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Ilyse Resnick, Luke Rinne, Christina Barbieri, and Nancy C. Jordan

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Children's Reasoning About Decimals and Its Relation to Fraction Learning and Mathematics Achievement

Ilyse Resnick
Penn State University Lehigh Valley

Luke Rinne, Christina Barbieri, and Nancy C. Jordan
University of Delaware

Reasoning about numerical magnitudes is a key aspect of mathematics learning. Most research examining the relation of magnitude understanding to general mathematics achievement has focused on whole number and fraction magnitudes. The present longitudinal study (N = 435) used a 3-step latent class analysis to examine reasoning about magnitudes on a decimal comparison task in 4th grade, before systematic decimals instruction. Three classes of response patterns were identified, indicating empirically distinct levels of decimal magnitude understanding. Class 1 students consistently gave correct responses, suggesting that they understood decimal properties even before systematic decimal instruction. Class 2 students were accurate when a 0 immediately followed the decimal, but were inaccurate when a zero was added to the end of the decimal string, suggesting a partial understanding of place value; their performance was also negatively influenced by a whole number bias. Class 3 students showed misunderstanding of both place value and a whole number bias. Class membership accurately predicted 6th grade mathematics achievement, after controlling for whole number and fraction magnitude understanding as well as demographic and cognitive factors. Taken together, the findings suggest students may benefit from instruction that emphasizes decimal properties earlier in school.

Educational Impact and Implications Statement

The present study showed that 4th-grade students use rule-based strategies to reason about decimal magnitude, even before they have had systematic instruction in school. In turn, level of decimal magnitude understanding was associated with later understanding of fractions and mathematics achievement in the intermediate grades. Findings highlight the importance of emphasizing decimal properties and suggest that decimal instruction could be presented earlier in school.

Keywords: decimal, fraction, magnitude, mathematics achievement

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Reasoning about symbolic whole number magnitude and symbolic fraction magnitude both contribute to broad mathematics achievement (e.g., Bailey, Hoard, Nugent, & Geary, 2012; Resnick et al., 2016; Siegler & Pyke, 2013; Siegler, Thompson, & Schneider, 2011, 2012). The role of decimal magnitude understanding in general mathematics achievement, however, has not been examined. If decimal, fraction, and whole number magnitude understanding each contribute independently to mathematics achievement, then reasoning about different types of magnitudes may require unique and important skills or processes. On the other hand, if decimal magnitude understanding does not predict mathematics achievement over and above whole number and fraction magnitude understanding, it would suggest that decimal magnitudes are processed in a way that is similar to fraction or whole number magnitudes.

To address this gap, the present longitudinal study examined how students reason about decimal magnitude before they have had systematic instruction on the topic as well as the contribution of early decimal magnitude reasoning to later mathematics achievement. We first characterized the ways that children interpret decimal magnitudes in fourth grade. By using a decimal comparison task, we sought to identify rule-based strategies reflecting different levels of decimal magnitude understanding. We then assessed whether decimal magnitude understanding in fourth grade predicts later fraction magnitude understanding in fourth grade and general mathematics achievement in sixth grade. To motivate the study, we review the literature on how whole number, fraction, and decimal magnitude understandings develop, the role of rule-based reasoning in understanding magnitudes in decimal and fraction format, and how such knowledge supports later mathematics achievement.

Ilyse Resnick, Department of Psychology, Penn State University Lehigh Valley; Luke Rinne, Christina Barbieri, and Nancy C. Jordan, School of Education, University of Delaware.

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Correspondence concerning this article should be addressed to Nancy C. Jordan, School of Education, University of Delaware, Newark, DE 19716. E-mail: njordan@udel.edu

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Development of Whole Number, Fraction, and Decimal Magnitude Reasoning

According to the integrated theory of numerical development, a unifying feature of mathematics learning is that all real numbers, including fractions and decimals, have magnitudes that can be ordered along the number line (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). When children learn mathematics, they gradually expand their mental number lines to include larger whole numbers as well as nonwhole rational numbers (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). To understand decimal and fraction magnitudes, students must reorganize their conceptions of the number line to accommodate new numerical properties (Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). It is not known precisely when students automatically activate numerical representations on the number line, although it has been suggested this occurs with whole numbers by 9 years of age (Laski & Siegler, 2014). It is likely that students use procedures or rules to activate representations of fractions and decimals through at least the intermediate grades.

Unfortunately, children often overgeneralize whole number properties when reasoning about rational numbers (Ni & Zhou, 2005). For example, students may overgeneralize that 0.25 is larger than 0.3 because 25 is larger than 3, or that 1/4 is bigger than 1/3 because 4 is larger than 3. Common decimal magnitude misconceptions have been identified by a number of researchers (Irwin, 2001; Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985; Stacey & Steinle, 1999). Misconceptions include an extension of whole number properties by identifying decimals with more places to the right of the decimal point as larger (e.g., $3.214 > 3.8$), misunderstanding place value by ignoring zeroes in the tenths place (e.g., $1.03 = 1.3$), and believing that adding a zero to the end of a decimal increases its magnitude (e.g., $5.40 > 5.4$). Children tend to retain the latter two misconceptions involving the role of zero longer (Irwin, 2001). Children also misapply properties of fractions to decimals (Durkin & Rittle-Johnson, 2014). For example, students may identify decimals with fewer places as larger (e.g., $1.2 > 1.353$) because fractions with smaller denominators are typically smaller than fractions with larger denominators (e.g., $1/10 > 1/1000$).

Role of Rule-Based Reasoning in Understanding Magnitudes in Decimal and Fraction Format

At any given time, most students possess multiple strategies for solving a problem and adaptively choose the strategy that maximizes speed and accuracy (Siegler, 1996). Adults and children use a range of rule-based strategies to reason about decimals (Resnick et al., 1989; Sackur-Grisvard & Leonard, 1985; Stacey & Steinle, 1999; Steinle & Stacey, 1998) and fractions (Fazio, DeWolf, & Siegler, 2016; Meert, Grégoire, & Noël, 2010; Rinne, Ye, & Jordan, 2017; Schneider & Siegler, 2010). These strategies incorporate both rule-based reasoning and holistic representation of magnitudes. For example, when identifying which of two fractions is larger, if both fractions have equal denominators, individuals may simply identify the fraction with the larger numerator as larger (Fazio et al., 2016). In this case, the fraction magnitude is not processed holistically; the problem is solved using an easier and more efficient strategy. However, when denominators and numerators are all different, individuals are more likely to reason holistically about the fraction magnitude (Rinne et al., 2017; Schneider & Siegler, 2010).

With respect to reasoning about fractions, rule-based strategies develop and change over time. In a 3-year longitudinal study of performance on fraction comparisons tasks (Rinne et al., 2017), students indicated the larger value within fraction pairs. Results showed that most fourth graders initially overgeneralized whole number properties and thought that larger numbers in both numerators and denominators yield larger fraction values. Eventually, some children began to choose fractions with *smaller* numbers in *both* numerators and denominators, believing that this translates to larger fraction values. That is, they applied an "inverse relationship" rule (i.e., the smaller the number, the larger the fraction), but failed to recognize that this rule holds only for the numbers in the denominator—the reverse is true for numbers in the numerator. However, children who at some point held this partial understanding tended to acquire normative fraction comparisons strategies earlier than did children who maintained the larger number bias. Thus, this partial understanding appears to represent a "stepping stone" on the way to normative strategy use.

Given the shared numerical properties of fractions and decimals, it is possible that a transitional partial understanding stage exists in decimal learning that is analogous to the one found in fraction learning. That is, the use of rule-based strategies for judging magnitude may develop in a similar fashion for each different kind of rational number representation. For both fractions and decimals, children may start with strategies that stem from simple generalizations of rules for whole numbers. Then, when learning rules that are distinct to a particular form of representation, children may initially misunderstand or misapply rules in a predictable, consistent way, only sometimes producing accurate judgments. As rules are fully learned, children will eventually exhibit normative magnitude judgments.

Relation Between Magnitude Understanding and Mathematics Achievement

It is widely viewed that reasoning about numerical magnitudes is foundational in mathematics learning (Halberda, Mazzocco, & Feigenson, 2008; Jordan et al., 2013; National Mathematics Advisory Panel, 2008; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013; Siegler & Lortie-Forgues, 2014; Siegler et al., 2011). Although the relation between nonsymbolic magnitude reasoning and mathematics achievement is not clear, there is a consistently strong relation between symbolic magnitude reasoning and mathematics achievement (De Smedt, Noel, Gilmore, & Ansari, 2013). Understanding whole number magnitude is important to early mathematics learning, while understanding of fraction magnitude supports understanding of more complex computations and algebraic concepts (Booth & Newton, 2012; Mix, Levine, & Huttenlocher, 1999; Siegler et al., 2011; Wu, 2009). Across this developmental continuum, understanding of numerical magnitude helps students assess the plausibility of their answers to arithmetic problems (Siegler, 2016).

There is one study that examines the role of decimal magnitude understanding in the context of algebra. DeWolf, Bassok, and Holyoak (2015) assessed concurrent predictors of seventh-grade algebra knowledge, including estimation of symbolic fraction,

decimal, and whole number magnitudes on the number line, a test of procedural fraction knowledge, and an assessment that focused on understanding of the relations between the numerator and denominator of fractions (referred to as the fractions relations task). Items on the fractions relations task assessed understanding fraction equivalence, division, inverse relations, multiplying by the reciprocal, and identifying part-to-part ratios versus part-to-whole ratios in countable sets. Among this set of predictors, only performance on decimal number line estimation and the fraction relations task emerged as independent concurrent predictors of algebra knowledge. DeWolf et al. argue that decimal magnitude tests provide a "purer" measure of numerical magnitude understanding than do fraction magnitude tests, because unlike fraction magnitudes, accessing decimal magnitudes does not explicitly require an understanding of mathematical relations between two numbers. That is, although bounded number line tasks involve proportional reasoning (Barth & Paladino, 2011), whole number and decimal magnitudes are often reasoned about as continuous one-dimensional quantities (e.g., a 1.25 L bottle of soda; DeWolf et al., 2015; Rapp, Bassok, DeWolf, & Holyoak, 2015). The bipartite structure of fractions, on the other hand, tends to elicit representation of magnitudes in terms of a two-dimensional relation between discrete objects (e.g., 3 candies in a box of 12 candies).

Present Study

The aims of the present study were to characterize fourth-grade students' use of rule-based strategies on a decimal comparison task and to assess whether these strategies support fraction magnitude understanding measured at the end of fourth grade as well as later mathematics achievement measured in sixth grade. At the time of this study, in the United States, fourth grade mathematics instruction typically covers fractions intensely and includes only limited instruction on decimals toward the end of the school year (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010; Tian & Siegler, 2017). We assessed children's decimal magnitude knowledge at the beginning of fourth grade, before they received any systematic instruction in school. We expected variability in children's understanding of decimals, because children typically have many informal experiences with decimals before systematic instruction (e.g., counting money; for a review see Mack, 1993).

We designed a decimal comparisons task to identify the rule-based strategies that children would be likely to use in the absence of a full understanding of decimal magnitudes. Children were asked to identify which of two decimals was larger across six blocks of four items, where each block was designed to capture an overgeneralization of whole number properties and/or misunderstanding of place value. Thus, patterns of performance across different blocks of items indicated what rule-based strategies (correct or incorrect) children were using to determine which decimal magnitude was larger. (The items, and their relation to whole number bias and place value bias, are presented in the materials section and summarized in Table 1.) If a child can access an accurate mental representation of decimal magnitude, they should be accurate across all blocks. If a child uses a rule-based strategy (i.e., applying whole number properties or misunderstanding place

Table 1
Description and Items in the Decimal Comparisons Task

Block	Block description	Items	Whole number bias	Place value bias
1	Different numbers Both in the tenths position	.2 or .4 .38 or .32 .08 or .03 .43 or .63	Helps performance	n/a
2	Same number In tenths vs. hundredths position	.04 or .4 .05 or .5 .9 or .09 .2 or .02	n/a	Hurts performance
3	Same number In tenths vs. thousandths position	.007 or .7 .1 or .001 .3 or .003 .006 or .6	n/a	Hurts performance
4	Different numbers Larger number in tenths position Smaller number in hundredths position	.04 or .5 .2 or .01 .9 or .08 .05 or .7	Helps performance	Hurts performance
5	Different numbers Larger number in hundredths position Smaller number in tenths position	.06 or .2 .09 or .8 .5 or .08 .1 or .03	Hurts performance	Hurts performance
6	Different numbers Both in the tenths position Smaller number has a zero in the hundredths position	.40 or .6 24.5 or 24.30 7.20 or 7.4 .8 or .60	Hurts performance	Hurts performance

Note. Whole number bias in the decimal comparisons task is to identify the decimal with the larger numerals as having the greater value regardless of the number's place value. Place value bias in the decimal comparisons task is to identify the decimal with more zeros between the decimal place and first digit larger than zero as having the greater value.

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value), we would expect him or her to be accurate only on items where the correct response is consistent with that strategy. For example, a student who holds a whole number bias would correctly identify that 0.4 is larger than 0.2 (because 4 is larger than 2), but incorrectly identify that 0.04 is larger than 0.2. If the rules the child used conflicted, we expected the child to perform at chance level. Take, for example, the item 0.04 versus 0.5; a whole number bias leads to a correct response, but a place value bias leads to an incorrect response. A child who misapplied whole number properties and did not understand place value would perform at chance on this item. Finally, similar to fractions, where some children show partial understandings on their way to normative development (Rinne et al., 2017), we expected some students to show partial knowledge of decimals, such as understanding place value (e.g., correctly identify $0.4 > 0.04$) while still possessing a whole number bias (e.g., incorrectly identify $0.40 > 0.6$).

Using a three-step latent class analysis (LCA; Asparouhov & Muthén, 2014; Vermunt, 2010), we first identified the latent class structure of fourth graders' performance across blocks on our decimal comparisons task. The three-step approach yields a model that relates class membership to other variables but does not allow those other variables to influence the latent class structure. Thus, we characterized rule-based strategies for reasoning about decimals, and then examined how other variables relate to this class structure. Similar to fraction magnitude understanding (Rinne et al., 2017) we hypothesized there would be at least three latent classes: good, partial, and poor understanding. However, there was no a priori hypothesis for if there would be one or two partial knowledge classes.

In our three-step model, we also assessed the relation between decimal class membership in fourth grade with covariates and mathematics achievement in sixth grade (controlling for the covariates). Covariates include fraction magnitude understanding, whole number magnitude estimation, general cognitive abilities (i.e., nonverbal reasoning, attentive behavior, working memory, receptive vocabulary, and reading fluency), and demographic factors (i.e., age, free lunch status, gender, and English Language Learner status). Cognitive predictor variables were chosen based on known relations to mathematics achievement (Geary, 2004) and fraction knowledge (Hansen, Jordan, & Rodrigues, 2017; Hecht & Vagi, 2010; Resnick et al., 2016; Ye et al., 2016). Given that in previous studies whole number and fraction magnitude understanding each uniquely predict later mathematics achievement, and considering the importance of developing effective rules for reasoning about different kinds of numerical properties, we hypothesized that decimal magnitude understanding will also account for unique variance in later mathematics achievement.

We also constructed a categorical structural equation model (SEM) to examine whether different types of rule-based strategies for determining decimal magnitude (assessed in the middle of fourth grade) support learning of rule-based strategies for solving fraction comparisons at the end of fourth grade, while controlling for fraction comparison strategies at the beginning of fourth grade. As discussed above, while decimals and fractions share numerical properties that are different from whole numbers, decimals and whole numbers have commonalities not shared by fractions: in particular, a structure that is grounded solely in base-10 syntax (i.e., unlike fractions, there is no "bipartite" structure). This commonality may lead students to maintain similar internal representations for decimals and whole numbers. Thus, it is not clear if understanding decimals would help support learning of fractions. To our knowledge, there have been no studies examining whether decimal understanding supports later fraction learning (for review see Tian & Siegler, 2017).

Method

Participants

Participants were part of a larger study on mathematics learning (Jordan et al., 2013). Students were recruited from nine elementary schools in two school districts in the same state that serve families from diverse socioeconomic backgrounds ($N = 435$). In the sample, 58.7% participated in a school free/reduced price lunch program, a proxy for low-income status; 46.9% was male; 52.1% identified as White, 39.4% Black, 5.7% Asian/Pacific Island, and 2.8% American Indian/Alaskan Native; and 16.3% additionally identified as Hispanic. Children's mean age at the start of the study was 8.83 years old. Eleven percent of the students were English language learners, and 11% were receiving special education services. Mathematics instruction in all schools was aligned with the Common Core State Standards (CCSS), starting in fourth grade.

Measures

General mathematics achievement. Mathematics achievement was assessed using a statewide standardized test, which was given to all students in sixth grade (American Institutes for Research, 2012). Assessment content included numeric reasoning, algebraic reasoning, geometric reasoning, and quantitative reasoning that is consistent with sixth-grade math standards. Internal consistency was high at $\alpha = .88$ (American Institutes for Research, 2012). Scores ranged from 0 to 1,300.

Decimal comparisons. Decimal magnitude knowledge was assessed with a decimal comparisons task. Students were asked to compare two decimals and circle the one with the greater value. There were six blocks of comparison items, with four items in each block. Each block corresponded to a comparison that focuses on a particular feature of decimal understanding. Blocks and items were presented in a fixed order, and are presented in Table 1. In Block 1, students compared decimals containing different digits located in the tenths position (e.g., 0.2 or 0.4); the correct response is consistent with whole number properties (e.g., 0.4 is larger than 0.2 and 4 is larger than 2). In Block 2, students compared decimals containing the same digit but located in either the tenths or the 100ths position (e.g., 0.04 or 0.4); the correct response is inconsistent with a place value bias, as this bias would lead to the incorrect belief that the number with more digits to the right of the decimal is greater (e.g., 0.4 is larger than 0.04 but 0.04 has more digits than 0.4). In Block 3, students compared decimals containing the same digit but located in either the tenths or 1000ths position (e.g., 0.007 or 0.7); like Block 2, the correct response is inconsistent with place value bias (e.g., 0.7 is larger than 0.007 but 0.007 has more digits than 0.7). In Block 4, students compared decimals containing different digits, with the larger-valued digit located in the tenths position and the smaller-valued digit located in the 100ths position (e.g., 0.04 or 0.5); the correct response is consistent with whole number properties (e.g., 0.5 is larger than

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0.04 and 5 is larger than 4) but inconsistent with place value bias (e.g., 0.04 has more digits than 0.5). In Block 5, students compared decimals containing different digits, with the larger numeral located in the 100ths position and the smaller numeral located in the tenths position (e.g., 0.06 or 0.2); the correct response is inconsistent with whole number properties and place value bias (e.g., 0.2 is larger than 0.06 but 6 is larger than 2 and 0.06 has more digits than 0.2). In Block 6, students compared decimals containing different digits in the tenths position, with the smaller-valued digit followed by a zero in the 100ths position (e.g., 0.8 or 0.60); like Block 5, the correct response is inconsistent with whole number properties and place value bias (e.g., 0.8 is larger than 0.60 but 60 is larger than 8 and 0.60 has more digits than 0.8). Internal reliability for the decimal comparison task for our sample was high ($\alpha = .95$).

Fraction comparisons. The fraction comparisons task had the same structure as the decimal comparisons task. Students were asked to compare two fractions and circle the one with the greater value. There were six blocks of four items presented in a fixed order. Each block corresponded to a comparison that focuses on a particular feature of fraction understanding (see Table 2). In Block 1, students compared unit fractions (e.g., 1/3 or 1/5). In Block 2, students compared fractions with the same denominator but different numerators (e.g., 3/5 or 4/5). In Block 3, students compared fractions with relatively larger numerators and denominators to fractions that had relatively smaller numerals in the numerators and denominators but were larger in magnitude (e.g., 50/100 or 16/17). In Block 4, students compared fractions with the same numerator but different denominators (e.g., 6/9 or 6/12). In Block 5, students compared reciprocal fractions (e.g., 5/7 or 7/5). In Block 6, students compared fractions with different numerators and denominators (e.g., 2/3 or 5/6). Internal reliability for the fractions comparisons task for our sample was high ($\alpha = .90$).

Control measures.

Matrix reasoning. The Matrix Reasoning subtest of the Wechsler Abbreviated Scale of Intelligence (WASI; Wechsler, 1999) assessed nonverbal ability. This test consisted of a 2×2 grid with geometric patterns in three cells, with the fourth cell empty. Students were asked to complete the sequence by selecting the missing geometric pattern from five response options. Students received one point for each correct trial with a total 35 possible points. Internal consistency was $\alpha > .90$ (Wechsler, 1999).

Attentive behavior. The SWAN Teacher Rating Scale (Swanson et al., 2006) is based on criteria for attention-deficit-hyperactivity disorder from the fourth edition of the *Diagnostic and Statistical Manual of Mental Disorders* (American Psychiatric Association, 1994). For each of the nine items, third-grade mathematics teachers rated each student's attentive behavior on a scale from 1 (*below average*) to 7 (*above average*). Lower scores represent ratings of poorer classroom attention. Internal consistency for this sample was high at $\alpha = .97$.

Working memory. The Counting Recall subtest of the Working Memory Test Battery for Children (WMTBC; Pickering & Gathercole, 2001) was used to assess working memory. Students were presented with a sequence of dot arrays, and asked to remember the number of dots presented in each array in the correct order. The number of dots ranged from sets of four dots to seven dots. After three correct trials out of six, the number of dot arrays increased by one. Students received one point for each correct trial,

Table 2
Description and Items in the Fraction Comparisons Task

Blocks	Block description	Items	Whole number bias
1	Unit fractions	1/3 or 1/2 1/55 or 1/57 1/4 or 1/5 1/10 or 1/100	Hurts performance
2	Different numerators Same denominator	7/12 or 9/12 5/7 or 6/7 24/48 or 28/48 2/10 or 4/10	Helps performance
3	Numerators and denominators with relatively larger numbers vs. relatively smaller numbers but larger in ratio	50/100 or 16/17 20/40 or 8/9 5/10 or 3/4 10/20 or 5/6	Hurts performance
4	Same numerator Different denominators	5/3 or 5/2 2/4 or 2/5 6/9 or 6/12 3/7 or 3/8	Hurts performance
5	Reciprocal fractions	3/2 or 2/3 8/4 or 4/8 5/14 or 14/5 5/6 or 6/5	n/a
6	Different numerators Different denominators	3/10 or 2/12 12/50 or 8/60 6/33 or 9/30 6/8 or 3/9	n/a

Note. Whole number bias in the fraction comparisons task is to identify the fraction with larger numerator and/or denominator regardless of ratio as having the greater value.

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out of 42 total possible points. Test–retest reliability on this standardized task is .61 (Pickering & Gathercole, 2001).

Reading fluency. The Sight Word Efficiency subtest of the Test of Word Reading Efficiency (TOWRE; Torgesen, Wagner, & Rashotte, 1999) was used. Students read aloud as many words as possible from a list of 104 written words within a 45-s time span. The score is the number of correctly read words. Test–retest reliability for this standardized measure is .97 (TOWRE; Torgesen et al., 1999).

Receptive vocabulary. Verbal ability was assessed using the Peabody Picture Vocabulary Test–Fourth Edition (PPVT; Dunn & Dunn, 2007). Students were shown four pictures and asked to point to the picture that depicted the spoken word said by the assessor. The student's score is the number of correctly identified pictures. Internal consistency is high at $\alpha > .96$ (Dunn & Dunn, 2007).

Whole number line estimation (WNLE). Students were asked to estimate where whole numbers should be placed on a number line segment according to their magnitude, labeled 0 on the left end and 1,000 on the right end (Siegler & Opfer, 2003). There were 22 items presented on a laptop in the following order: 56, 606, 179, 122, 34, 78, 150, 938, 100, 163, 754, 5, 725, 18, 246, 722, 818, 738, 366, 2, 486, and 147. Students were asked to show the presenter where 0 and 1,000 were located on the number line and then completed one practice round using the number 150. Consistent with Opfer and Siegler (2007), students received verbal feedback after the practice round. Percent absolute error (PAE) was calculated for each estimate by dividing the absolute difference between the estimated and actual magnitudes by the numerical range of the number line (1,000), and then multiplying by 100. Students were assigned a single score by taking the mean percent absolute error across all estimations. Internal consistency for this sample was high at $\alpha = .91$.

Demographic variables. Free/reduced price lunch program status, gender, age, and English language learner status were assessed through school records.

Testing Timeline and Procedure

All behavioral tasks were administered in school by trained assessors. Matrix reasoning/nonverbal ability, TOWRE reading fluency, PPVT receptive vocabulary/verbal ability, and whole number line estimation (WNLE) were administered individually in the winter of third grade. Working memory (WMTBC) was assessed individually during spring of third grade. Students' teachers rated attentive classroom behavior (SWAN) in third grade.

The fraction comparisons task was given in the fall and the spring of fourth grade, and the decimal comparisons task was administered between these time points in the winter of fourth grade. The decimal and fraction tasks were administered in a group setting via paper and pencil. The mathematics achievement test was administered by the school district following published guidelines in the spring of sixth grade.

Data analytic strategy

As noted previously, the decimal and fraction comparisons tasks contained blocks of items that assessed different aspects of decimal and fraction understanding, respectively. Chance performance was 50% correct on each item. Preliminary analyses indicated that students tended to answer either all or most of the items within a given block correctly or incorrectly, resulting in a bimodal distribution within block type. This finding suggests that students were using a strategy that influences performance at the block level, as expected. Thus, students' performance on each block of four items was dichotomized; students were assigned a score of "1" if they answered three or four items correctly (exceeding chance performance) or a score of "0" if they answered two or fewer items correctly. This step simplifies the data and makes it more appropriate for latent class analysis (LCA).

A three-step LCA approach (Asparouhov & Muthén, 2014; Vermunt, 2010) was used to assess the use of rule-based strategies on the decimal comparison task and investigate whether these strategies predict later mathematics achievement. The first step of the three-step procedure was to identify the underlying latent class structure without influence from covariates. In the second step, error terms were derived for individual's assignments to a most likely latent class. In the third step, most likely latent class membership values were treated as indicator variables for a new latent class model, which was then used to examine the relation between class membership and other auxiliary variables (i.e., demographic and cognitive covariates) and distal outcomes (i.e., mathematics achievement).

To inform our decision regarding how many classes to include in each LCA model, we used the Bootstrapped Likelihood Ratio Test (BLRT; Nylund, Asparouhov, & Muthén, 2007), as well as the value of the entropy statistic, which indicates how separate or distinct the latent classes are. The entropy statistic ranges from 0–1, with values larger than 0.6 indicating sufficient latent class separation (Collins & Lanza, 2010). Finally, we also based the number of classes on previous mathematics learning literature (e.g., Jordan, Kaplan, Ramineni, & Locuniak, 2009; Rinne et al., 2017), the characteristics of each block of items, and the interpretability of the resulting latent class structure.

Demographic variables included age in months, gender (female = 1), income status as indicated by enrollment in the free or reduced lunch program (low = 1), and English language learner status (ELL = 1). Demographic variables and cognitive competencies were treated as covariates. The distal outcome was sixth grade mathematics achievement on a standardized state test. All LCA models were tested using M*Plus* 7.1 (Muthén & Muthén, 1998–2014).

We also built a categorical SEM to assess the relation between the rule-based strategies used for decimal comparisons and strategies used for a fraction comparison task. Specifically, latent class membership for the decimal comparisons task in the winter of fourth grade was used to predict later (spring) fourth grade latent class membership for the fraction comparisons task, while controlling for earlier (fall) fourth grade performance on the fraction comparisons task.

It should be noted that the present study analyzes some of the same data on fraction comparisons as Rinne et al. (2017), but our analysis addresses unique research questions using a variety of additional measures. In Rinne et al.'s (2017) study, an LCA was developed to assess understanding of fraction magnitudes. Here, we used a similar analysis to investigate broader relations between decimal magnitude understanding and other areas of mathematics achievement. Fraction class membership was included as one covariate in the three-step LCA as well a baseline predictor in the

categorical SEM. Additionally, our sample had a different pattern of missing data compared with Rinne et al. (2017) because of our focus on a different set of test times and the inclusion of additional covariates. Thus, the results of the LCA of fraction comparisons in the present article, though similar to those found by Rinne et al. (2017), reflect our unique focus on a different set of research questions.

Results

To determine whether missing data patterns for student outcomes were systematically related to covariates, we tested for covariate-dependent missingness (Little & Schenker, 1995) among participants' latent classification probabilities for the fraction and decimal comparisons tasks. Results were not significant, $\chi^2[140] = 59.78$, $p > .25$, indicating that data were missing at random.

Latent Class Analysis of Decimal Comparisons Task

Identification of decimal comparison latent class structure. We conducted a three-step LCA of students' response patterns for the six decimal comparison blocks. A bootstrapped likelihood ratio test (BLRT) indicated that a three-class model fit significantly better than a two-class model ($2^*\Delta LL = 53.345$, $df = 7$, $p < .001$). A four-class model could not be identified because of the extraction of too many latent classes. Given that the entropy value for the three-class model was very high (.95), we utilized the three-class model for all subsequent analyses.

For the purposes of the LCA, performance on each block of four items was scored as 1 when performance was above chance (3 or 4 correct) and 0 when performance was at or below chance (2 or fewer correct). However, the left half of Table 3 shows the average proportion of correct responses (out of 4) for each item block within each latent class. These proportions are labeled as "high," "low," or "chance" to highlight patterns of performance across blocks.

Students in Class 1 consistently performed near ceiling regardless of item block (i.e., comparison type), suggesting that even in fourth grade, some children broadly understood decimal properties ($n = 55$, 12.97%). Students in Class 2 ($n = 82$, 19.34%) were highly accurate on blocks where having a whole number bias would not negatively influence performance, but an understanding of place value was required, as in Block 2 (e.g., 0.04 vs. 0.4) and Block 3 (e.g., 0.007 vs. 0.7). On these items, a zero was added immediately to the right of the decimal point on one of the decimals within each pair. However, these students were consistently inaccurate on items where zeros were added to the *end* of the decimal string, resulting in a negative impact of whole number bias, as in Block 6 (e.g., 0.40 vs. 0.6). This suggests that students in Class 2 understand some aspects of place value in decimal representations: they understood that adding a zero immediately to the right of the decimal point lowered the overall value, but they failed to understand that adding zeros at the end of the decimal string does not change its value. Rather, it appears as though they are mistakenly interpreting zeros that come at the *end* of the decimal string as yielding increased numerical magnitudes, just as they would for whole numbers (e.g., $0.60 > 0.6$ because $60 > 6$). On items where a whole number bias would hurt and zeros were *added immediately after* the decimal, as in Block 5 (e.g., 0.06 or 0.2), these students performed at chance. They were also highly accurate when comparisons were consistent with whole number properties, as in Block 1 (e.g., 0.2 or 0.4) and Block 4 (e.g., 0.04 or 0.5). Taken together, this suggests these students possess a whole number bias, but have a partial understanding of decimal place value.

Not surprisingly because of fourth graders' limited exposure to decimals, the majority of children fell into Class 3 ($n = 287$, 67.69%), which is characterized by little knowledge of decimal magnitudes as reflected by both a whole number bias and a misunderstanding of place value after the decimal (i.e., tenths, 100ths, 1000ths). Class 3 children were accurate only when whole number properties could be used to compare the decimals and when place value was constant, as in Block 1 (e.g., 0.08 or 0.03). Responses were inaccurate when comparisons were either inconsistent with whole number properties, as in Block 6 (e.g., 0.40 or 0.6), or required an understanding of place value after the decimal, as in Block 2 (e.g., 0.04 or 0.4), Block 3 (e.g., 0.007 or 0.7), and Block 5 (e.g., 0.06 or 0.2). Students in Class 3 performed at chance when a whole number bias and a misunderstanding of place value yield opposing responses, as in Block 4 (e.g., 0.04 or 0.5). Sub-

Table 3
Mean Proportion Correct (Out of 4) by Item block, Latent Class, and Outcome Measure (Decimals vs. Fractions)

	Decimal latent class structure: Winter fourth grade			Fraction latent class structure: Fall fourth grade		
Block	Good understanding $n = 55$	Partial understanding $n = 82$	Poor understanding $n = 287$	Good understanding $n = 80$	Partial understanding $n = 47$	Poor understanding $n = 304$
1	High (.75)	High (.99)	High (.99)	High (.94)	High (.88)	Low (.50)
2	High (.96)	High (.97)	Low (.40)	High (.79)	Low (.17)	High (.94)
3	High (.98)	High (.95)	Low (.50)	High (.98)	High (.76)	Low (.30)
4	High (.95)	High (.96)	Chance (.52)	High (.87)	High (.86)	Low (.20)
5	High (.99)	Chance (.55)	Low (.10)	Chance (.61)	Low (.12)	Chance (.64)
6	High (.94)	Low (.15)	Low (.30)	High (.93)	Low (.18)	Chance (.55)

Note. Good understanding is defined as getting most items correct. Partial understanding is defined as having a pattern of responses consistent with a whole number bias, but understanding place value. Poor understanding is defined as having a pattern of responses consistent with having a whole number bias and weak understanding place value. Value in parentheses is mean percent correct.

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sequently, we refer to these three classes as *good understanding* (Class 1), *partial understanding* (Class 2), *and poor understanding* (Class 3).

Effects of covariates on decimal comparison latent class membership. The LCA model allows us to examine the relation between covariates and the patterns of performance across blocks (i.e., students' level of understanding of decimal magnitudes). The three-step LCA ensures that covariates do not unduly influence the formation of the latent classes; thus, the effects reported here describe relations between covariates and latent class membership wherein the latent class structure is independent of covariate values. The poor understanding class (Class 3) is treated as the reference group because most students were in this latent class. Thus, effects of covariates indicate the relative odds of being in the good understanding class (Class 1) or the partial understanding class (Class 2), respectively, compared with being in the poor understanding class.

Effects of covariates are shown in Table 4. Compared with students with a poor understanding of decimal magnitudes (Class 3), students with a good understanding (Class 1) tended to exhibit better (i.e., lower error) WNLE scores. That is, students with better WNLE accuracy (measured in third grade) were significantly more likely in the winter of fourth grade to be in the good understanding class (Class 1) than the poor understanding class (Class 3). Better receptive vocabulary scores also predicted a higher likelihood of membership in Class 1 versus Class 3. Different covariates significantly predicted the chances of being in the partial understanding class (Class 2) versus the poor understanding class (Class 3). Better nonverbal ability and a good understanding of fraction magnitudes (i.e., membership in fraction understanding Class 1) predicted membership in the partial understanding class for decimal comparisons.

Latent Class Analysis of Fraction Comparisons Task

To analyze the relationship between decimal magnitude understanding and fraction magnitude understanding, we conducted an LCA of students' response patterns for the six blocks of the fraction comparisons task given in the fall of fourth grade. Previous work with an overlapping data set (Rinne et al., 2017) found that a three-class model fit well with a broader sample of students on this task. Regarding the present sample, a BLRT showed that the fit improvement for the three-class model over a two-class model was significant ($2^{*}\Delta LL = 77.535$, $df = 7$, $p < .001$). Results are shown in the right half of Table 3.

Class 1 ($n = 51$; 18%) included students whose high performance across blocks indicates that they understand the properties of fractions. The performance of students in Class 2 ($n = 29$; 11%), on the other hand, indicates a misunderstanding that *smaller* digits—regardless of whether they appear in the numerator or denominator—produce larger magnitudes. This "inverse-relationship" (Rinne et al., 2017) strategy results in a pattern of responses that contrasts with a whole number bias (i.e., the child would always choose the fraction that has larger numerals). Thus, performance was near ceiling in blocks in which having a whole number bias should lead to incorrect responses, Block 1 (e.g., 1/3 or 1/5), Block 3 (e.g., 50/100 or 16/17), and Block 4 (e.g., 6/9 or 6/12). Performance was near floor in Block 2 where having a whole number bias should lead to correct responses (e.g., 3/5 or 4/5). Performance was near

Table 4
Effects of Covariates on Decimal Comparisons Class Membership

Decimals class	Covariate	Coefficient	*SE*	*p*-value	Odds ratio
Decimals Class 1 (vs. Class 3)	WNLE	−.070	.035	.049*	.932
	Attentive behavior	.014	.022	.528	1.014
	Receptive vocabulary	.029	.015	.049*	1.029
	Nonverbal ability	.001	.070	.991	1.001
	Reading fluency	−.029	.020	.147	.971
	Working memory	.010	.010	.316	1.010
	Age	.032	.039	.419	1.033
	Female	.429	.350	.220	1.536
	Low SES	−.188	.364	.605	.829
	English language learner	.203	.537	.706	1.225
	Fractions Class 1 (vs. Class 3)	.874	.547	.110	2.396
	Fractions Class 2 (vs. Class 3)	.215	.570	.706	1.240
Decimals Class 2 (vs. Class 3)	WNLE	−.091	.061	.134	.913
	Attentive behavior	.031	.024	.197	1.031
	Receptive vocabulary	.002	.021	.922	1.002
	Nonverbal ability	.339	.124	.006**	1.404
	Reading fluency	.040	.034	.232	1.041
	Working memory	−.001	.013	.916	.999
	Age	.030	.060	.612	1.030
	Female	−.082	.530	.877	.921
	Low SES	.721	.580	.214	2.056
	English language learner	−.314	.819	.702	.731
	Fractions Class 1 (vs. Class 3)	2.244	.574	<.001***	9.431
	Fractions Class 2 (vs. Class 3)	1.110	.852	.193	3.034

Note. WNLE = whole number line estimation; SES = socioeconomic status.
* $p < .05$. ** $p < .01$. *** $p < .001$.

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chance in Block 5 (e.g., 5/7 or 7/5) and Block 6 (e.g., 2/3 or 5/6) in which numerators and denominators differ in opposite directions so there would be no influence of whole number bias or a small number strategy. Taken together, membership in Class 2 indicated a partial understanding that larger digits can lead to smaller magnitudes for fractions. Class 3 (n = 196; 71%) included students who—like those in Class 3 for the decimal comparisons task—are strongly biased by their whole number knowledge, such that they believe that fractions with larger digits always produce larger fraction magnitudes, regardless of whether they appear in the numerator or denominator. This results in near-floor performance for Blocks 1, 3, and 4, near ceiling performance for Block 2, and again, near chance performance for Blocks 5 and 6.

Effects of Early Decimal Comparison Latent Class Membership on Later Mathematics Achievement

We also investigated the relation between fourth grade decimal comparison latent class membership and a distal outcome—sixth grade general mathematics achievement—after controlling for variables that may also contribute to overall mathematics achievement: cognitive and demographic variables, WNLE, and fraction comparison class membership. For this analysis, we constructed a model that included both direct effects of covariates on the distal outcome (mathematics achievement) and class-specific covariate effects. Results showing the combination of these effects for each class are given in Table 5.

To compare mean mathematics achievement scores across classes, we conducted Wald tests on differences between modeled means (i.e., sample means weighted by estimated class probabilities). Relative to the mathematics achievement of students with a poor understanding of decimal magnitudes (Class 3; M = 774.564), students with a good understanding of decimal magnitudes (Class 1) had significantly higher achievement scores, M = 813.186, Wald $\chi^2(1) = 6.16$, $p = .013$. Although students with a partial understanding (Class 2) also exhibited higher mean mathematics achievement scores, M = 849.572, the difference between Class 2 and Class 3 did not reach significance, Wald $\chi^2(1) = 2.657$, $p = .103$. The difference in achievement scores between the partial and good understanding

Table 5
Model of Sixth Grade Mathematics Achievement Scores, by Decimal Understanding Class

Decimals class	Covariate	Coefficient	*SE*	*p*-value
Decimals Class 1	WNLE	−2.041	1.913	.286
(M = 889.20)	Attentive behavior	.021	1.006	.983
	Receptive vocabulary	−.364	.701	.604
	Nonverbal ability	−4.012	6.863	.559
	Reading fluency	−2.455	1.082	.023*
	Working memory	.931	.866	.282
	Age	−5.172	3.340	.122
	Female	4.741	27.806	.865
	Low SES	−68.669	22.158	.002**
	English language learner	−9.257	23.198	.690
	Fractions Class 1 (vs. Class 3)	83.549	32.214	.010*
	Fractions Class 2 (vs. Class 3)	44.347	27.554	.108
Decimals Class 2	WNLE	−2.262	1.650	.170
(M = 813.19)	Attentive behavior	1.284	1.736	.460
	Receptive vocabulary	1.006	1.688	.547
	Nonverbal ability	.576	2.296	.802
	Reading fluency	.331	.687	.629
	Working memory	.302	.380	.427
	Age	.740	2.997	.805
	Female	10.990	32.902	.738
	Low SES	−18.303	14.993	.222
	English language learner	12.113	27.573	.660
	Fractions Class 1 (vs. Class 3)	19.455	30.992	.530
	Fractions Class 2 (vs. Class 3)	11.260	24.440	.645
Decimals Class 3	WNLE	−2.604	.582	<.001***
(M = 774.56)	Attentive behavior	1.837	.439	<.001***
	Receptive vocabulary	1.108	.352	.002*
	Nonverbal ability	1.922	1.440	.182
	Reading fluency	.395	.359	.271
	Working memory	.091	.200	.650
	Age	.922	.715	.197
	Female	17.244	7.086	.015*
	Low SES	−1.682	8.138	.836
	English language learner	9.205	10.140	.364
	Fractions Class 1 (vs. Class 3)	44.842	11.677	<.001***
	Fractions Class 2 (vs. Class 3)	10.139	12.266	.408

Note. WNLE = whole number line estimation; SES = socioeconomic status.
* $p < .05$. ** $p < .01$. *** $p < .001$.

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classes was not significant, Wald $\chi^2(1) = .018$, $p = .894$. Overall, these results show that decimal understanding class in fourth grade accounts for unique variance in sixth grade mathematics achievement beyond that attributable to all other variables, including prior fraction understanding.

As seen in Table 5, effects of covariates on sixth grade mathematics achievement differed across decimal understanding classes. Most notably, membership in the "good understanding" *fraction* comparison class (Class 1) significantly predicted mathematics achievement among students in decimal understanding Class 1 and Class 3 (but not Class 2). This shows that fourth grade fraction understanding, like decimal understanding, uniquely predicts general mathematics achievement in sixth grade, even after controlling for other variables. Among students with a poor understanding of decimal magnitudes in fourth grade (that includes most students), WNLE, attentive behavior, and receptive vocabulary also independently predicted later mathematics achievement. Females in this group also exhibited a slight advantage in later mathematics achievement scores. Among students in decimal understanding Class 1 (good understanding) in fourth grade, low socioeconomic status (SES) was associated with poorer sixth grade mathematics achievement scores. There was also a small but significant *negative* association between reading fluency and later mathematics achievement, though the weakness of this effect along with the lack of any apparent explanation suggests that this negative relation may be spurious.

Relation Between Decimal and Fraction Comparison Class Structures

We constructed a categorical SEM to investigate whether early decimal understanding contributes to the development of normative fraction concepts over the course of instruction. Specifically, we examined whether fourth grade winter decimal comparison class membership predicted fourth grade spring fraction comparison class membership, while controlling for prior fraction comparison class membership assessed in the fall of fourth grade. We controlled for prior knowledge of fractions to rule out the possibility that any observed relation between early decimal understanding and later fraction understanding is simply a function of common prior knowledge and cognitive ability relevant to both domains.

The results of the categorical SEM are displayed in Table 6. As in previous models, the reference class is Class 3 (i.e., the poor understanding class) for both the decimal comparisons task and the fraction comparisons task. Membership in fraction comparison Class 1 in the spring of fourth grade was strongly predicted by prior fraction class membership, as well as membership in either decimal comparison Class 1 or decimal comparison Class 2. This result shows that having either a good or partial understanding of decimals is associated with later learning of fractions. Prior fraction and decimal comparison class memberships did not predict membership in the smaller number bias class for fraction comparisons in the spring of fourth grade.

Discussion

The ability to reason about numerical magnitudes facilitates mathematics learning (National Mathematics Advisory Panel, 2008). To date, however, much more work has focused on students' understanding of whole number and fraction magnitudes than decimal magnitudes. In the present study, we identified three classes of early decimal magnitude understanding based on student performance on different types of decimal comparisons in the middle of fourth grade. We then showed that decimal comparison class membership predicted later mathematics achievement in sixth grade, while controlling for whole number line estimation, fraction knowledge, and other cognitive processes and demographics. Finally, we found that decimal comparison class membership in the winter of fourth grade predicted fraction comparison class membership at the end of fourth grade, even after controlling for fraction knowledge at the beginning of fourth grade.

Identification of Decimal Comparison Latent Class Structure

Three classes of student performance on the decimal comparisons task emerged. Students in Class 1 (good understanding) consistently gave correct responses, suggesting that even in fourth grade they broadly understood decimal properties. Students in Class 2 (partial understanding) were highly accurate when a whole number bias would not influence performance but an understanding of place value after the decimal was required. Students in Class 3 (poor understanding) showed little knowledge of decimal magnitudes, as reflected by both a whole number bias and a misunderstanding of place value after the decimal.

Because students in Class 1 were highly accurate, they could be accessing an internal representation of magnitude. However, students in Class 2 and 3 did not appear to reason about decimal magnitudes holistically (Zhang, Fang, Gabriel, & Szűcs, 2016);

Table 6
Categorical SEM Results

Outcome	Predictor	Coefficient	*SE*	*p*-value	Odds ratio
Spring fourth grade fractions Class 1 (vs. Class 3)	Decimal Class 1	1.771	.543	.001	5.877
	Decimal Class 2	.967	.357	.007	2.630
	Fall fourth grade fraction Class 1	2.970	.532	<.001	19.492
	Fall fourth grade fraction Class 2	.940	.428	.028	2.560
Spring fourth grade fractions Class 2 (vs. Class 3)	Decimal Class 1	1.011	.707	.153	2.748
	Decimal Class 2	.348	.436	.424	1.416
	Fall fourth grade fraction Class 1	1.010	.749	.178	2.746
	Fall fourth grade fraction Class 2	1.197	.461	.009	3.310

Note. SEM = structural equation model.

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rather, given their performance described above, they applied (or misapplied) rules regarding whole number and place value. When different rules conflicted with one another (e.g., .06 vs. .4–place value helps performance but whole number bias hurts performance), students with a partial (Class 2) or poor (Class 3) understanding performed at chance, suggesting they simultaneously possessed different strategies that competed with one another for application. These findings support an overlapping waves theory of development (Siegler, 1996), which suggests that at any given time children can access a number of strategies to solve a problem, even though some strategies may be more effective than others depending on the context.

The finding that children applied rule-based strategies to reason about decimal magnitudes, rather than constructing holistic representations of magnitudes, contrasts somewhat with the findings of DeWolf, Grounds, Bassok, and Holyoak (2014, 2015). DeWolf and colleagues argue that decimals and whole numbers are processed as (continuous) unidimensional magnitudes whereas fractions are processed as (discrete) relational structures. However, DeWolf et al.'s studies are with older populations (i.e., seventh graders and adults). It is possible that, as with fractions (see Bonato et al., 2007; Schneider & Siegler, 2010), older children and adults have multiple ways of reasoning about decimal magnitudes, including an analog continuous representation as well as rule-based strategies that are strategically deployed based on task demands. The older children and adults in DeWolf and colleagues' studies may be using an analog representation while also possessing rule-based strategies that would apply in particular circumstances. That DeWolf et al. found that older children and adults use an analog representation to reason about decimal magnitudes, while we found that younger children (who are on the cusp of systematic learning about decimals) use rule-based strategies, suggests a developmental progression. That is, when learning about any kind of new magnitude, children begin by learning relevant rules before later gaining the ability to construct analog representations.

It should be noted that DeWolf et al. (2014) used a decimal number line estimation task with their seventh-grade students while DeWolf et al. (2015) used a magnitude comparison task with adults. The similar results for both studies suggest that differences in performance between DeWolf's studies and the current study are not because of task demands but rather differences in reasoning. Nevertheless, a study using the same task (e.g., magnitude comparison) across different ages would characterize a developmental progression.

In a longitudinal study on fraction magnitude understanding from fourth through sixth grades, Rinne et al. (2017) found a developmental progression in fraction magnitude understanding, with students commonly moving from poor to partial to good understandings. In the present study, possession of partial understandings of both decimal and fraction magnitudes was positively related to the subsequent acquisition of a good understanding of fraction magnitudes. These parallel findings suggest that for both fractions and decimals, the ability to recognize that the quantitative relationship between particular digits and the overall magnitude of the representation depends on the location of the digit may constitute an important insight, supporting subsequent development toward a normative understanding. Even though children with partial understanding of decimal magnitude misuse rule-based strategies, they are more likely to shift to good understanding of later fraction magnitude than are those who persist with faulty whole number strategies.

The finding that having only a partial understanding of decimals predicts later fraction understanding highlights the potential importance of transitional knowledge. That is, the type of partial or transitional knowledge that students hold as they become more knowledgeable in a specific area may help determine whether students are on their way to developing accurate representations of decimals or if some kind of intervention is needed. Relatedly, Loehr and Rittle-Johnson (2017) taught third- and fourth-grade students decimals, such as 0.25, using formal place value labels (e.g., referring to the decimal as "two tenths and five hundredths"), informal labels (e.g., referring to the decimal as "point two five"), or having no labels. Students exposed to formal labels correctly solved more decimal magnitude comparison items that required an understanding of the role of zero within decimals as well as place-value structure (e.g., 0.4 vs. 0.40). However, they also had lower performance on a decimal number line estimation task than did those who were exposed to no labels. Loehr and Rittle-Johnson (2017) suggest that the lower performance on certain decimal outcomes for students exposed to formal labels may indicate a disruption or change in their conceptualization of decimals, reminiscent of Piaget's concept of disequilibrium (Piaget, 1964). Exposure to formal decimal labels may have challenged students' way of thinking in a way that helped them notice place-value structure within decimals. Although children developed the understanding of the role of zero, this new knowledge was intermediate or transitional along the way to a more complete understanding of decimals. Students understood that adding a zero to the end of the decimal string did not increase its magnitude, but their representation of its exact magnitude on a number line was still limited. McNeil (2007) also noted that U-shaped development of children's performance on mathematical equivalence problems (e.g., 7 + 4 + 5 = 7 + ___) indicates a transitional period during which the nature of children's understanding of equivalence is changing.

In the present study we found that students with a partial understanding performed differentially depending on the particular type of decimal understanding being assessed by block (e.g., the role of zero to the immediate right of the decimal point as opposed to the role of zero at the end of the decimal string). We detailed three specific classes of response patterns that indicated qualitatively different forms of decimal understanding, each more sophisticated than the next, which predicted future fraction understanding and mathematics achievement. As proposed by *overlapping waves theory* (Siegler, 1996), both frequency and use of strategies or ways of thinking about a specific concept shift overtime with increased experience. Incorrect or inefficient strategies should reduce in frequency of use with more efficient correct strategies and ways of thinking becoming more prominent. Our results are consistent with this progression, such that those with partial decimal understanding demonstrated a basic understanding that adding a zero immediately to the right of the decimal point decreases its value without understanding the role that a zero added to the end (right) of the decimal string plays. Even still, these students are more likely to have an accurate understanding of fractions later on than those who fully rely on whole-number rules.

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Effects of Covariates on Decimal Comparison Latent Class Membership

Different covariates predicted membership in Class 1 and Class 2 versus Class 3. Accuracy in whole number line estimation and a larger receptive vocabulary each predicted membership in Class 1 (good understanding), but not membership in Class 2 (partial understanding). This suggests children in Class 1 are using accurate magnitude representations to solve the decimal comparison task. A strong representation of whole number magnitudes, as reflected by WNLE acuity, provides an organizational structure for reasoning about decimal and fraction magnitudes (Hansen, Jordan, Siegler, et al., 2015; Resnick et al., 2016). The finding that whole number line estimation skills did not predict possession of a partial understanding of decimals (vs. a poor understanding) suggests that students with partial or poor decimal magnitude knowledge may have inadequate mental number line representations or do not access their mental number line when completing decimal comparison tasks.

Rather, students in Class 2 had better nonverbal ability and understanding of fraction magnitude compared with Class 3. This suggests an important relationship between fraction and decimal magnitude understanding in particular, beyond what whole number magnitude can contribute. This relation may center on the use of rules to solve comparison problems in the absence of the ability to construct holistic magnitude representations. Understandings of rational number magnitudes that are specific to decimal and fraction formats may mutually support one another as children learn to integrate new numerical representations into their mental number lines.

Whole number line estimation, receptive vocabulary, nonverbal reasoning, and fraction magnitude understanding have been widely implicated in the development of whole number and fraction understandings in previous research (e.g., Bailey, Siegler, & Geary, 2014; Fuchs et al., 2006; Geary et al., 2008; Hansen et al., 2017; Hecht & Vagi, 2010; Vukovic et al., 2014). Two previous studies that examined cognitive predictors of decimal understanding yielded largely convergent findings. Seethaler, Fuchs, Star, and Bryant (2011) found that calculation skill, nonverbal reasoning, concept formation, working memory with numbers, and language ability all uniquely predicted rational number computation. However, because their measure of rational number computations included both fractions and decimals, it is difficult to determine specific contributions to decimal skill. In the other study, Malone, Loehr, and Fuchs (2017) found that nonverbal reasoning was associated with correct use of place value labels but not reasoning about decimal magnitudes. Overall, despite using different measures, our findings as well as those of other researchers generally suggest that a variety of mathematics competencies and cognitive abilities are intertwined with the development of rational number knowledge.

Effects of Early Decimal Comparison Latent Class Membership on Later Mathematics Achievement

Early decimal comparison class membership, fraction comparison class membership, and whole number line estimation acuity each uniquely predicted performance on a statewide standardized achievement test in sixth grade. The relative independence of the predictors suggests that reasoning about magnitudes is not a single skill, but rather is dissociable based on the form of the numerical representation and children's prior knowledge of those representations. Understanding whole number magnitudes may support later mathematics achievement because early whole number competencies allow children to make connections among mathematics relations and procedures (Gersten, Jordan, & Flojo, 2005). Understanding fraction magnitudes may support prealgebra and algebra performance specifically because of their bipartite structure (Booth & Newton, 2012; DeWolf et al., 2015), and mathematics achievement more generally because understanding fractions requires students to reorganize, and subsequently deepen, their understanding of numerical properties (Siegler et al., 2011). Understanding decimal magnitude may similarly provide students with an opportunity to deepen their understanding of numerical properties as students learn the rules for how decimals operate, particularly rules related to place value. Indeed, whole number, fraction, and decimal magnitude understanding may all independently contribute to mathematics achievement because each involves the need to flexibly apply different sets of numerical rules in different situations to properly represent the magnitudes of numerical representations.

Limitations

We aimed to assess students' early conceptions of decimal magnitudes. Although we observed some differences, a slightly older sample (e.g., at the end of fourth grade, after decimal instruction) and more sensitive measures (e.g., open-ended interviews) might increase variability in useful ways. Open-ended interviews, or think aloud protocols, would shed light on students' explicit problem-solving strategies. Additionally, we were unable to document precisely when systematic fraction and decimal instruction took place within the classroom. However, the Common Core State Standards in mathematics, which our participating schools followed, stipulate that fractions be taught intensively throughout fourth grade and decimals be introduced only later in the fourth-grade year (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010), which is typical of U.S. classrooms (Tian & Siegler, 2017). Moreover, further data on children's experiences outside of school might reveal that early understanding of decimals is associated with parental involvement and experiences with decimals at home.

In the present study, we assessed decimal magnitudes at a single time point using a single measure. More longitudinal data are needed to examine the developmental progression from poor to partial to normative understanding. Future research should examine whether learning decimal magnitudes begins with learning rules, and whether learning of those rules follows a developmental sequence, which in turn would support the learning of an analog representation of magnitude. In addition, items on our decimal and fraction comparisons measures were blocked by shared features (e.g., all items that involved both a whole number bias and a place value bias were blocked together). This design may have made certain features, and subsequently associated rule-based strategies, more salient, encouraging students to use a specific strategy they might not have otherwise used.

A final consideration is that because the sixth-grade state mathematics achievement measure used in the present study was consistent with grade-level benchmarks, it surely contained fraction

and decimal content. Unfortunately, we did not have access to the individual items from the school district. We do know, however, that the mathematics achievement measure contained a wide range of topics, such as numeric reasoning, algebraic reasoning, geometric reasoning, and quantitative reasoning. Future work can address these issues by using a mathematics achievement measure that is free of specific fraction or decimal content. Measures of motivation and informal decimal and fraction knowledge also might be included.

Implications and Conclusions

The current findings have practical as well as theoretical implications. First, the finding that children's level of decimal understanding is associated with later fraction knowledge, even after controlling for prior fraction knowledge, has implications for instructional sequencing of mathematical content. Fractions are typically taught before decimals in many countries, including the United States (Tian & Siegler, 2017). Some researchers suggest that decimals are easier to learn compared with fractions and, thus, advocate for decimals to be taught before or in concert with fractions (DeWolf et al., 2014, 2015; Hurst & Cordes, 2016; Iuculano & Butterworth, 2011; Zhang, Wang, Lin, Ding, & Zhou, 2013). A recent review concludes there is no research that directly supports this assertion (Tian & Siegler, 2017). Our data, however, do provide correlational evidence that decimal knowledge, which relies on understanding of place value, may support fraction learning. Understanding of place value, the knowledge that the spatial location of a given numeral within a multidigit set provides information regarding the relation between digits and the overall magnitude, may help students see there is a meaningful relation between the spatial location of the numerator and denominator. In addition, because decimals are more readily mapped onto a mental number line compared with fractions (DeWolf et al., 2014, 2015), learning decimals first, and then mapping fractions onto decimals, may help extend the continuous representation of decimals onto fractions, better connecting fractions to the number line. It seems likely that the relation between decimal and fraction knowledge is bidirectional, with each skill supporting the other (Rittle-Johnson, Schneider, & Star, 2015). More research using an experimental design is required to systemically assess order of instruction.

Our work also highlights the potential usefulness of decimal magnitude comparison tasks for characterizing how students reason about magnitude. As noted earlier, children and adults use a variety of strategies to reason about magnitude and use them adaptively based on task demands (e.g., Schneider & Siegler, 2010; Siegler, 1996). Magnitude comparison tasks complement number line approaches by using item characteristics to identify the strategies students use when reasoning about magnitude when their number line knowledge may be incomplete, hard to access, or not the most efficient strategy. Our decimal comparison task was designed to identify the presence of whole number bias and understanding of place value. However, comparison tasks can be used to identify other strategies and representations (e.g., misapplying fraction or negative properties to decimals).

In conclusion, our study is the first to provide evidence that early decimal magnitude knowledge uniquely supports later mathematics achievement, over and above fraction and whole number magnitude knowledge. The findings underscore the need for examining how understandings of different symbolic representations of numbers are distinct from one another, yet related, and how each kind of representation can be used to support more complex mathematics learning.

References

American Institutes for Research. (2012). *DCAS 2011–2012 Technical Report*. Retrieved from http://www.doe.k12.de.us/cms/lib09/DE01922744/Centricity/Domain/111/Vol1_Annual_TechRep.pdf

American Psychiatric Association. (1994). *Diagnostic and statistical manual of mental disorders* (4th ed.). Washington, DC: Author.

Asparouhov, T., & Muthén, B. (2014). Auxiliary variables in mixture modeling: Three-step approaches using M plus. *Structural Equation Modeling, 21,* 329–341. http://dx.doi.org/10.1080/10705511.2014.915181

Bailey, D. H., Hoard, M. K., Nugent, L., & Geary, D. C. (2012). Competence with fractions predicts gains in mathematics achievement. *Journal of Experimental Child Psychology, 113,* 447–455. http://dx.doi.org/10.1016/j.jecp.2012.06.004

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2014). Early predictors of middle school fraction knowledge. *Developmental Science, 17,* 775–785. http://dx.doi.org/10.1111/desc.12155

Barth, H. C., & Paladino, A. M. (2011). The development of numerical estimation: Evidence against a representational shift. *Developmental Science, 14,* 125–135. http://dx.doi.org/10.1111/j.1467-7687.2010.00962.x

Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance, 33,* 1410–1419. http://dx.doi.org/10.1037/0096-1523.33.6.1410

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology, 37,* 247–253. http://dx.doi.org/10.1016/j.cedpsych.2012.07.001

Collins, L. M., & Lanza, S. T. (2010). *Latent class and latent transition analysis*. Hoboken, NJ: Wiley.

De Smedt, B., Noel, M. P., Gilmore, C., & Ansari, D. (2013). How do symbolic and non-symbolic numerical magnitude processing skills relate to individual differences in children's mathematical skills? A review of evidence from brain and behavior. *Trends in Neuroscience and Education, 2,* 48–55. http://dx.doi.org/10.1016/j.tine.2013.06.001

DeWolf, M., Bassok, M., & Holyoak, K. J. (2015). From rational numbers to algebra: Separable contributions of decimal magnitude and relational understanding of fractions. *Journal of Experimental Child Psychology, 133,* 72–84. http://dx.doi.org/10.1016/j.jecp.2015.01.013

DeWolf, M., Grounds, M. A., Bassok, M., & Holyoak, K. J. (2014). Magnitude comparison with different types of rational numbers. *Journal of Experimental Psychology: Human Perception and Performance, 40,* 71–82. http://dx.doi.org/10.1037/a0032916

Dunn, D. M., & Dunn, L. M. (2007). *Peabody picture vocabulary test: Manual* (4th ed.). Minneapolis, MN: Pearson.

Durkin, K., & Rittle-Johnson, B. (2015). Diagnosing misconceptions: Revealing changing decimal fraction knowledge. *Learning and Instruction, 37,* 21–29. http://dx.doi.org/10.1016/j.learninstruc.2014.08.003

Fazio, L. K., DeWolf, M., & Siegler, R. S. (2016). Strategy use and strategy choice in fraction magnitude comparison. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 42,* 1–16. http://dx.doi.org/10.1037/xlm0000153

Fuchs, L. S., Fuchs, D., Compton, D. L., Powell, S. R., Seethaler, P. M., Capizzi, A. M., . . . Fletcher, J. M. (2006). The cognitive correlates of third-grade skill in arithmetic, algorithmic computation, and arithmetic word problems. *Journal of Educational Psychology, 98,* 29–43. http://dx.doi.org/10.1037/0022-0663.98.1.29

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Geary, D. C. (2004). Mathematics and learning disabilities. *Journal of Learning Disabilities, 37,* 4–15. http://dx.doi.org/10.1177/00222194040370010201

Geary, D. C., Boykin, A. W., Embretson, S., Reyna, V., Siegler, R., Berch, D. B., & Graban, J. (2008). Report of the task group on learning processes. In U.S. Department of Education. (Ed.), *The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities, 38,* 293–304. http://dx.doi.org/10.1177/00222194050380040301

Halberda, J., Mazzocco, M. M. M., & Feigenson, L. (2008). Individual differences in non-verbal number acuity correlate with maths achievement. *Nature, 455,* 665–668. http://dx.doi.org/10.1038/nature07246

Hansen, N., Jordan, N. C., & Rodrigues, J. (2017). Identifying persistent learning difficulties in fractions: A longitudinal study of student growth from third through sixth grade. *Contemporary Educational Psychology, 50,* 45–59. http://dx.doi.org/10.1016/j.cedpsych.2015.11.002

Hansen, N., Jordan, N. C., Siegler, R. S., Fernandez, E., Fuchs, L., Gersten, R., & Micklos, D. (2015). General and math-specific predictors of sixth-graders' knowledge of fractions. *Cognitive Development, 35,* 34–49. http://dx.doi.org/10.1016/j.cogdev.2015.02.001

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102,* 843–859. http://dx.doi.org/10.1037/a0019824

Hurst, M., & Cordes, S. (2016). Rational-number comparison across notation: Fractions, decimals, and whole numbers. *Journal of Experimental Psychology: Human Perception and Performance, 42,* 281–293. http://dx.doi.org/10.1037/xhp0000140

Irwin, K. (2001). Using everyday knowledge of decimals to enhance understanding. *Journal for Research in Mathematics Education, 32,* 399–420. http://dx.doi.org/10.2307/749701

Iuculano, T., & Butterworth, B. (2011). Understanding the real value of fractions and decimals. *Quarterly Journal of Experimental Psychology, 64,* 2088–2098. http://dx.doi.org/10.1080/17470218.2011.604785

Jordan, N. C., Hansen, N., Fuchs, L. S., Siegler, R. S., Gersten, R., & Micklos, D. (2013). Developmental predictors of fraction concepts and procedures. *Journal of Experimental Child Psychology, 116,* 45–58. http://dx.doi.org/10.1016/j.jecp.2013.02.001

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early math matters: Kindergarten number competence and later mathematics outcomes. *Developmental Psychology, 45,* 850–867. http://dx.doi.org/10.1037/a0014939

Laski, E. V., & Siegler, R. S. (2014). Learning from number board games: You learn what you encode. *Developmental Psychology, 50,* 853–864. http://dx.doi.org/10.1037/a0034321

Little, R. J. A., & Schenker, N. (1995). Missing data. In G. Arminger, C. C. Clogg, & M. E. Sobel (Eds.), *Handbook of statistical modeling for the social and behavioral sciences* (pp. 39–75). New York, NY: Plenum.

Loehr, A. M., & Rittle-Johnson, B. (2017). Putting the "th" in tenths: Providing place-value labels helps reveal the structure of our base-10 numeral system. *Journal of Cognition and Development, 18,* 226–245. http://dx.doi.org/10.1080/15248372.2016.1243118

Mack, N. K. (1993). Learning rational numbers with understanding: The case of informal knowledge. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 85–106). Hillside, NJ: Erlbaum.

Malone, A. S., Loehr, A. M., & Fuchs, L. S. (2017). The role of domain-general cognitive abilities and decimal labels in at-risk fourth-grade students' decimal magnitude understanding. *Learning and Individual Differences, 58,* 90–96. http://dx.doi.org/10.1016/j.lindif.2017.05.007

McNeil, N. M. (2007). U-shaped development in math: 7-year-olds outperform 9-year-olds on equivalence problems. *Developmental Psychology, 43,* 687–695. http://dx.doi.org/10.1037/0012-1649.43.3.687

Meert, G., Grégoire, J., & Noël, M. P. (2010). Comparing the magnitude of two fractions with common components: Which representations are used by 10- and 12-year-olds? *Journal of Experimental Child Psychology, 107,* 244–259. http://dx.doi.org/10.1016/j.jecp.2010.04.008

Mix, K. S., Levine, S. C., & Huttenlocher, J. (1999). Early fraction calculation ability. *Developmental Psychology, 35,* 164–174. http://dx.doi.org/10.1037/0012-1649.35.1.164

Muthén, L. K., & Muthén, B. O. (1998–2014). *Mplus user's guide* (7th ed.). Los Angeles, CA: Author.

National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). Common core state standards for mathematics. *Common Core State Standards Initiative*. Retrieved from http://www.corestandards.org/assets/CCSSI_Mathematics%20Standards.pdf

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the national mathematics advisory panel*. Washington, DC: U.S. Department of Education.

Ni, Y., & Zhou, Y. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist, 40,* 27–52. http://dx.doi.org/10.1207/s15326985ep4001_3

Nylund, K. L., Asparouhov, T., & Muthén, B. O. (2007). Deciding on the number of classes in latent class analysis and growth mixture modeling: A Monte Carlo simulation study. *Structural Equation Modeling, 14,* 535–569. http://dx.doi.org/10.1080/10705510701575396

Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology, 55,* 169–195. http://dx.doi.org/10.1016/j.cogpsych.2006.09.002

Piaget, J. (1964). Development and learning. In R. E. Ripple & V. N. Rockcastle (Eds.), *Piaget rediscovered: A report on the conference on cognitive studies and curriculum development* (pp. 7–20). Ithaca, NY: Cornell University.

Pickering, S., & Gathercole, S. E. (2001). *Working memory test battery for children (WMTB-C)*. London, United Kingdom: The Psychological Corporation.

Rapp, M., Bassok, M., DeWolf, M., & Holyoak, K. J. (2015). Modeling discrete and continuous entities with fractions and decimals. *Journal of Experimental Psychology: Applied, 21,* 47–56. http://dx.doi.org/10.1037/xap0000036

Resnick, I., Jordan, N. C., Hansen, N., Rajan, V., Rodrigues, J., Siegler, R. S., & Fuchs, L. S. (2016). Developmental growth trajectories in understanding of fraction magnitude from fourth through sixth grade. *Developmental Psychology, 52,* 746–757. http://dx.doi.org/10.1037/dev0000102

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20,* 8–27. http://dx.doi.org/10.2307/749095

Rinne, L. F., Ye, A., & Jordan, N. C. (2017). Development of fraction comparison strategies: A latent transition analysis. *Developmental Psychology, 53,* 713–730. http://dx.doi.org/10.1037/dev0000275

Rittle-Johnson, B., Schneider, M., & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review, 21,* 1–11.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organization in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction, 2,* 157–174. http://dx.doi.org/10.1207/s1532690xci0202_3

Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number-space mappings: What underlies mathematics achievement? *Journal of*

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Experimental Child Psychology, 114, 418–431. http://dx.doi.org/10.1016/j.jecp.2012.10.012

Schneider, M., & Siegler, R. S. (2010). Representations of the magnitudes of fractions. *Journal of Experimental Psychology: Human Perception and Performance, 36,* 1227–1238. http://dx.doi.org/10.1037/a0018170

Seethaler, P. M., Fuchs, L. S., Star, J. R., & Bryant, J. (2011). The cognitive predictors of computational skill with whole versus rational numbers: An exploratory study. *Learning and Individual Differences, 21,* 536–542. http://dx.doi.org/10.1016/j.lindif.2011.05.002

Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. New York, NY: Oxford University Press.

Siegler, R. S. (2016). Magnitude knowledge: The common core of numerical development. *Developmental Science, 19,* 341–361. http://dx.doi.org/10.1111/desc.12395

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., . . . Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23,* 691–697. http://dx.doi.org/10.1177/0956797612440101

Siegler, R. S., & Lortie-Forgues, H. (2014). An integrative theory of numerical development. *Child Development Perspectives, 8,* 144–150. http://dx.doi.org/10.1111/cdep.12077

Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science, 14,* 237–250. http://dx.doi.org/10.1111/1467-9280.02438

Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental Psychology, 49,* 1994–2004. http://dx.doi.org/10.1037/a0031200

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62,* 273–296. http://dx.doi.org/10.1016/j.cogpsych.2011.03.001

Stacey, K., & Steinle, V. (1999). A longitudinal study of children's thinking about decimals: A preliminary analysis. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 233–240). Haifa, Israel: PME.

Steinle, V., & Stacey, K. (1998). The incidence of misconceptions of decimal notation amongst students in grades 5 to 10. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times: Proceedings of the 21st annual conference of the mathematics education research group of Autralasia* (pp. 548–555). Sydney, New South Wales, Australia: MERGA.

Swanson, J., Schuck, S., Mann, M., Carlson, C., Hartman, K., Sergeant, J., & McCleary, R. (2006). *Categorical and dimensional definitions and evaluations of symptoms of ADHD: The SNAP and SWAN rating scales*. Irvine, CA: University of California.

Tian, J., & Siegler, R. S. (2017). Which type of rational numbers should students learn first? *Educational Psychology Review, 30,* 351–372. http://dx.doi.org/10.1007/s10648-017-9417-3

Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (1999). *TOWRE: Test of word reading efficiency*. Austin, TX: Pro-Ed.

Vermunt, J. K. (2010). Latent class modeling with covariates: Two improved three-step approaches. *Political Analysis, 18,* 450–469. http://dx.doi.org/10.1093/pan/mpq025

Vukovic, R. K., Fuchs, L. S., Geary, D. C., Jordan, N. C., Gersten, R., & Siegler, R. S. (2014). Sources of individual differences in children's understanding of fractions. *Child Development, 85,* 1461–1476. http://dx.doi.org/10.1111/cdev.12218

Wechsler, D. (1999). *Wechsler abbreviated scale of intelligence*. San Antonio, TX: The Psychological Corporation.

Wu, H. (2009). What's sophisticated about elementary mathematics? *American Educator, 33,* 4–14.

Ye, A., Resnick, I., Hansen, N., Rodrigues, J., Rinne, L., & Jordan, N. C. (2016). Pathways to fraction learning: Numerical abilities mediate the relation between early cognitive competencies and later fraction knowledge. *Journal of Experimental Child Psychology, 152,* 242–263. http://dx.doi.org/10.1016/j.jecp.2016.08.001

Zhang, L., Fang, Q., Gabriel, F. C., & Szűcs, D. (2016). Common magnitude representation of fractions and decimals is task dependent. *Quarterly Journal of Experimental Psychology, 69,* 764–780. http://dx.doi.org/10.1080/17470218.2015.1052525

Zhang, L., Wang, Q., Lin, C., Ding, C., & Zhou, X. (2013). An ERP study of the processing of common and decimal fractions: How different they are. *PLoS ONE, 8,* e69487. http://dx.doi.org/10.1371/journal.pone.0069487

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