structures (e.g., whole group vs. small group) and both high and low pedagogical integrity to the curriculum design. We look across the cases to highlight the role of the teacher's approach to technology use, implementation of Predict-Check-Explain (PCE) inquiry cycles, and discourse practices. In addition, the cases illustrate important differences in teacher orientations towards their own role and towards the use of the curriculum.

Chapter 4 discusses implications of the findings. We conclude that giving students access to new technological tools and having them work in groups to complete tasks as sequenced in the curriculum, is not sufficient to ensure *opportunity to learn* in the ways intended by the curriculum designers. The way that the teacher presented, facilitated and orchestrated discussion around the tasks, resulted in the technology being used procedurally, for exploration without any guidance, or for exploration along with reasoning, explanation and connections to important mathematical ideas. Whether or not teachers made structural or pedagogical adaptations was less important than the capacity for teachers to be able to do so with integrity to the curricular design. Teachers play a central role in enactment of technology-based curricula. We therefore underscore the need for sustained efforts to build teacher capacity, including opportunities for teachers to develop and refine new instructional practices, as well as develop new orientations to teaching and learning.

Chapter 1 Context for the Study

Recent visions of mathematics instruction focus on engagement in productive struggle as students make sense of and reason about mathematical ideas to develop conceptual and procedural understanding (Common Core State Standards Initiative, 2010; Hiebert et al., 1996). These new views of mathematics instruction are often regarded as ambitious, highlighting the challenges involved in taking on new roles, responsibilities, and practices for both teachers and students (Franke, Kazemi, & Battey, 2007; Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). Innovative curricula are viewed as an essential lever for reform in these directions, providing support for new standards and instructional methods (Remillard, 2005).

The growth of new digital tools and technology-based activities provides new opportunities for students to explore mathematical concepts and ideas through dynamic visual representations and simulations not possible with traditional paper and pencil exercises or chalkboard demonstrations. Technology enables transformation in learning experiences by supporting students in reasoning about mathematical concepts through experiences in intrinsically engaging environments (Roschelle, Noss, Blikstein, & Jackiw, 2017). Dynamically linked mathematical representations afford opportunities for the development of deep conceptual understanding of mathematical ideas promoted by current reforms. In this report, we explore the implementation of SunBay Digital Mathematics (SunBay Math), a set of middle-school curriculum replacement units centered on teacher facilitation of the use of technology-based, dynamically linked representations, to help students learn core mathematical concepts.

These replacement units reflect a shift in the teacher's role to engage students in inquiry and facilitate discussions to develop a shared understanding of important mathematical ideas. We explored what happens during implementation when teachers use curriculum units that reflect new pedagogical approaches and incorporate technology. Questions guiding our exploration included: Does providing teachers and students access to technology-based activities lead to increased opportunities for deep engagement and opportunities to learn? If so, under what conditions? Do teachers' instructional practices reflect the intent of the curriculum and technology?

This study is part of a larger external evaluation of the impact of SunBay Math in two large districts in Florida, which included a mixed methods implementation study and a randomized control trial.[1] Findings are described in a companion report, *The i3 Validation of SunBay Digital Mathematics* (Sirinides et al., forthcoming). The in-depth analysis from the evaluation showed a positive impact on the teachers' understanding and tolerance of student struggle as a part of the math learning process, as well as an increase in the teachers' comfort with using technology for math instruction. However, many teachers did not fully implement the SunBay Math units and/or lessons, making it difficult to assess the true impact of the innovative technology-based curriculum on student learning.

[1] In 2014, SunBay Math's developer, SRI International, was awarded an i3 validation grant as part of the Investing in Innovation (i3) program of the Office of Innovation and Improvement (OII), U.S. Department of Education. The Helios Education Foundation also supported the project with a matching grant. As part of this award, the Consortium for Policy Research in Education (CPRE) at the University of Pennsylvania, served as an independent evaluator, assessing the impacts and implementation of SunBay Math under the i3 project in two districts.

The present study draws on prior work that has shown that teacher practices can mediate the effects of curriculum on student learning (Tarr et al., 2008; Taylor, 2016). It examines dimensions of variation in SunBay Math lesson enactments and teacher instructional practices that are associated with that variation. It also looks at pedagogical and structural factors associated with students' opportunities to engage in reasoning and making conceptual connections to the mathematics. This report aims to contextualize and further explain the effects of teacher adaptations on the implementation of technology-based middle-school mathematics curriculum.

Conceptual Framework

SunBay Math is comprised of professional development and instructional resources that enable teachers to engage students in sense-making and reasoning—important priorities in current efforts towards reform in mathematics education. Because "instruction consists of interactions among teacher and students around content, in environments" (Cohen, Raudenbush, & Ball, 2003, p. 122), the effects of instructional resources are always mediated by teacher and student knowledge, beliefs and action. Teachers and students adapt curriculum materials during implementation and these adaptations may enhance or detract from curriculum developers' intent (Henningsen & Stein, 1997; Otten & Soria, 2014; Remillard, 2005; Stein, Grover, and Henningsen, 1996; Stigler & Hiebert, 2004). Furthermore, a high level of cognitive demand is difficult to maintain and often declines during enactment, particularly with curricula focused on reasoning and problem-solving, where enactment is more dependent upon the interactions between teachers and students (Stein, Remillard, & Smith, 2007).

Implementation of curricular interventions is often studied in terms of fidelity and measured by observable features such as the extent of use, coverage, or faithfulness to the text (Brown, Pitvorec, Ditto, & Kelso, 2009). Yet research demonstrates that curricula may be used in ways that incorporate superficial features but do not reflect the underlying pedagogical principles of the curriculum design (Coburn, 2003). Brown (2009) conceptualizes teachers not as adopters or users, but as designers, emphasizing the "dynamic interplay" between the teacher, technology and written curriculum materials. Teachers interact with the curriculum to select, interpret, and reconcile the intended goals with their own goals and the perceived constraints, as well as to accommodate instruction to their students. In addition, teachers use curriculum in different ways: they may follow it as a script; they may work from the materials but adapt; or they may improvise by relying more on their own ideas and resources than what is in the written curriculum (Remillard & Bryans, 2004). Important to this process is the teacher's *pedagogical design capacit*y, or "skill in perceiving the affordances of the materials and making decisions about how to use them to craft instructional episodes that achieve her goals" (Brown, 2009, p. 29). This view of the teacher's relationship with the curriculum places the teacher in an active role of participation with the curriculum, where there is a bi-directional or dynamic interchange between the teacher and the text (Remillard, 2005).

In conceptualizing the implementation of SunBay Math in classrooms, we view teachers as participating with the curriculum (Remillard, 2005) and draw upon recent work that problematizes the practice of evaluating implementation of curricular interventions only in terms of fidelity. Chval, Chávez, Reys, and Tarr (2009) propose an alternative construct called *textbook integrity* that includes not only regular use of the curricular materials and how much of it is used but also the consistency of the instructional strategies with the pedagogical orientation of the materials as written. Similarly, at the lesson level, Brown et al. (2009) propose that there are two different forms of fidelity to consider: *fidelity to the* *literal lesson* or the degree to which the lesson is implemented as written and *fidelity to the authors' intended lesson*, defined as "the degree of alignment between the author's intended opportunities to learn and the opportunities to learn in the enacted lesson" (p. 373). In defining the authors' intended *opportunity to learn*, they draw upon Hiebert's (2003) notion of considering not just exposure to the content, but also the nature of students' engagement with content in particular ways (e.g., reasoning, communicating, exploring).

In our study, we focus specifically on lesson enactment and draw upon Chval et al.'s (2009) notion of integrity along with Brown et al.'s (2009) conceptualization of fidelity to the author's intended lesson, to consider two dimensions in the enactment of SunBay Math lessons: *structural integrity* and *pedagogical integrity*. Structural integrity is adherence to the curriculum in terms of the observable parts of a lesson and activity structures (e.g., the Warm-Up vs. the Main part of the lesson, whole class vs. small group learning). Pedagogical integrity refers to the interactions and conditions put in place for students to engage with the content of the lesson in ways envisioned by the curriculum designers (e.g., reasoning about mathematical ideas through exploration with dynamically linked representations). One way to think about these distinctions is that the structural integrity of a lesson could be determined by watching a lesson but without hearing any teacher or student voices; whereas pedagogical integrity requires listening to what teachers and students are saying and how they are interacting and responding, both to each other and to the content of the lesson. By focusing on these constructs of structural and pedagogical integrity in this empirical study, we aim to identify the conditions and features of lesson enactment that help students make conceptual connections to the mathematics. This analysis may help improve the implementation of SunBay Math and similar technology-based curricular interventions.

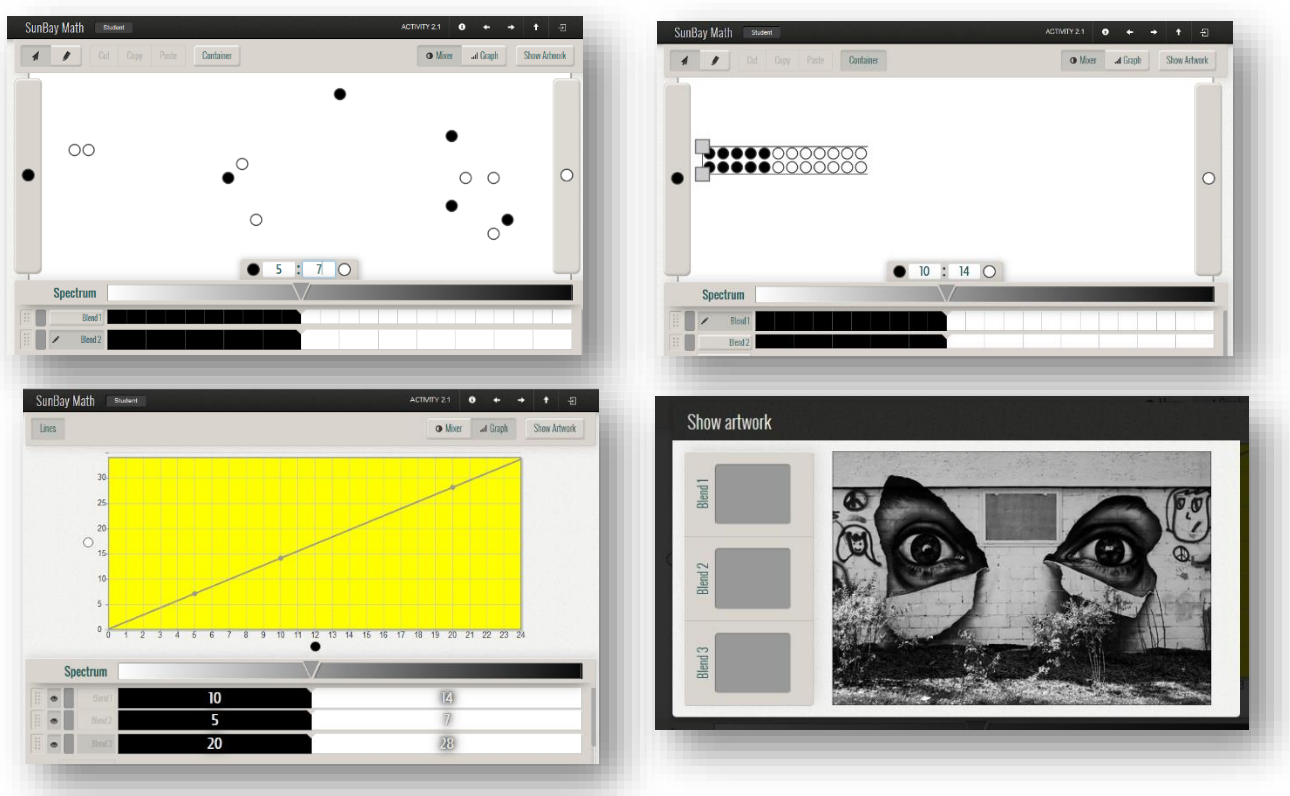
Curriculum Design

SunBay Math was designed as a *curricular activity system* in which technology-based dynamic mathematical representations, standards-based instruction, and teacher professional development are integrated to meet the needs of local educational contexts (Vahey, Knudsen, Rafanan, & Lara-Meloy, 2013). The program consists of two- to three-week long curriculum replacement units designed to improve students' understanding of key math concepts (functions, ratio and proportion, algebraic expressions and geometric transformations) and incorporate effective instructional practices and mathematical practices from the Common Core State Standards for Math (CCSSM). By introducing teachers and students to new ways of learning and interacting with complex mathematical concepts and practices through a modular design, the units were designed as replacement units so that they could be easily implemented and also help facilitate instructional reform (Davis & Krajcik, 2005; Ball & Cohen, 1996). The theory behind the use of replacement units is that once teachers begin to see the resulting depth of student learning and engagement, they will be motivated to make broader changes in their math instruction, and perhaps even use their regular curriculum materials differently (e.g., focus on conceptual understanding, provide opportunities for collaborative learning, support productive struggle, and so forth).

SunBay Math's dynamic technology-based tools feature multiple, linked mathematical representations that are designed to support student understanding of both underlying concepts and procedures. Each unit is designed around the use of one of four dynamic web-based tools for generating and manipulating mathematical relationships through simulations. For example, *Ratio Visualizer*—explored in more detail in the case studies in Chapter 4—includes dynamic visual and graphical representations of ratio and proportions through a simulation of paint mixing with *pips* or units of color to make various shades of paint. There are also tools for dynamic, linked representations of algebraic expressions, geometric transformations and linear functions.

Shown below in Figure 1 are four dynamically linked representations in the *Ratio Visualizer* tool used in the sixth-grade unit on ratio and proportion. The mixer (shown in the top images) allows the user to enter any number of paint pips (dots) in each color. The resulting color blend is shown by a triangular marker on the *spectrum* at the bottom of the screen. Directly below the spectrum, the *blend bar* shows the number of pips of each color in a ratio bar that corresponds to the spectrum. In the first image, the ratios 10:14 and 5:7 are represented on both the blend bars and the spectrum. If the ratios were not equivalent, the blend bars would not line up. Users can also link to and manipulate the *container* representation to organize the pips into rows and columns. In the image on the top right, the container has been resized to show 10:14 as two equal rows of 5:7, which is also shown on the spectrum and blend bars. The container therefore allows the user to see the unit ratio, or the ratio in its simplest form, within any ratio. Multiple ratios and equivalences can also be represented in a graph and table format as shown in the image at the bottom left. Finally, at any time the user can select *Show Artwork* to pull up a picture of the artwork along with images of the color blends that have been entered (bottom right).

Figure 1. *Ratio Visualizer* Dynamic Representations: Linked Mixer, Spectrum, Container, Graph and Artwork



Users can manipulate specific inputs (e.g., adding units of paint to create a new color, or in geometry, stretching a shape to alter its angles) to produce linked changes across the multiple visual representations. This linked structure allows students to explore underlying mathematical relationships by observing how different inputs produce different responses across the various representations. These tools therefore allow for *representational fluency*, or the ability to make meaning of and move between different representations of a mathematical concept (Zbiek, Heid, Blume, & Dick, 2007).

The SunBay Math units consist of 7 to 10 investigations, most of which are designed to be completed in one 45-minute class period. Each investigation contains four parts: (1) a short Warm-Up section (problems designed to preview the math needed for the lesson), (2) the Main section (a set of tasks designed to be completed through inquiry, using the technology), (3) the Wrap-Up (teacher facilitated discussion to make the math explicit and consolidate learning) and (4) Problem-Solving (problems that can be started in class and then assigned for homework). Each unit has a student workbook in which students can read the tasks and record their responses while working with the technology. The SunBay Math Teacher Guides include sections on the mathematical goals, standards, key ideas, implementation, as well as suggested timings, tips, teacher questions and student answers for each investigation.

SunBay Math is designed to support students in an inquiry cycle of predicting, evaluating and understanding mathematical change through exploration with dynamic, linked representations. The Predict-Check-Explain (PCE) cycle is designed to allow students to develop and justify conjectures or hypotheses about the impacts of various inputs across different representations, to test the hypotheses using the technology, and then to reflect on and discuss the accuracy of their hypotheses and the insights they developed through the cycle. As one member of the development team explained:

> Predict requires you to bring your prior knowledge—you have to start with whatever understanding you have and then you check with the software. It's not the teacher who's telling you that you're correct or not correct, it's the software that actually represents it. Then you have to explain why things are happening in the way that they are, from the software.

The designers of SunBay Math present this approach as a contrast to purely symbolic approaches to traditional mathematics instruction, where mathematical rules are learned in isolation and then applied to various situations. Classroom teachers facilitate this process by guiding students' progression through multiple PCE cycles during each lesson.

In addition to the structural components of a SunBay Math lesson (i.e., using the technology and workbooks and completing the lesson sequence), SunBay Math expects teachers to engage learners in a manner that positions them to think through problems themselves, as opposed to doing most of the intellectual work for students. The teacher is positioned in the role of a *facilitator*, leveraging the perspectives and ideas of individual students while guiding the entire class toward deep conceptual understanding of mathematical concepts. As one of the curriculum developers stated, the teacher's job is to "get the students to learn it in a way that makes sense, not to memorize it."

This intentional framing of teachers as facilitators pushes against common practices found in many math classrooms where teachers tell or show students procedures and then have students complete practice problems, often without students acquiring any deep conceptual understanding of what they are learning (Stigler & Hiebert, 1997). The teacher's role in SunBay Math involves actively leading whole class introductions to build on students' prior knowledge and prepare them to complete the tasks successfully through the PCE cycle. Students then engage in PCE cycles with the technology in small groups, as teachers circulate through the classroom. They listen in on students' dialogues about their experimentations with the technology, and interject with strategic questions, probing students' thinking to lead them toward mathematical reasoning, while simultaneously ensuring that they move through the investigations in a timely manner. At the end of the lesson, teachers again orchestrate whole class discussions to elicit student strategies and reasoning around the important ideas, steering student contributions toward a shared understanding of the central mathematical concepts. Thus, the teacher's role involves some restraint in not directly telling students what they need to see and learn, but also careful listening to build on and support students' mathematical reasoning through questioning and probing to ensure that they learn important mathematics. A curriculum developer explained that in a SunBay Math lesson you should see "students talking more than teachers," while teachers are asking lots of open-ended questions with a "repeated emphasis on 'why'." Consequently, implementing the curriculum in line with the developer's intent is a matter of orchestrating specific structural components and activity structures as well as relatively sophisticated pedagogical moves.

Implementation

As part of the scale-up and evaluation of SunBay Math, two units each for Grades 6, 7 and 8 were implemented in 30 middle schools randomly selected from two large school districts (District 1 and District 2) with diverse, high-need student populations in Florida. During 2016–2017, the second year of implementation when the evaluation took place, the District 1 student population comprised 39% African-American and 32% Hispanic students, with 61% eligible for free or reduced-price lunch. The District 2 student population comprised 32% Hispanic and 28% African-American students, with 59% of students eligible for free or reduced-price lunch.

The developers of SunBay Math created the online tools and supporting materials as well as "train the trainer" modules for district-level facilitators who had considerable experience as math teachers and/or school-based math coaches. This training focused on the content of each unit, while also being designed to give the facilitators a sense of the experience students should have when engaging with the technology and the tasks. The facilitators then provided 2.5 days of professional development sessions for teachers of Grades 6–8 math (1.5 days prior to using their first SunBay Math unit, and an additional half-day prior to implementing any new unit for the first time), which reflected not only the content and experiential aspects, but also some additional focus on pedagogical strategies. Teachers were expected to implement two 2-week SunBay Math modules during the school year in place of the regular curricular materials, over the two-week periods when the corresponding concepts or standards were covered in the district pacing guidelines.

During the school year, teachers also received on-site support from the facilitators who were available to provide coaching to teachers via conferencing, observation, reflection, modeling, or any other type of support that teachers requested. All but two schools also had access to support from one or more school-based teacher leaders who regularly met with implementing teachers to reflect on and discuss SunBay Math implementation. This model of implementation therefore included several layers of support and potential points of translation from the curriculum developer's vision to the actual enactment in classrooms.

Chapter 2: Methods

Data Collection and Participants

Videorecorded lesson enactments were collected from a purposive sample of 26 Grade 6–8 classrooms in schools from both Florida districts implementing SunBay Math during Year 2 of the evaluation. Drawing on the expertise of the district facilitators, the sample was designed intentionally to elicit a range of lesson enactments in relation to district, grade level, unit, teacher experience and the facilitator's view of the overall strength of the implementing teacher. Participation in video recording was voluntary and as Tables 1 and 2 show, the resulting sample reflects variation in all but one dimension: there were only 4 teachers from District 2. Only one lesson from each teacher was recorded; the sample was not selected to be representative of the larger implementation nor representative of the particular teacher's approach to instruction, but rather to capture and explore variation in the enactment of SunBay Math lessons in classrooms.

Table 1. Characteristics of Teachers who were Videorecorded (n = 25/26*)

Role		District		Teaching Experience		SunBay Math Experience		Highest Degree Received	
Teacher	22	District 1	22	1–5 years	6	1 year	10	Bachelors	16
Teacher Leader	3	District 2	4	6–11 years	5	2 years	15	Masters	9
				12+ years	14				

*We did not have information on one teacher's background.

Table 2. Videorecorded Lesson Enactments

SunBay Math Topics and Units	Grade	Videorecorded Lessons	Number of Investigations
Ratio & Proportion			
True Colors Murals	6	10	6
Managing the Soccer Team	7	2	1
Algebraic Expressions			
Little X Games	6	2	2
3D Design Studio	7	7	3
Transformational Geometry			
Transformation Nation	8	5	3

The 26 videorecorded lessons reflect 15 different SunBay Math investigations from five of the six SunBay Math units. Table 2 shows the variation in the sample of 26 video recordings collected across grade levels and units. While the sixth- and seventh-grade units were in the second year of implementation, the eighth-grade units were being implemented for the first time.

Data Analysis

Analysis of the lesson enactment data proceeded in three stages. In the first stage, our goal was to characterize the dimensions on which lesson enactments varied in relation to both components unique to SunBay Math and to overall instructional quality. Four rubrics were selected from the Instructional Quality Assessment (IQA) (Boston, 2012; Matsumura, Slater, Junker, & Peterson, 2006), a validated toolkit that is designed to measure the quality of instruction in relation to cognitive demand (see Appendix A). The IQA *Potential of the Task* rubric is designed to measure the level of cognitive demand in the lessons as per the teachers' guide. *Implementation of the Task* holistically assesses the level of thinking students actually engaged in throughout the lesson, while *Rigor of Discussion* and *Rigor of Teacher Questions* relate specifically to the whole class discussion that occurs after students have had the opportunity to engage in mathematical work. While only a portion of any given SunBay Math lesson is designed to be whole group, it is during these discussions that students have opportunities to engage in reasoning and explanation, consolidate learning, and draw connections between different strategies. As the developer of the IQA (2012) asserts, "The whole-group discussion provides an opportunity for teachers to advance the mathematical understandings of *all* students" (Boston, 2012, p. 83).

We also viewed a subset of videos to determine dimensions that varied in lesson enactment in relation to central characteristics of SunBay Math as it is designed (i.e., elements that might not be as important in the enactment of other curricular materials). Four categories emerged from this analysis: 1) the presence of the three core components of a SunBay Math investigation (Warm-Up, Main, and Wrap-Up), 2) the enactment of the PCE cycle, 3) the approach to technology use and 4) the activity structure of the lesson in terms of balance between teacher facilitation and opportunities for student collaboration. Through an iterative process, we constructed four SunBay Math Enactment rubrics to capture levels of variation along these dimensions as shown in Appendix B, piloted them with a subsample of videos, and then further refined them.

In the second stage of analysis, a team of three researchers applied the four SunBay Math Enactment rubrics and four IQA rubrics to the entire sample of 26 videorecorded lessons. The team first watched, coded and discussed 12 videos as a group to establish consistency in coding. Each video was then coded by at least two members of the research team. On a weekly basis, all three video coders met to discuss and resolve any discrepancies in the coding.

In the third stage of analysis, we aligned the rubric levels with the key indicators of the curriculum design, or the curriculum designer's intent (Brown et al., 2009), in order to operationalize our constructs of opportunity to learn, structural integrity and pedagogical integrity.

Opportunity to learn was captured by the IQA rubric for Implementation of the Task, which assesses the overall opportunity students have to engage in high-level thinking and reasoning. Structural integrity was revealed in the extent to which the lesson began with a Warm-Up and a whole group discussion of the task, followed by students working in small groups on additional tasks with the technology while the teacher circulated, concluding with a whole class discussion or Wrap-Up. Pedagogical integrity to the curriculum designers' intent was indicated in the use of technology for exploration and the incorporation of the PCE cycle throughout the lesson, along with the scores on the IQA for teacher questions and student discussion. These last two IQA rubrics hone in on teacher and student actions in shaping mathematical discourse during whole group discussions.

These seven rubrics were then used to define high, moderate and low levels of opportunity to learn, structural integrity and pedagogical integrity, in relation to the curriculum design, as shown in Table 3, and described in more detail below.

Table 3. Dimensions of Opportunity to Learn, Structural Integrity and Pedagogical Integrity to the Curriculum Designer's Intent

	Opportunity to Learn	**Structural Integrity**		**Pedagogical Integrity**		
Level of Integrity	*IQA Rubric Levels for Implementation*	*Balance of Activity Structures*	*Three-part Investigation Structure*	*Approach to Technology Use*	*PCE*	*IQA Rubric Levels for Questioning and Student Discussion*
High	Doing mathematics or procedures with connections	Mostly small group	All three connected components	Exploration with connections	Consistent cycles	Doing mathematics or procedures with connections
Moderate	Procedures without connections	Mostly whole group	At least two connected components	Exploration without guidance	At least one cycle present	Procedures without connections
Low	Memorization	Entirely small or whole group	Only one component	Procedural use	Partial cycle	Memorization
None	Absence	NA	No components	Not used	None	Absence

Determining Levels of Opportunity to Learn

The IQA Implementation of the Task rubric (see Appendix A) focuses on the overall level of cognitive demand that students are engaging in during the lesson. At the highest score level of 4 (doing mathematics), students are "exploring and understanding the nature of mathematical concepts, procedures, and/or relationships" and there is explicit evidence of student reasoning and justification. At level 3 (procedures with connections), students are "creating meaning for mathematical concepts, procedures, and/or relationships" but there is no explicit evidence of student reasoning or justification. Levels 3 and 4 were considered to be high in relation to the curriculum designers' intent, as both of these levels reflect an overall focus on students' reasoning and making conceptual connections. Level 2, procedures without connections, was considered a moderate opportunity to learn, where students are using procedures that are "explicitly called for" and focusing on producing correct answers rather than connections. Finally, level 1, where students are recalling or reproducing memorized facts, rules, formulae, or definitions, was considered to be a low opportunity to learn.

Determining Levels of Structural Integrity

The two dimensions of structural integrity were assessed by applying the rubrics for: 1) balance of activity structures and 2) investigation components. The SunBay Math lessons are written to reflect a combination of whole class discussion, group work and individual work. Enactments that had some teacher-facilitated whole group portions with extended opportunities for students to work in groups, most closely reflected the balance of activity structures as written in the curriculum, and were considered high integrity. If the enacted lesson was mostly whole group but had at least some opportunities for student collaboration, it was considered to be at a moderate level of integrity, while those that were entirely whole group or entirely small group were considered to be of low integrity to the curriculum designer's intent. Additionally, lesson enactments that had the three main components from the same investigation (i.e., Warm-Up, Main, and Wrap-Up) were considered high integrity, while those that had two or one were considered medium and low respectively.

Determining Levels of Pedagogical Integrity

The four dimensions of pedagogical integrity were assessed by application of four of the rubrics: 1) approach to technology use, 2) PCE cycles, 3) teacher questions and 4) rigor of discussion.

For approach to technology use, our category of Exploration with Connections was considered the highest level of integrity to the authors' intent. This category includes teachers who intentionally guided and focused students on using the dynamic representations to reason, explain and construct arguments about the mathematics without following a specific procedure or solution path. Moderate integrity was characterized by the category Exploration without Guidance where teachers provided opportunities for students to use the technology to solve tasks and potentially to make connections with the dynamic representations, but there was no explicit evidence of teacher guidance or evidence that students were engaging at that level. Procedural Use of Technology was considered low integrity, where the dynamic representations in the technology were used by teachers and students primarily to confirm a solution already determined by a procedure or where students were directed by the teacher to a specific solution path or method.

For PCE, completion of all or most PCE cycles in the lesson enactment represented the highest level of integrity to the curriculum designer's intent, since this was a central feature of the design of the investigations and tasks. Moderate integrity was characterized by the presence of at least one PCE cycle during the lesson, while incomplete cycles (e.g., Predict and Check without Explanation) were considered low integrity.

The third and fourth dimensions of pedagogical integrity were determined by converting the IQA scores on the rubrics for Teacher Questions and Rigor of Discussion (see Appendix A) into levels of low, moderate and high pedagogical integrity.

The two IQA rubrics consist of a scale from 0 to 4, representing a continuum of increasing cognitive demand through opportunities for students to engage in reasoning, connections and explanations. Table 4 illustrates how we used the descriptors of these rubric scores to convert them into low, moderate and high levels of integrity to the curriculum design. As the authors of the IQA state, "a demarcation line exists between score levels 2 and 3 that separates high- and low-level cognitive demands, talk moves, or expectations in each dimension" (Boston 2012, pp. 85–86). Therefore, ratings on these rubrics had to be at least at level 3 to be considered high pedagogical integrity.

The Rigor of Discussion rubric measures the level of reasoning, explanation and justification that students engage in during whole group portions of the lesson. This is influenced by the questions that teachers pose to elicit student thinking. However, even when teachers pose high-level questions, students do not always respond with complete or substantive explanations. Together, these two rubrics provide a snapshot of the discourse in terms of the level of discussion occurring in a lesson. For these dimensions of discourse, we considered the highest level of Doing Mathematics to be going above and beyond the intent of the curriculum as designed, and designated that score as High+ in terms of pedagogical integrity.

Table 4. IQA Score Levels in Relation to Integrity to the Curriculum Design

Integrity	**Score**	**Level of Cognitive Demand**	**Characteristics of Discourse**
High+	4	Doing mathematics	Explicit explanations, connections or reasoning; high quality talk moves and student responses
High	3	Procedure with connections to meaning	Substantive attempts to engage students in explanations, connections or reasoning, but these may be incomplete
Medium	2	Procedures without connections to meaning	Focus on rote or procedural skills; weak or minimal attempts at talk moves
Low	1	Memorization	Focus on facts, rules, or formulas; absence of talk moves or more than one-word responses from students
NA	0	Absence	Absence of whole group discourse

To determine if there were relationships between students' engagement and opportunity to learn in the enacted lesson and the dimensions of pedagogical and/or structural integrity, we analyzed the levels of integrity of each dimension (shown in Table 3) across the individual lesson enactments. We then purposefully selected four lesson enactments from the same or similar investigations in one unit to serve as case studies of lesson enactment (Patton, 1990). The rationale for the selection of the four lesson enactments and additional methods of case study analysis are described in Chapter 4, the case study section of the report.

Chapter 3: Structural and Pedagogical Integrity of Enacted Lessons

In this study, we examined the enactment of SunBay Math lessons in terms of: (1) opportunities for student engagement with mathematical concepts and; (2) teacher practices associated with enacting these lessons according to the curriculum designers' intent. When we compared the IQA scores for Potential of the Task with Implementation of the Task (see Appendix C), we found that in line with many other studies of lesson enactment, most of the lessons (20 out of 26) were enacted at a lower level of cognitive demand than the level at which they were written in the curriculum materials (Cohen, 1990; Henningsen & Stein, 1997; Lloyd, 1999; Stigler & Hiebert, 2004). The preponderance of video-recorded lessons were enactments at a level 2 on the Implementation of the Task IQA rubric, or Procedures without Connections (n = 16), indicating that proceduralizing high-level tasks was common.

Based on our sample of classroom observation data, we also found that teacher adaptations were common, both in the ways the lesson was structured for students and in the instructional moves that shaped students' interaction with the content of the lesson. Table 5 presents the frequency of levels of pedagogical and structural integrity along with the overall level of implementation across the 26 observations.

Table 5. Levels of Pedagogical Integrity, Structural Integrity, and Opportunities to Learn

	NA*	Absent 0	Low (1)	Med (2)	High (3/4)
Pedagogical Integrity					
Approach to technology use	2	2	12	5	5
Predict, check, explain	3	1	10	7	5
Teacher questions (IQA)		0	8	12	6
Student discussion (IQA)		4	9	9	4
Structural Integrity					
Balance of activity structures		0	6	14	6
Investigation components		0	3	13	10
Opportunity to Learn					
Implementation of the Task (IQA)		0	0	16	10

Notes: The eighth-grade transformational geometry unit had some introductory investigations that did not explicitly require the use of technology or PCE; therefore these lessons were given NA for those rubric scores.

As Table 5 illustrates, the lesson enactments varied across all dimensions of pedagogical and structural integrity as well as opportunity to learn. Most of the lessons were implemented at a procedural level (16), while 10 were implemented at a high level of cognitive demand (procedures with connections or doing mathematics). We further explored the associations of these dimensions of structural and pedagogical integrity with the opportunities for students to engage in reasoning and make conceptual connections to the mathematics. When the levels of implementation are placed side-by-side with the levels of pedagogical and structural integrity, shown in Figure 2 as a heat map, some illuminating patterns emerge. In Figure 2 the lessons are ordered first by opportunity to learn (level of implementation) and then by pedagogical integrity.

Figure 2. Opportunity to Learn, Pedagogical Integrity and Structural Integrity Across Lesson Enactments

		Opportunity to Learn	Pedagogical Integrity				Structural Integrity			
Grade	Content Focus of Unit	Level of Implementation	Approach to Technology use	PCE	Questions	Discussion	Balance of Activity Structures	Investigation Components	Year of implementation	
6	Prop	H+	H	H	H+	H	M	M	2	High Implementation and High Pedagogical integrity
6	Prop	H	H	H	H	H	H	H	2	
6	Prop	H	H	H	H	H	H	H	2	
6	Prop	H	H	H	H	H	M	M	2	
6	Alg	H	H	M	M	L	M	H	2	
6	Prop	H	L	M	H	M	M	M	2	High Implementation and Moderate/High Pedagogical integrity
8	Geo	H	NA	NA	H	M	M	H	1	
8	Geo	H	M	NA	M	M	M	H	1	
8	Geo	H	M	M	M	M	H	M	1	
8	Geo	H	NA	NA	M	L	M	H	1	
7	Alg	M	M	H	M	M	M	H	2	Procedural Implementation and Moderate/Low Pedagogical integrity
6	Alg	M	M	M	M	M	H	M	2	
6	Prop	M	L	M	M	M	M	H	2	
8	Geo	M	L	M	M	L	M	L	1	
7	Alg	M	L	M	L	L	L	H	2	
7	Prop	M	M	L	M	M	M	M	1	
6	Prop	M	L	L	M	—	H	M	2	
6	Prop	M	L	L	M	L	L	M	2	
6	Prop	M	L	L	M	—	H	M	2	
7	Alg	M	L	L	L	M	W	M	2	
7	Alg	M	L	L	L	L	L	M	2	Procedural Implementation and Low Pedagogical integrity
7	Alg	M	L	L	L	—	L	L	1	
7	Alg	M	L	L	L	L	M	H	1	
7	Alg	M	—	L	L	L	L	M	1	
7	Prop	M	—	—	L	L	M	M	2	
6	Prop	M	L	L	L	—	L	L	1	

Only two lesson observations had evidence of both high pedagogical integrity and high structural integrity. The 10 lesson enactments that were rated at a high level of opportunity to learn (in the the top half of Figure 2) had either moderate or high levels of pedagogical integrity: 9 of the 10 were from either the sixth-grade proportions unit or the eighth-grade transformational geometry unit. In contrast, all of the enactments of lessons from the seventh-grade algebra and proportions units were rated at a low level of opportunity to learn and also had moderate to low levels across the dimensions of pedagogical integrity.

This suggests that the unit itself (the mathematical content and/or the nature of the technology) may be an important factor in the nature of the transformations that are made during enactment, though the sample size is too small to be conclusive. Transformational geometry is a topic that lends itself to conceptual exploration while algebraic expressions is a topic that has traditionally been taught procedurally in middle school. In interviews conducted for the larger evaluation study, many teachers commented that students had a harder time understanding and making sense of the visual representations in the technology for the algebraic units. Alternatively, the dynamic mathematical representations in the proportions and geometry units may have been more conducive to conceptual exploration than those in the algebraic units. While it is beyond the scope of this study to make that determination, our findings suggest that the affordances for conceptual connections in the curriculum unit and/or related technological tools is an important consideration.

To mitigate the influence of the curricular unit on implementation with integrity, we looked more closely at the variation within units. In the sixth-grade proportions unit, we had the most videorecorded lessons (10) as well as the most variation in the enacted levels of opportunity to learn, pedagogical integrity and structural integrity. The sixth-grade *True Colors Murals* unit focuses on understanding the multiplicative relationships within and between ratios and using these relationships to recognize equivalence, find missing values and compare ratios. Figure 3 shows the relationships between the investigation and the opportunity to learn, pedagogical integrity and structural integrity across the 10 lesson enactments from that unit.

As Figure 3 shows, pedagogical integrity appears highly associated with the maintenance of high levels of implementation or opportunity to learn. Adaptations teachers made to the lesson structure (Investigation Components) and the grouping of students (Balance of Activity Structures) in that same unit were less influential in maintaining high levels of cognitive demand for student engagement than the adaptations they made to pedagogical practices.

Figure 3. Opportunity to Learn, Pedagogical Integrity and Structural Integrity Across Lesson Enactments from Grade 6 True Colors Murals Unit

	Opportunity to Learn	Pedagogical Integrity				Structural Integrity		
	Level of Implementation	Approach to Technology use	PCE	Questions	Discussion	Balance of Activity Structures	Investigation Components	Year of implementation
2	H+	H	H	H+	H	M	M	2
1	H	H	H	H	H	H	H	2
2	H	H	H	H	H	H	H	2
2	H	H	H	H	H	M	M	2
5	H	L	M	H	M	M	M	2
7	M	L	M	M	M	M	H	2
2	M	L	L	M	—	H	M	2
5	M	L	L	M	L	L	M	2
6	M	L	L	M	—	H	M	2
4	M	L	L	L	—	L	L	1

Pedagogical practices which support the development of students' sense-making and reasoning are essential to maintaining or raising the cognitive demand of the lesson, regardless of whether the lesson was enacted in a whole group or small group activity structure. Figure 3 shows that the enactment of the lessons in the first five rows was implemented at a high level and reflected the SunBay Math curriculum designers' intended pedagogical emphasis on student exploration, reasoning and explanation around the dynamic mathematical representations in the technology. However, structural integrity to the lessons as written—that is, having students work in groups and following the lesson structure—did not, on its own, appear to provide these same opportunities to learn for students. Four of the five lessons rated as having a low level of implementation also have high levels of structural integrity in either balance of activity structures or investigation components.

A closer examination of the five lessons enacted at lower levels of implementation (Procedures without Connections on the IQA) reflects an overall focus on learning procedures, getting correct answers and completing tasks, rather than on sense-making or reasoning. These lessons manifested lower levels of teacher questioning and discussion, and most were also low on PCE and the approach to technology use, reflecting a transformation of the SunBay Math activities and practices to align with a more traditional and procedural approach to mathematics instruction. In most cases, the PCE cycle was transformed into PC (Predict and Check) or sometimes only Check. Students had opportunities to use the technology, but they were using it primarily to check the veracity of their solutions rather than to explore the mathematics.

To further explore the associations between pedagogical integrity and structural integrity to the overall level of implementation[2], Table 6 presents statistical associations between each of the rubric scores calculated at Kendall's tau (Kendall & Gibbons, 1990; Kendall, 1938).[3] The correlation matrix presents coefficients for the full sample (n = 26) in the shaded triangle above the diagonal, and the same coefficients for the reduced sample of lesson enactments from the *True Colors Mural* unit (n = 10).

Table 6 Statistical Associations among Class Observation Rubric Scores

Rubrics	LOI	ATU	PCE	QUE	DSC	BAS	INC
Level of implementation	1.000	0.676	0.644	0.709	0.555	0.307	0.338
Approach to technology use	0.796	1.000	0.721	0.676	0.630	0.412	0.338
Predict Check Explain	0.821	0.866	1.000	0.695	0.673	0.319	0.385
Questions	0.937	0.746	0.800	1.000	0.615	0.480	0.229
Discussion	0.785	0.828	0.956	0.795	1.000	0.370	0.324
Balance of activity structures	0.131	0.289	0.250	0.215	0.179	1.000	0.204
Investigation components	0.214	0.393	0.510	0.335	0.521	0.510	1.000

Shaded triangle above the diagonal is Kendall's τ for the full sample (n = 26). Unshaded lower triangle is for the *True Colors Mural* videos only (n = 10).

We find that for both full and reduced samples, the level of implementation or opportunity to learn was significantly and positively associated with all four dimensions of pedagogical integrity: technology use, PCE, questioning and discussion. The coefficients ranged from 0.56 to 0.71 for the full sample, and were larger for the *True Colors Mural* unit lessons, suggesting that instances of reduced pedagogical integrity corresponded with decreased opportunities for students to engage in high-level thinking and reasoning. The dimension of pedagogical integrity with the highest estimated relationship to opportunities to learn, was questioning. In contrast, the correlations between the two dimensions of structural integrity and opportunitites to learn were not statistically significant. Thus reductions in structural integrity did not correspond with decreased opportunities for students to engage in high-level thinking and reasoning.

We also looked for evidence of the relationships of these dimensions to each other. Table 6 shows a relatively large correlation between Approach to Technology Use and PCE. This suggests that when teachers use PCE as intended in the curriculum design, technology is often used in a way that supports student exploration of mathematical concepts. The two dimensions of structural integrity show no statistically significant association either with each other or with pedagogical integrity.

Looking across these dimensions of enactment together suggests that the Approach to Technology Use, PCE, and discourse practices (teacher questioning and student discussion) are high-leverage pedagogical practices that can maintain or lower the overall level of implementation of SunBay Math in terms of cognitive demand, or opportunity to learn, for students. They are also highly associated and

[2] Level of implementation was measured by the score on the IQA Implementation of the Task rubric (see Appendix A).

[3] Because of the small sample size and ordinality of the variables, a non-parametric estimator was used to test for independence based on ranks. Exploratory analysis was also conducted using Spearman's rho correlation as well as multiple correspondence analysis. The results did not differ from those presented in magnitude or statistical significance. Kendall's estimator is based on concordant and discordant pairs and usually smaller than values of Spearman's rho correlation. Significance tests for tau are also more accurate with smaller sample sizes.

complementary practices. Dimensions of structural integrity are less influential in determining the nature of students' engagement with mathematical ideas.

Chapter 4: Case Studies of Lesson Transformations

We now turn to examples of four of these video enactments from the *True Colors Murals* unit to explore how the same content can be implemented to reflect different levels of both pedagogical and structural integrity, and to illustrate in context the particular teacher practices and orientations which are associated with different opportunities to learn for students. The case studies were purposefully chosen for contrast in both structural and pedagogical integrity to the written curriculum as shown in Table 7 below.

Table 7. Case Studies of Lesson Enactments from *True Colors Murals*

Structural Integrity	Pedagogical Integrity	
	Low	High
Low	Ms. Elmore (Inv. 4)	
Moderate	Ms. Deegan (Inv. 2)	Ms. Aguila (Inv. 2)
High		Ms. Chancellor (Inv. 2)

Three of the lessons were enactments of the same investigation (Investigation 2 on using doubling and halving to find equivalent ratios) while the fourth was an enactment of a similar investigation (Investigation 4 on generating equivalent ratios with discrete and continuous quantities). These four teachers had between six and 15 years of experience.[4] Ms. Aguila and Ms. Chancellor both had leadership roles in their schools (Math Lead and Department Chair, respectively). It was Ms. Elmore's first year of teaching SunBay Math units. From the video records, a narrative of each lesson enactment was constructed, telling the story of the lesson from start to finish, highlighting each of the elements of structural and pedagogical integrity within the context of the specific lesson, and constructing rich descriptions with excerpts of teacher and student dialogue.

As highlighted in the last chapter, teacher questioning was a high-leverage pedagogical practice closely associated with maintaining or lowering the cognitive demand of students' opportunity to learn. We therefore further analyzed transcripts of the video recordings of these four lesson enactments to provide a more nuanced picture of the nature and role of teacher questions. All questions posed by the teacher were marked and further analyzed to better understand teacher questioning and students' opportunities for mathematical engagement in these contrasting implementations of SunBay Math lessons. We adapted the methodology and typology of teacher questions in mathematics from Boaler and Brodie (2004) to code instances of teachers posing questions, either in whole class discussions or while students were working in groups or independently (see Table 8). Following Boaler and Brodie's (2004) methods, we considered "utterances that had both the form and function of questions, and which were mathematical" (p. 777). Repeated questions during one teacher turn, even if altered slightly, were counted as one question. Each transcript was coded by two researchers and any discrepancies

[4] All teacher and student names are pseudonyms.

were resolved through discussion. The preponderance of these question types are reported as a percentage of the total questions asked in each case study and later summarized in the cross-case analysis.

Table 8. Question Types from Boaler & Brodie (2004)

Question Type	Characteristics
Gathering information	Requires immediate answer, rehearses known facts/procedures, or enables students to state facts/procedures
Inserting terminology	Once ideas are under discussion, enables correct mathematical language to be used to talk about them
Exploring mathematical meanings and relationships	Points to underlying mathematical relationships and meanings;. Makes links between mathematical ideas and representations
Probing	Asks students to articulate, elaborate, or clarify ideas
Generating discussion	Solicits contributions from other members of class
Linking and applying	Points to relationships among mathematical ideas and mathematics and other areas of study/life
Extending thinking	Extends the situation under discussion to other situations where similar ideas may be used
Orienting and focusing	Helps students to focus on key elements or aspects of the situation in order to enable problem-solving
Establishing context	Talks about issues outside of math in order to enable links to be made with mathematics

The *True Colors Mural* unit is comprised of 10 investigations, each designed to be completed in 45 to 50 minutes, revolving around the real-world context of mixing paint for a company that makes murals. The unit begins with an introduction to ratio through the context of mixing red and yellow pips to make and compare different shades of orange. In Investigation 2, students use the technology to make equivalent blends (ratios) from black and white paint, explore the different representations (see Figure 1), and learn the symbolic notation for writing ratios. The tasks in Investigation 2 are constructed to allow students to discover that the numbers in the ratio can be doubled or halved to create an equivalent ratio, and that they can also build up ratios additively with the container to create equivalent ratios. At the end of the lesson, students are asked to generalize from the problems they have completed to explain how to create blends that are the same shade of grey. The investigation therefore offers students opportunities to identify patterns, make explicit connections between representations, strategies, and mathematical concepts, and form and justify generalizations based on this work without necessarily using a known procedure. Later, in Investigation 4, students have the opportunity to use what they have learned about equivalence and unit ratios to solve problems that involve both continuous and discrete quantities.

Ms. Chancellor: Students Working in Groups with High Pedagogical Integrity

The enactment of *True Colors* Investigation 2 in Ms. Chancellor's sixth-grade classroom provides an example of students working collaboratively in small groups while using dynamic mathematical representations in the software to make conceptual connections to the mathematics.

The lesson was enacted with high structural integrity, closely matching what was described in the written curriculum, with all the suggested investigation parts as well as the appropriate balance of activity structures: whole class discussions facilitated by the teacher at the beginning and end; as well as an extended period of time for group work, where students worked collaboratively using the technology to solve problems at their own pace while the teacher circulated.

The lesson was also enacted with high levels of pedagogical integrity. Ms. Chancellor's interactions with students during small group work with the technology emphasized and reinforced the use of technology with an inquiry approach. While students were working on the tasks, she continually prompted them to use the different representations in the software to check their predictions, "see what happens," and make connections. Rather than telling them if a solution was correct, she reminded them, "See if you can get it to line up in the software," or "Test it and find out. See if it matches up."

Through her interactions with students, Ms. Chancellor reinforced the PCE cycle, encouraged exploration and the use of different representations in the technology, and reinforced norms for working collaboratively. She emphasized and defined the predict and check parts of the PCE cycles throughout the lesson. For example, before having students work in groups on the tasks, she gave the following directions to orient them around the PCE cycle:

> When it says predict, you make a guess. Don't check it until you get to the check piece. You're not going to get in trouble if your prediction was wrong. [It's] just to make you think about it before you start.

Ms. Chancellor also oriented students towards collaboration around the mathematics. While they were working, she often reminded them to talk to their groupmates, at one point saying to the whole class, "Remember to use your groupmates as a resource," and in another instance saying to a student, "Did you share that logic with your group members?" During the group work portion of the class, she also encouraged students to continue exploring mathematical relationships they were discovering**.** When students noticed the doubling or halving pattern, she responded either by saying, "Hmm, interesting…" or by redirecting them to share this observation with their group members. She answered procedural questions or questions about the software that students raised, but also posed questions back to them that required reasoning, for instance, "Why do you think it's seven?" and "What do you notice about the original numbers and the new numbers?" Through these responses to students, she communicated the expectation that they should be reasoning about the tasks and the mathematical relationships with their groupmates.

Ms. Chancellor posed high-level questions throughout the lesson. In the whole group portions of the lesson, the majority of the questions Ms. Chancellor posed (63%) were focused on gathering information but she also asked some questions that required students to explore meaning (15%) or probed their thinking (12%). However, once a student responded to these higher-level questions, she tended to either restate or expand on the explanation, or follow up with more directive questions. (In other words, there was still room for improvement in terms of engaging students in high-level reasoning and explanation.)

In sum, this enactment of Investigation 2 reflected collaborative small group work with teacher facilitation of opportunities for students to develop understanding of mathematical procedures (doubling and halving a ratio to create an equivalent ratio) that was connected to visual and dynamic mathematical representations in the technology.

Ms. Deegan: Students Working in Groups with Low Pedagogical Integrity

Like the case of Ms. Chancellor, the enactment of Investigation 2 in Ms. Deegan's classroom reflected substantial time for students to work in groups buttressed by whole class discussions. However, the teacher practices in facilitating and supporting student work on the tasks led to a focus on completion rather than on exploration or connections. This case offers an example of how a teacher may transfer responsibility for learning to the students without providing adequate support to ensure that they engage with mathematical ideas at a high level of reasoning or sense-making.

While the lesson was enacted with a high level of structural integrity to the balance of activity structures, it did not incorporate all investigation components: there was no Wrap-Up discussion. After having students finish the Problem Solving section from the previous lesson and completing the Warm-Up, Ms. Deegan began the Main portion of the investigation by reading the first task to the students. During the whole class introduction, she posed questions that elicited factual responses from students and then followed up on their responses only to confirm when the answer was correct. For the next half hour, students worked in their groups to answer the tasks using their iPads and workbooks.

In terms of pedagocial integrity, Ms. Deegan's facilitation of student work did not maintain a focus on using technology for exploration, engaging in PCE, or discussion of mathematical ideas. While students worked in groups, Ms. Deegan circulated and checked in with various groups to gauge and encourage their progress through the tasks, as well as to answer questions. She monitored student work by directing them to work through the tasks in the workbook, which meant that students worked on different parts of the PCE cycle that were embedded in the tasks. Unlike Ms. Chancellor, she did not remind the whole class what it meant to predict, check or explain. When a girl was asking for help with a predict question, Ms. Chancellor responded: "It's what you think is going to happen. What do you think is going to happen? I don't know. You got to tell—they want *you* to do it." Similarly, when a student asked for help on an explain question by asking, "Can I just say I know because I compared them to the other grays?", Ms. Deegan confirmed that her response was fine as long as it was her explanation rather than push her to focus on the mathematical relationships. In this way, she was encouraging student autonomy but not providing support or direction to help them focus on inquiry or on the important mathematical ideas.

Ms. Deegan circulated while students were working in groups, but her interactions focused almost entirely on logistics and completion rather than on using technology to explore and reason about mathematical relationships. When students asked questions, she responded by reading the problems aloud or by directing them to the next step, which often involved using the dynamic representations: "So now, what does it say now? Keep reading the next part. You guys make the container yet?" As she circulated, her comments focused on checking for progress on the workbook page rather than looking at the quality or reasonableness of their answers. She continually urged them to keep moving through the problems: “You have two more pages to go actually. If you get this done, you’ll be ahead of the game.”

While students were working in groups, Ms. Deegan occasionally asked for confirmation that they understood the task, but then did not press further: “How are you guys doing over here? Understanding? Ok good.” A few minutes before the end of the period, she told students to finish up the question they were working on and turn in their iPads. There was no time spent working on or discussing the Wrap-Up to Investigation 2.

In sum, this implementation represented a case of a teacher providing a lot of time for students to work together and with a great deal of autonomy. Students may or may not have engaged in mathematical reasoning, making connections, or exploring meaning while they were working in their groups with the technology, but their interactions with the teacher were almost exclusively focused on procedural logistics and completion. The fact that there were no instances of whole class discussion around the important mathematical ideas of the investigation makes it difficult to ascertain whether all students did indeed have opportunities to engage in meaning-making or develop an understanding of the mathematical ideas embedded in the tasks.

Ms. Aguila: Whole Group Teacher Facilitation with High Pedagogical Integrity

The same lesson was enacted in Ms. Aguila’s sixth-grade classroom with more whole group teacher facilitation than what was recommended by the SunBay Math curriculum, reflecting a lower level of structural integrity. However, this enactment received the highest overall rating for level of implementation and teacher questioning. The case study highlights several important teacher practices that maintained and complimented the pedagogical integrity of the lesson as intended.

During both the Problem-Solving (homework) review and Warm-Up, she consistently called on several students for each problem and followed up with probing questions, such as “What did you do?” and “Why does that work?” Often when a student would give a response she would ask the rest of the class if they agreed and would then call on another student to explain why. She ended the Warm-Up by asking the class to look for patterns across the problems and eliciting several responses. This was not a suggestion in the teacher guide, but it emphasized the importance of doubling and halving, a main focus of this investigation.

During the Main portion of the investigation, Ms. Aguila led the class through each task: typically, by having a student read the question aloud, then clarifying the task, giving students a few minutes to work together on the iPad, asking a student to share or come up and demonstrate the solution to the class, and finally engaging the class in further demonstration or discussion. In this way, the class moved through all four of the tasks in the Main section at the same pace, and the solution to each question was discussed as a class before moving on to a new question. Despite the teacher exerting control over the pace and direction of the work, there were several elements that supported a high level of cognitive demand throughout the lesson.

First, Ms. Aguila engaged students in complete cycles of PCE, emphasizing the meaning of prediction (“There’s no right or wrong answer—you’re predicting”) and eliciting multiple answers, correct and incorrect. She then had students check their responses with the technology-based tools (e.g., “Check the spectrum—were both triangles on the same spot?”). Once the solution had been verified with the software and demonstrated, she then asked multiple students to verbalize an explanation before having them write one down in their workbooks. She also provided support for how students should explain by drawing on the technology without telling them exactly what to write: “What strategy did you use? How are those numbers related? Did you use the container? Did you use the spectrum? Write down how you know.”

Throughout the lesson, Ms. Aguila prompted students to use different representations when checking their predictions with the technology. She encouraged them to click on the artwork, spectrum, container, and mixer to check if the blends they created were equivalent. During whole class discussions, she posed questions about dynamic demonstrations of different mathematical representations. She also constructed an organized list of equivalent ratios on the board, which she continually added to and referenced throughout the lesson to highlight the multiplicative relationships between the quantities. In addition, Ms. Aguila introduced some of her own questions and a comparison to another familiar context—mixing chocolate milk—to enhance student understanding. While Ms. Aguila’s use of written representations on the board and the introduction of familiar contexts were adaptations of the SunBay Math materials, these adaptations maintained or enhanced the cognitive demand of the tasks and reflected the overall mathematical goals of the lesson as written.

Ms. Aguila asked a variety of question types throughout the lesson, including questions that focused on mathematical meanings and relationships (Boaler & Brodie, 2004). She frequently asked questions that generated student thinking and reasoning, such as: “How come that works? What do the numbers tell you? What was your strategy? Is there a pattern? Why do you think? How do you know they are the same color?” Finally, Ms. Aguila elicited multiple responses for each question, even after the correct answer had been provided. This is something we rarely saw in other video-recorded lessons. In this class, incorrect answers were both elicited and probed.

In sum, the lesson enactment by Ms. Aguila represents an implementation that maintained a high level of cognitive demand not only by following the intended curriculum but, in particular, through the questioning and pedagogical strategies that she brought to the lesson. It reflected her deliberate use of the resources in the curriculum along with her own pedagogical moves and orientation, to construct a dialogic learning experience where students engaged in reasoning around the important mathematical ideas.

Ms. Elmore: Whole Group Teacher Facilitation with Low Pedagogical Integrity

While the last case is an enactment of Investigation 4 rather than Investigation 2, it offers an important contrast in that the teacher practices focused more directly on procedural knowledge and solutions, something that was common in the larger sample of lesson enactments. The enactment of this lesson had low structural integrity in terms of both lesson components (having neither a connected Warm-Up nor a Wrap-Up) as well as the balance of activity structures (entirely whole group), along with low pedagogical integrity across all dimensions. This enactment reflected significant adaptations to the lesson, but in a way that detracted from the mathematical goals and the integrity of the lesson as designed.

In Investigation 4, students are expected to use what they have learned about ratio (including unit ratios) to generate equivalent ratios, find missing values and use table and graph representations. The tasks are designed to be open to multiple strategies. However, Ms. Elmore spent the first 15 minutes of class having students complete and then review an exit ticket[5], while highlighting how to find the unit rate through division. In this way, the lesson began with a procedural focus on the steps of a specific strategy before students had any opportunity to explore relationships between proportional quantities with the technology.

The first task in the investigation involves using the dynamic mathematical representations in the technology to complete a table of equivalent ratios for making different amounts of clay with a given ratio of 3 cups of flour to 1 cup of salt. After reading the introduction and task aloud, Ms. Elmore underlined parts of the task and demonstrated how to write the given ratio. She then told them to enter the given ratio into the mixer tool while she did the same on the projection screen, and asked students to figure out how many white pips they needed to create the same ratio with 12 black pips. Very quickly a student offered that it would be 3 white pips, explaining, "I divided 12 by 3 and I got 4." Ms. Elmore restated and expanded on this solution strategy by highlighting the calculations and recording them on the projected version of the workbook.

She then put the solution (12 black pips and 4 white pips) into the mixer tool in the technology and asked students if it was correct, emphasizing how to make sure the spectrum marker was lined up. In this way, the technology was used to check the answer found through division. She then asked the students how else they could check to make sure the blends were equivalent. When two students responded by restating the multiplicative relationship between the numbers, Ms. Elmore suggested: "Could we check the container?" However, rather than have students explore or demonstrate what the container could show, she answered her own question by stating: "Yes. Ok, that was another way."

The next entry in the table required determining how many cups of salt would be used for 1 cup of flour. Students struggled to provide the correct answer to this question, first answering 12, then 1, then 3, and finally 6. Rather than allow students to explore any of these predictions on the software, Ms. Elmore asked them to look for a pattern in the numbers and again led them to the procedure of dividing by 3 through more direct questioning while she recorded it on the projected screen:

Ms. Elmore: You guys could check those responses with the iPad and see if those blends are equal, but I want you to think about if there is a relationship between the flour and salt? Just the first row. How do you think they got from 12 to 4? How do you think they got from 3 to 1?

Student: Divided by 3.

Ms. Elmore: Divided by 3. That could work! 12 divided by 3 is 4. And 3 divided 3 is 1, so if you continue that pattern it should be 1 divided by 3, am I right or wrong?

Instead of having students finish the table ("We'll fill it in later"), Ms. Elmore moved on to the next task which involved using the graphing tool. Ms. Elmore first reviewed parts of the linked graph using the projected display and then explained how each point represented the ratios of 3 to 1 and 12 to 4 that had been entered. As the class returned to the question in the workbook, she said, "Explain what each point means." Ms. Elmore told students to note what she was writing on the screen, "We are going to

[5] Exit tickets were created by one of the districts to complement the units with formative assessment opportunities.

put ‘each point shows one of the’—please write this down—'clay blends ratios from the table above.’” For students, answering the explanation question consisted of copying down the teacher’s response.

At the end of the lesson Ms. Elmore directed students to hit the button marked “lines” and then posed and answered her own question:

> What happened? Do you see the difference now? That’s another way to check that your ratios are equal. Ok? The fact that both of these points are on the same line, they’re showing they have the same blend of 3 blacks to 1 white ratio. 3 blacks to 1 white ratio. So they share the same unit ratio and they create the same color blend.

As these examples show, there was mention and some demonstration of the different representations—the spectrum, the container, and the graph. However, the focus was on demonstration of the visual representations as confirmation of the procedure or for verification of the answer, rather than to test out predictions, explore, or look for relationships.

While Ms. Elmore posed some of the questions that were designated as “predict” or “explain” from the guide, the overall focus was on using the numbers and mathematical procedures to solve the tasks and writing down the correct explanation. The majority of questions (63%) focused on gathering information. While she did ask some questions that focused on exploring meaning, in every case she followed up with more directive questions that funneled directly to a numerical answer or one she provided herself. Overall, this lesson enactment represented a teacher-directed lesson centered on correctness and procedural knowledge. Ms Elmore provided direct instruction around a specific method (using division to find a unit rate) before having students work on the tasks, and answered some of the higher-level questions herself, only asking students to provide numerical answers. In turning the focus away from mathematical exploration towards the application of a procedure, she significantly lowered the cognitive demand of the task for students.

Cross Case Analysis: Unpacking Pedagogical Integrity

These four lesson enactments illustrate differences in the ways that the written curriculum is transformed through the pedagogical integrity of specific teaching practices and interactions between teacher and students in the classroom. First, we study these cases for the variation in the specific elements that are unique to SunBay Math lessons—PCE cycles and the use of the interactive technology-based tools to develop representational fluency (Zbiek et al., 2007). We then explore teacher questioning and student discussion in terms of more nuanced discourse practices. Finally we examine different teacher orientations that can be seen in these cases, specifically in terms of the role they adopted as a teacher and their stance towards the use of the curriculum units.

SunBay Math Practices

All four lessons incorporated opportunities for students to engage in the parts of the PCE cycle embedded within the tasks in the curriculum. Ms. Aguila continually reinforced norms for what it meant to predict, check and explain as she taught the lesson. Ms. Chancellor also defined what it meant to predict, and in her interactions with small groups indirectly emphasized the inquiry stance embodied in PCE by posing questions to students (“What do you notice?”; “Hmmm, interesting…”). In both cases, high levels of integrity to PCE provided important guidance for students in how they should interact with the tasks and representations in the technology. Ms. Deegan, on the other hand, referenced the explanation aspect of PCE as what “they” wanted you to do and stated that as long as it was the students' own explanation, it was acceptable ("This is your thoughts"). Her overall focus on PCE was more on completing each of the questions than on engaging in inquiry. Ms. Elmore did not directly address the meaning of PCE. Since she was teaching Investigation 4 it may be that students were already familiar with this routine. However, by walking her students through procedures for solving the problems before using the software, she in effect circumvented the PCE inquiry cycle. In both of these cases, the low levels of integrity to PCE meant that while students were using the technology, there was no guidance as to how they should be engaging with the dynamic mathematical representations to answer the questions.

In all four lesson implementations, students had some opportunities to use the software to make connections and develop understanding. However, Ms. Aguila and Ms. Chancellor directly supported the students' use of technology for exploration and developing representational fluency by prompting them to use different representations to check their predictions and look for relationships. Both teachers posed questions to focus students on using the technology to reason about the mathematics. Ms. Aguila also demonstrated these relationships to the whole class by using the representations dynamically (i.e., clicking between two blends and switching back and forth between the spectrum and the container to highlight mathematical relationships visually). It was unclear in Ms. Deegan's and Ms. Elmore's lessons whether students were making these connections through their use of the technology. In Ms. Elmore's class those opportunities were limited by focus on the use of representations to check the veracity of answers found through procedures. Table 9 summarizes the variation in both PCE and the approach to technology use across all four cases.

Table 9. SunBay Math Practices Across Cases

SunBay Math Practice	Ms. Aguila	Ms. Chancellor	Ms. Deegan	Ms. Elmore
PCE	Reinforcement of meaning	Reinforcement of meaning and redirection	Completion	Circumvention
Approach to Technology Use	Demonstration, exploration and supporting representational fluency	Exploration and supporting representational fluency	Use without guidance	Demonstration and procedural use

Discourse Practices

The more in-depth analysis of teacher questioning in these cases yields dramatic differences in the prevalence of different types of questions across the four enactments. Table 10 illustrates the six most common types of questions in order of prevalence across the four cases. As Table 10 shows, Ms. Aguila and Ms. Chancellor asked a lot more questions and posed more probing questions and more questions that focused on exploring meaning. Gathering information comprised the vast majority of questions in Ms. Deegan's lesson and most of the questions in Ms. Chancellor and Ms. Elmore's lessons. Ms. Aguila's lesson had nearly an equal balance between gathering information and probing and was the only case where there were any questions in the categories extending thinking or linking and applying.

Table 10. Types of Questions Posed in Lesson Enactment Case Studies (Boaler & Brodie, 2004)

Teacher	Total no. of Questions	Gathering	Probing	Exploring meaning	Generating Discussion	Inserting terminology	Other[6]
Aguila	93	37(40%)	32(34%)	11 (12%)	8 (9%)	3 (3%)	2 (2%)
Chancellor	124	79 (63%)	15 (12%)	19 (15%)	2(2%)	4 (3%)	5 (4%)
Deegan	37	33 (89%)	1 (3%)	1 (3%)	0	2 (5%)	0
Elmore	41	26 (63%)	3 (7%)	8 (20%)	3 (7%)	1 (2%)	0

The way in which teachers responded to incorrect student responses also varied in these lesson enactments, reflecting different opportunities for students to engage in productive struggle. Ms. Aguila and Ms. Elmore both elicited incorrect responses during whole class discussions, but Ms. Aguila elicited additional responses even after a student offered the correct response, thereby ensuring incorrect responses would get out into the discussion. Ms. Elmore elicited them only when the first student she called upon gave an incorrect response. More importantly, once elicited, Ms. Aguila probed the student's thinking, thereby communicating confidence in students' ability to reason about mathematics. In contrast, Ms. Elmore either did not acknowledge an incorrect response or redirected the student to the correct procedure and response. Both moves fail to signal confidence in students' capacity to reason through the problem. While Ms. Chancellor did not elicit any incorrect responses during whole group discussion, the way she responded to questions while students were working in groups encouraged them to figure it out together. This emphasis not only helped to maintain the opportunity for productive struggle but also encouraged collaboration among the group. Table 11 summarizes these four discourse practices across the cases.

[6] Orienting and focusing, extending thinking, and linking and applying.

Table 11. Summary of Discourse Practices Across Cases

Discourse Practice	Ms. Aguila	Ms. Chancellor	Ms. Deegan	Ms. Elmore
Questioning	High level, varied, and focused on mathematical meaning and relationships	High level, focused on mathematical meaning and relationships	Low level—encouragement and direction	Low level-procedural
Student explanations	Multiple strategies	Single strategy	None	Single strategy
Response to student explanations	Restated, expanded on, prompted for more	Expanded on, follow up with more directive questions	N/A	Redirected toward desired response
Incorrect responses	Elicited and probed	Orients to group	Leads to correct response	Ignores or leads to correct response

Teacher Orientation

The questions teachers asked and the way in which they responded to student contributions also illustrate differences in the role of the teacher in facilitating student learning. Ms. Aguila's orientation as a facilitator was to elicit student's ideas and then build on those ideas (by probing, inserting examples, or focusing them on patterns) to help them make connections to the mathematics. Ms. Chancellor continually prompted students to make sense of the mathematics while they were working collaboratively on the tasks. She took a less active role, however, in pressing on or connecting their ideas during whole class discussions. Ms. Deegan took on the role of a manager during whole class discussions. While students worked through the tasks, she made sure they were moving through each question and demonstrated correct responses, but she did not address the quality of their explanations. Ms. Elmore took the most directive approach, modeling strategies for solving problems, highlighting vocabulary and written formats, and modeling explanations.

These four cases also illustrate significant differences in the orientation of the teachers towards the use of the curriculum. Ms. Chancellor followed the curriculum with the most integrity, both in terms of structure and pedagogy. Her lesson followed the recommended timing for the parts of the investigations and the balance of activity structures. She emphasized PCE cycles and oriented students towards collaboration and productive struggle.

The enactment in Ms. Aguila's class, which had low structural integrity with high pedagogical integrity, however, suggests that structural integrity is less important for overall implementation. This is further supported by the fact that the enactment of Ms. Deegan's lesson reflected structural integrity without pedagogical integrity; while students completed the tasks as written in the curriculum, her role as a teacher—encouraging autonomy with the goal of completion—only partially reflected the goals of SunBay Math. As a result, the opportunity for students to engage in productive struggle around important mathematical ideas was diminished.

Ms. Aguila adapted the lesson in terms of both structure and pedagogy, but these adaptations were in line with the goals of SunBay Math and in many cases enhanced the lesson as written (e.g., eliciting and probing correct and incorrect responses, introducing her own representations and examples, and posing a variety of question types). Ms. Aguila demonstrated a high degree of what Brown (2009) defines as pedagogical design capacity, or "skill in perceiving the affordances of the materials and making decisions about how to use them to craft instructional episodes that achieve her goals" (p. 29).

Ms. Elmore also adapted the lesson in ways that reflected her goals, but her goals were to ensure that students learned correct procedures and obtained correct answers, instead of encouraging them to engage in productive struggle around important mathematics. Her practices reflected a procedural orientation towards mathematics learning and teaching. This may have been a result of having less opportunity for professional support as it was her first year of implementing SunBay Math. Importantly, the practices of Ms. Deegan and Ms. Elmore when teaching these SunBay Math lessons did not produce evidence of students' capabilities to engage in high-level reasoning and sense-making. For some teachers implementing SunBay Math, completion of all the tasks in the order they are presented may take precedence over ensuring that each lesson had a coherent beginning, middle and end around the instructional goal. Table 12 summarizes the orientations to teacher role and curriculum use across the cases.

Table 12. Summary of Teacher Orientations Across Cases

Discourse Practice	Ms. Aguila	Ms. Chancellor	Ms. Deegan	Ms. Elmore
Teacher role	Eliciting, building on, and connecting student ideas and mathematics	Prompting student meaning making	Management without connections	Modeling correct strategy and knowledge
Use of Curriculum	Adapting	Offloading	Offloading and adapting	Improvising and adapting

Finally, Table 13 combines the teacher practices and orientations discussed in these case studies. Looking at the practices within each case, one can see a thread that links them into an overall instructional approach: Ms. Aguila's practices focus on engaging students in high-level reasoning, while Ms. Chancellor's focus on facilitating collaborative reasoning. Ms. Deegan's practices focus on completion, while Ms. Elmore's focus on procedural knowledge and correctness.

Table 13. SunBay Math Practices, Discourse Practices, and Teacher Orientations Across Cases

		Ms. Aguila	Ms. Chancellor	Ms. Deegan	Ms. Elmore
SunBay Math Practices	PCE	Reinforcement of meaning	Reinforcement of meaning and redirection	Completion	Circumvention
	Approach to Technology Use	Demonstration, exploration and supporting representational fluency	Exploration and supporting representational fluency	Use without guidance	Demonstration and procedural use
Discourse Practices	Questioning	High level, varied, and focused on mathematical meaning and relationships	High level, focused on mathematical meaning and relationships	Low level—encouragement and direction	Low level—procedural
	Student explanations	Multiple strategies	Single strategy	None	Single strategy
	Response to student explanations	Restated, expanded on, prompted for more	Expanded on, follow up with more directive questions	N/A	Redirected toward desired response
	Incorrect responses	Elicited and probed	Orients to group	Leads to correct response	Ignores or leads to correct response
TeacherOrientation	Teacher role	Eliciting, building on, and connecting student ideas and mathematics	Prompting student meaning making	Management without connections	Modeling correct strategy and knowledge
	Use of curriculum	Adapting	Offloading	Offloading and adapting	Improvising and adapting

Chapter 5: Discussion and Implications

The results of this study suggest that elements of structural integrity—giving students access to technological tools and having students work in groups to complete the tasks as sequenced in the curriculum—is not sufficient to ensure opportunity to learn in the ways intended by the curriculum designers.

Developing curricular materials that can support and enhance students' opportunity to learn is a laudable and important goal. But while it may seem that activities embedded in technology remove, at least to some degree, the effect of the teacher, our study found significant variation in teacher instructional practices that had important consequences. Because of the way the teacher presented and orchestrated discussion around the tasks, the technology was either used procedurally (e.g., to find answers or check previously determined answers), used for exploration but without any guidance, or used for exploration along with reasoning, explanation and connections to important mathematical ideas. Just like more traditional text-based curricula, technology is adapted in practice and can be transformed by teachers and students. These findings highlight the importance of the *technological pedagogical integrity* of an enacted lesson, or how teachers frame, make use of, and support the purpose of the technological tools and/or what students should be doing with them.

The presence and nature of whole group introductions and conclusions to bookend the work around the task is equally important for students' opportunity to learn. A limitation of the study is that we are not able to ascertain how individual students or groups of students were discussing or making sense of the activities. Yet this limitation also points to a larger issue: what students learn in group work is likely incomplete and relatively unsophisticated, but it is through whole class discussion that the teacher can elicit, guide and consolidate student thinking in relation to the important mathematical ideas (Sleep, 2012; Stein, Engle, Smith, & Hughes, 2008). In the absence of high quality discussion focused on the important mathematical ideas, the learning that results is more or less dependent on the strengths and weaknesses of individual students and interactions that may or may not occur with the teacher.

Finally, the findings suggest that teachers' repertoires of instructional practices influence their ability to use and adapt the curriculum in ways that reflect the principles of teaching and learning of SunBay Math. All teachers adapted the curriculum in some way, albeit to different degrees and in relation to different dimensions (timing, activity structure, PCE cycles, use of technology, or explanation and discussion). In some cases, these adaptations did not disrupt the overall emphasis on engaging students in high-level reasoning around mathematical concepts and ideas. In other cases, they enhanced it. However, in many cases teacher adaptations subverted the design and intent of the materials. Teachers will always adapt curriculum to some degree. The question for reform efforts based on curriculum as a vehicle for change then becomes how to build teacher capacity for decision-making around technology-based curriculum that is responsive to the particular context, but still aligns with the curriculum developers' intent and/or district priorities.

Being able to use and adapt curricular materials to construct a lesson that coheres around the learning goals requires what Brown (2009) calls *pedagogical design capacity*. For example, although Ms. Aguila did not get to the discussion of the Wrap-Up question in the written curriculum, she had students share strategies for the last task in a way that accomplished the same goals. In this way, Ms. Aguila demonstrated her pedagogical design capacity by presenting a coherent lesson with clear learning goals. Similarly, while she directed students through the technology-based tasks in a more teacher-directed way than was suggested by curriculum design, she orchestrated high quality discussion linked to the learning goals throughout the lesson. Whether or not teachers make adaptations to the curriculum seems less important, then, than whether or not they have the capacity to make changes in response to local context that keep the overall intent of the curriculum design intact. As Taylor (2013) argues, rather than looking for "teacher-proof curriculum," we might be better off developing "curriculum-proof teachers" who can use and adapt curriculum in ways that expand learning opportunities for students.

Implications

New learning technologies are being promoted as a vehicle for instructional reform in mathematics, and evidence suggests that technology can be used to make learning more student-centered, increase the authenticity of mathematical activity, provide feedback that promotes reflection, and shift the authority for determining mathematical truth from the teacher or text to the students (Heid, 1997). The idea that technology can be a catalyst for these shifts, however, rests on the assumption that teachers have the capacity—beliefs, knowledge, and practices—to support and orchestrate the use of technological tools for meaningful mathematics instruction.

As a set of replacement units, SunBay Math is designed to position students as active learners and users of technology. The curriculum itself helps to orchestrate this positioning both in the way the technology operates and in the way the tasks and investigations are structured. However, this study emphasizes the central role teachers play in enacting the curriculum and key areas of leverage for SunBay Math professional development and ongoing support for high-level implementation: teacher capacity and instructional practices.

The appropriation of new pedagogical practices (Grossman, Valencia, Evans, Thompson, Martin, & Place, 2000) often involves undoing old practices—not just taking on new ones—and these old practices may be deeply entrenched (Munter & Correnti, 2017). In order to shift the work of problem-solving and "figuring it out" to students, teachers must organize the classroom and lesson structure differently, establish new norms and expectations, use different discourse moves to elicit student thinking, support student thinking and reasoning around challenging tasks, and expertly weave together multiple strategies and levels of understanding towards the learning goal. Supporting students to make connections to mathematical ideas through dynamic mathematical representations in the technology, adds another layer of complexity.

These practices were evident to some degree in the higher-level lesson enactments, but in lower-level enactments we saw students work in groups with the technology without guidance or direction, teachers asking questions that focused on gathering information and then following up with more directive questions to steer students towards the correct answer or procedure, or teachers demonstrating a procedure or method before having students use the technology to complete practice problems. These practices are common in more traditional and didactic models of math instruction, and in fact, had been promoted by both districts in recent years. It is not surprising then that we saw these patterns persist in the second year of SunBay Math implementation. Learning and adopting new practices takes time, and occurs at different rates for different teachers (Grossman et al., 2000; Desimone, Porter, Garet, Yoon,& Birman, 2002). While expectations change, and new curricula is provided, teachers are rarely given adequate time or opportunities to develop, practice and refine new instructional practices in their own classroom settings. This study highlights that these instructional practices are essential for providing high quality learning experiences for students with technology.

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Appendix A. IQA Rubrics (Boston, 2012)

Potential of the Task Rubric

Score	Description of Score Level
4	**The task has the potential to engage students in exploring and understanding the nature of mathematical concepts, procedures, and/or relationships, such as:** • Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR • Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts. The task must explicitly prompt for evidence of students' reasoning and understanding. For example, the task **MAY** require students to: • solve a genuine, challenging problem for which students' reasoning is evident in their work on the task; • develop an explanation for why formulas or procedures work; • identify patterns and form and justify generalizations based on these patterns; • make conjectures and support conclusions with mathematical evidence; • make explicit connections between representations, strategies, or mathematical concepts and procedures; • follow a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship.
3	**The task has the potential to engage students in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the task does not warrant a "4" because:** • the task does not explicitly prompt for evidence of students' reasoning and understanding; • students may be asked to engage in doing mathematics or procedures with connections, but the underlying mathematics in the task is not appropriate for the specific group of students (i.e., too easy or too hard to promote engagement with high-level cognitive demands); • students may need to identify patterns but are not pressed for generalizations or justification; • students may be asked to use multiple strategies or representations but the task does not explicitly prompt students to develop connections between them; • students may be asked to make conjectures but are not asked to provide mathematical evidence or explanations to support conclusions.
2	The potential of the task is limited to engaging students in using a procedure that is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. **There is little ambiguity about what needs to be done and how to do it.** The task does not require students to make connections to the concepts or meaning underlying the procedure being used. **Focus of the task appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem-solving strategy, practicing a computational algorithm)**. **OR** The task does not require student to engage in cognitively challenging work; the task is easy to solve.
1	**The potential of the task is limited to engaging students in memorizing or reproducing facts, rules, formulae, or definitions**. The task does not require students to make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced.
0	**Students did not engage in a mathematical activity**.

Implementation of the Task Rubric

<table>
<tr><td>Score</td><td>Description of Score Level</td></tr>
<tr><td>4</td><td>Students engaged in exploring and understanding the nature of mathematical concepts, procedures, and/ relationships, such as:
Doing mathematics: using complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example); OR
Procedures with connections: applying a broad general procedure that remains closely connected to mathematical concepts.
There is explicit evidence of students' reasoning and understanding. For example, students may have:
solved a genuine, challenging problem for which students' reasoning is evident in their work on the task;
developed an explanation for why formulas or procedures work;
identified patterns and formed generalizations based on these patterns;
made conjectures and supported conclusions with mathematical evidence;
made explicit connections between representations, strategies, or mathematical concepts and procedures;
followed a prescribed procedure in order to explain/illustrate a mathematical concept, process, or relationship.</td></tr>
<tr><td>3</td><td>Students engaged in complex thinking or in creating meaning for mathematical concepts, procedures, and/or relationships. However, the implementation does not warrant a "4" because:
there is no explicit evidence of students' reasoning and understanding;
students engaged in doing mathematics or procedures with connections, but the underlying mathematics in the task was not appropriate for the specific group of students (i.e., too easy <u>or</u> too hard to sustain engagement with high-level cognitive demands);
students identified patterns but did not make generalizations;
students used multiple strategies or representations but connections between different strategies/representations were not explicitly evident;
students made conjectures but did not provide mathematical evidence or explanations to support conclusions.</td></tr>
<tr><td>2</td><td>Students engaged in using a procedure that was either specifically called for or its use was evident based on prior instruction, experience, or placement of the task. There was little ambiguity about what needed to be done and how to do it. Students did not make connections to the concepts or meaning underlying the procedure being used. Focus of the implementation appears to be on producing correct answers rather than developing mathematical understanding (e.g., applying a specific problem solving strategy, practicing a computational algorithm).

OR There is evidence that the mathematical content of the task is at least 2 grade-levels below the grade of the students in the class.</td></tr>
<tr><td>1</td><td>Students engage in memorizing or reproducing facts, rules, formulae, or definitions. Students do not make connections to the concepts or meaning that underlie the facts, rules, formulae, or definitions being memorized or reproduced.</td></tr>
<tr><td>0</td><td>The students did not engage in mathematical activity.</td></tr>
</table>

Rigor of Teacher Questions Rubric

4	The teacher consistently asks academically relevant questions that provide opportunities for students to elaborate and explain their mathematical work and thinking (probing, generating discussion), identify and describe the important mathematical ideas in the lesson, or make connections between ideas, representations, or strategies (exploring mathematical meanings and relationships).
3	At least two times during the lesson, the teacher asks academically relevant questions (probing, generating discussion, exploring mathematical meanings and relationships).
2	There are one or more superficial, trivial, or formulaic efforts to ask academically relevant questions, probing, generating discussion, exploring mathematical meanings and relationships) (i.e., every student is asked the same question or set of questions) or to ask students to explain their reasoning. OR only one strong effort is made to ask academically relevant questions.
1	The teacher asks procedural or factual questions that elicit mathematical facts or procedure or require brief, single-word responses.
0	The teacher did not ask questions during the lesson, or the teacher's questions were not relevant to the mathematics in the lesson.

Student Discussion Following Task Rubric

4	Students show/describe written work for solving a task and/or engage in a discussion of the important mathematical ideas in the task. During the discussion, students provide complete and thorough explanations of why their strategy, idea, or procedure is valid; students explain why their strategy works and/or is appropriate for the problem; students make connections to the underlying mathematical ideas (e.g., "I divided because we needed equal groups"). OR Students show/discuss more than one strategy or representation for solving the task, and provide explanations of why the different strategies/representations were used to solve the task.
3	Students show/describe written work for solving a task and/or engage in a discussion of the important mathematical ideas in the task. During the discussion, students provide explanations of why their strategy, idea, or procedure is valid and/or students begin to make connections BUT the explanations and connections are not complete and thorough (e.g., student responses often require extended press from the teacher, are incomplete, lack precision, or fall short on making explicit connections). OR Students show/discuss more than one strategy or representation for solving the task, and provide explanations of how the different strategies/representations were used to solve the task but do not explain why they were used.
2	Students show/describe written work for solving the task (e.g., the steps for a multiplication problem, finding an average, or solving an equation; what they did first, second, etc.) but do not engage in a discussion of why their strategies, procedures, or mathematical ideas work. [There are presentations of students' work but no discussion.] OR Students show/discuss only one strategy or representation for solving the task.
1	Students provide brief or one-word answers (e.g., fill in blanks); OR Student's responses are non-mathematical.
0	There was no discussion of the task.

Appendix B. SunBay Math Enactment Rubrics

Lesson Structure Rubric

3 components	Warm-Up, Main, Wrap-Up from one investigation occur in one class period
2 components	At least 2 of the 3 components from the same investigation are present
1 component	Only 1 of 3 components from the investigation are present
None	Lesson does not incorporate any of the three SunBay Math components

Balance of Activity Structures Rubric

Mostly Whole Group	Lesson is mostly whole group and teacher-facilitated with some opportunities for students to work in groups
Mostly Small Group	Lesson has some teacher-facilitated whole group portions with extended opportunities for students to work in groups
Whole Group	Lesson is almost entirely whole group teacher-led with no opportunities for students to work in groups
Small Group	Lesson is almost entirely students working in groups while the teacher circulates

Technology Use Rubric

Exploration with connections	Student exploration and use with the teacher helping to make connections to the mathematics
Use without guidance	Opportunity for exploration without guidance, connections, or explanation from the teacher; purpose or evidence of student use is more or less unclear
Procedural use	Technology is used in a procedural way (e.g., to check answers); technology is not used in a way that fosters student exploration or engagement in reasoning about the mathematical content
No use	Lesson does not incorporate any technology; or it is unclear the extent to which it was used

Predict-Check-Explain (PCE) Rubric

Consistent PCE	Lesson incorporates all three aspects of the cycle: there are opportunities for students to predict, check, and explain and students engage in multiple PCE cycles while using the technology.

One PCE	There is at least one complete PCE cycle enacted in the lesson, either in whole group or paired work, but PCE cycle is not consistent across the lesson.
Partial PCE	Two connected aspects of the PCE cycle are incorporated (e.g., students predict and check, but never explain)
No PCE	Lesson does not reflect any aspects of the PCE cycle

Appendix C. Task Implementation Scores in Relation to Potential of the Task on the IQA