Pre-Service Teachers' Difficulties with Problem Solving

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This paper reports the results of an investigation into the ways pre-service teachers engaged in structured problem solving as part of their first-year mathematics education course. The purpose of this study was to determine the preferred problem solving strategies of pre-service teachers and the types of difficulties they experienced. The written discourse of 179 pre-service teachers indicated difficulties with being able to articulate the strategies they used in their solution processes. The results also showed that pre-service teachers did not readily use models and relied largely on numerical procedures.

The proficiency strands in the Australian Curriculum: Mathematics describe how content is explored and developed to provide a meaningful basis for the development of mathematical concepts. Problem solving is one of the proficiency strands characterised as "…the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively" (Australian Curriculum, Assessment and Reporting Authority [ACARA], n. d.). A central activity of mathematics teaching and learning is to develop the ability to solve a wide variety of problems (Stacey, 2017).

Success in problem solving is influenced by students' beliefs about the nature of mathematics learning (Schommer-Aikins, Duell & Hutter, 2005). Prior experience shapes the amount of time and effort that will be invested in a problem (Schoenfeld, 1985). Other affective factors such as motivation, perceived personal control, perceived usefulness of mathematics (Schoenfeld, 1989) and maths anxiety (Ramirez, Chang, Maloney, Levine & Beilock, 2016), shape how a student engages in problem solving. A lack of exposure often results in many students having difficulties planning and applying procedures when faced with non-routine problems (Mataka, Cobern, Grunert, Mutambuki & Akom, 2014). Students have come to separate the mathematics they know and experience in their classrooms from the discipline of creativity, problem solving, and discovery, about which they seldom experience (Schoenfeld, 1989).

Problem solving can be considered as a set of skills worthy of instruction in its own right. However, developing instructional models for problem solving is a difficult process. Some instructional models reduce the complexity of mathematical tasks and students' opportunities to grapple with content and misrepresent the flexible and non-routine nature of problem solving (Boaler, 2001; Stein, Grover, & Henningsen, 1996). Creating the 'right' instructional context, and providing the appropriate kinds of modeling and guidance, is challenging for teachers (Schoenfeld, 2016). Teachers also often provide a rationale for avoiding problem solving based on arguments that the curriculum requires students to master facts, procedures and algorithms (Wilson, Fernandez & Hadaway, 1993). One consequence of experiencing mathematics in such a way is that students learn that answers and methods to problems will be provided to them and are not expected to figure out the methods for themselves (Schoenfeld, 2016).

Nonetheless, the fundamental goal of mathematics instruction is to develop transferable skills, knowledge, and abilities that students can apply to tasks not explicitly covered in the curriculum (Wilson, Fernandez & Hadaway, 1993). "Effective teaching is dependent on

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presenting students with engaging tasks for which they make their own decisions on solving strategies, rather than following procedures (Sullivan, 2011, p. 64)”. Students who are able to solve problems think critically within instructional models that emphasise thinking processes above mathematical content procedures (Snyder & Snyder, 2008).

> Mathematics instruction should provide students the opportunity to explore a broad range of problems and problem situations, ranging from exercises to open-ended problems and exploratory situations. It should provide students with a broad range of approaches and techniques (ranging from the straightforward application of the appropriate algorithmic methods to the use of approximation methods, various modeling techniques, and the use of heuristic problem solving strategies) for dealing with such problems. (Schoenfeld, 2016, p. 32)

This study seeks to determine the type of strategies pre-service teachers used and difficulties they experienced when solving problems that involve fractions. Studies into pre-service teachers’ problem solving ability are scarce, mainly because of the difficulties in accounting for the type of instructional approaches they encountered as students (Mataka, Cobern, Grunert, Mutambuki & Akom, 2014). Developing pre-service teachers’ problem solving skills by providing them with the necessary tools that they can later utilise is important because they will be responsible for cultivating these skills in their own students. Teachers need learning opportunities to develop their own content knowledge and skills to solve mathematical problems themselves (Sullivan, 2011).

Problem Solving Heuristics

Solving problems requires a base content knowledge of mathematics and a repertoire of problem solving heuristics. Early researchers identified heuristics as essential methods for guiding the systematic discovery of mathematical proofs (Neth & Gigerenzer, 2015). In problem solving, heuristics can be considered somewhat synonymously with terms such as strategies, approaches, methods and techniques used in the context of doing mathematics. Efforts to teach novices must take into account that problem solving processes and heuristics develop slowly over time (Lester, 1994).

Many formulations of problem solving frameworks depict Polya’s four stages of *understanding the problem*, *devising a plan, carrying out the plan,* and *looking back*. These stages are often seen as a series of linear steps with an emphasis on getting answers rather than teaching students how to think (Wilson, Fernandez & Hadaway, 1993). Polya’s stages are actually cyclic in nature involving passing through one stage, going back and checking before proceeding on to a possible solution path (Mataka, Cobern, Grunert, Mutambuki & Akom, 2014). Reconsidering and re-examining solution processes and results is an important step in consolidating knowledge and developing skills to solve problems. Students need to understand that their thinking and the strategies they use in obtaining a solution are just as important as getting the correct answer. According to Lester (1994), teaching students about Polya’s framework does little to improve students’ abilities to solve problems. What is important is that teachers value problem solving as part of a systematically planned instructional program where students solve many problems and learn to communicate their thinking.

Teaching problem solving involves exposing students to particular strategies. Producing drawings, for example, allows a problem context to be ‘seen’ and modelled which in turn facilitates problem solutions (Bakar, Way & Bobis, 2016). Providing occasions for mathematical modeling engages students in learning situations that develop a deeper, conceptual knowledge of mathematics (Boaler, 2001).

Structured Problem Solving

Structured problem solving is a powerful way of developing mathematical concepts and skills – a major instructional approach in Japanese mathematics lessons (Takahashi, 2006). Students work on a problem individually before sharing their solutions with others. The teacher leads a whole class discussion to allow students opportunities to share and learn from each other and encouraging them to think about problems, highlighting that there is often more than one solution process (see Figure 1). Students think more deeply about mathematical content when they are exposed to problems they haven't previously been shown how to solve, challenging them to find their own solutions and justify their reasoning (Sullivan, Askew, Cheeseman, Clarke, Mornane, Roche & Walker, 2015).

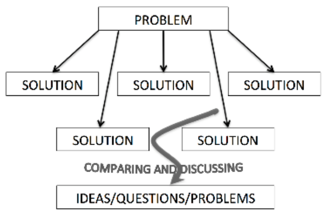


Figure 1. Structured problem solving (Takahashi, 2006, p. 39).

Method

This study involved 179 first-year pre-service teachers across 7 tutorial groups from one university in Melbourne. Problem solving was a common feature of their course content employing a structured problem solving approach (Takahashi, 2006) where problems were attempted on a weekly basis. Their solution processes were discussed in classes as a means of identifying particular problem solving heuristics and emphasising the stages of Polya's framework. In addition, pre-service teachers engaged in a 2-hour tutorial exploring further problems as small group tasks with a specific focus on *The Manchester Warehouse* problem (Booker, Bond, Sparrow & Swan, 2014) where they were guided through a range of problem solving approaches.

> The manchester warehouse was having a sale on beach towels. On Monday, it sold 1/3 of its beach towels, on Tuesday it sold 1/2 of what was left from Monday, and on Wednesday it sold 3/4 of what was left from Tuesday. If 3 beach towels were not sold, how many beach towels did the warehouse have when the sale started?

The Fashion Warehouse problem below is an adaptation of *The Manchester Warehouse* problem developed by one of the researchers and was used to assess pre-service teachers' problem solving ability at the duration of their course.

> Problem solving is a process that involves analysing the problem, exploring means to a solution and trying various solution strategies. Provide two strategies that can be used to solve the following problem.
>
> The fashion warehouse was having a sale on sunglasses. On Tuesday, it sold 1/5 of its sunglasses. On Wednesday, it sold 1/2 of what was left. On Thursday, it sold 3/4 of what was left from Wednesday. If 6 sunglasses were not sold, how many sunglasses did the Fashion warehouse have when the sale started?

Pre-service teachers' written work samples were analysed for the effective use of problem solving methods and the application of numerical approaches to the inherent fractional content in the question. The initial evaluation considered the proportion of pre-service teachers that could solve the problem using two problem solving strategies. The analysis also examined the effectiveness of their strategies in achieving a correct mathematical solution. The process of mathematizing involves explaining one's actions and choices using a common mathematical discourse (Sfard, 2008). Further analysis therefore examined aspects of pre-service teachers' written discourse for effective specification and use of problem solving heuristics. The data was summarised into tables using percentages and pseudonyms are used throughout the data analysis.

Results

Pre-service teachers used a range of strategies, including a combination of numerical calculations (fractions or percentages), whole number trial and error methods, discrete or region models, tables, and generating and using algebraic equations to solve the problem. Table 1 shows the number of effective strategies employed by pre-service teachers. Two different strategies would have been graded at the highest level for this task.

Table 1
Use of Problem Solving Strategies

Number of effective strategies	% of responses
Two different strategies	25
One strategy	15
Ineffective strategies	45
Unrelated mathematical discourse	12
No attempt	3

This data shows that 40% of pre-service teachers were able to solve the problem using at least one strategy. More than half could not produce a mathematically acceptable solution. Joanne's response is similar to the 12% of pre-service teachers that provided unrelated mathematical discourse indicating little understanding of the problem solving strategies applicable to the problem (see Figure 2).

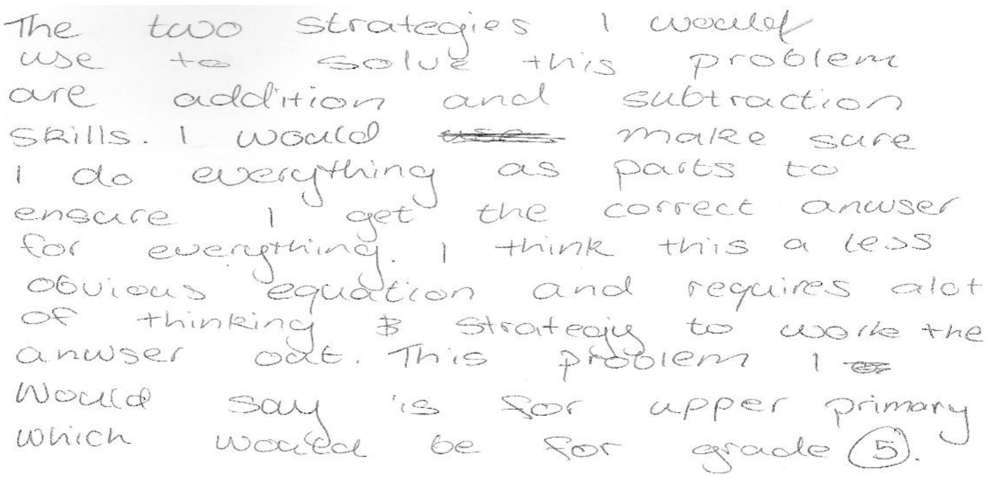

The two strategies I would use to solve this problem are addition and subtraction skills. I would ~~use~~ make sure I do everything as parts to ensure I get the correct anwser for everything. I think this a less obvious equation and requires alot of thinking & strategy to work the anwser out. This problem I would say is for upper primary which would be for grade (5).

Figure 2. Joanne's discourse about effective problem solving strategies.

A detailed analysis of the problem solving strategies used by pre-service teachers revealed numerical approaches as well as distinct problem solving heuristics. Table 2 shows the effectiveness of each of their chosen strategies whether pre-service teachers specified them or not.

Table 2
Effectiveness of Problem Solving Strategies used in Written Submissions

Strategies		% of responses	
	Correct solution	minor errors	Incorrect solution
Calculations using fractions	24	9	13
Calculations using percentages	3	1	1
Whole number (trial & error)	12	2	9
Discrete model	7	1	2
Region model	12	3	2
Used tables	2	2	3
Algebraic	3	1	2

Only 24% of pre-service teachers confidently worked with fractions without having to convert them to percentages or use whole number approaches. Paul's submission is indicative of the difficulties most of them had with applying fractions (see Figure 3). As in Paul's case, when novices arrive at a numerical answer they are usually satisfied and rarely see if the answer makes sense (Heller, Keith & Anderson, 1992).

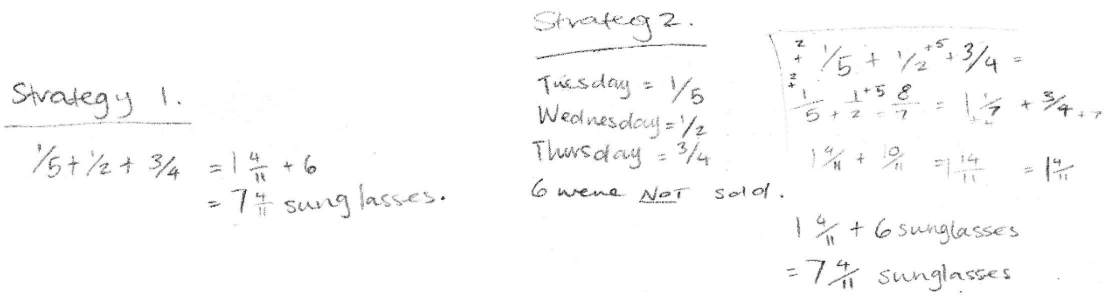


Figure 3. Paul's application of fractions.

When analysing the cohort's ability to name and described their chosen methods only 13% of them were also able to correctly specify the two strategies and effectively use them to solve the problem. Conversely, 27% of them specified at least one strategy but could not effectively use them to solve the problem as shown in Table 3.

Table 3
Specification of Problem Solving Strategies

Number of strategies specified	Number of effective strategies	% of responses
2	2	13
2	1	6
1	1	9
2	0	9
1	0	18

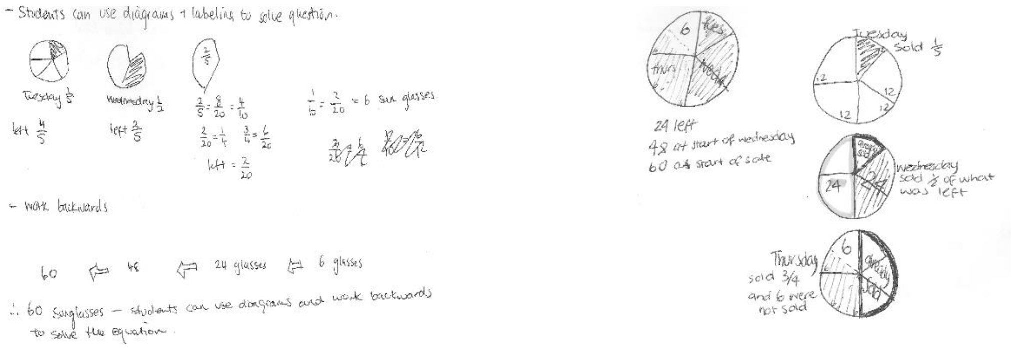
While 45% of pre-service teachers could not specify a strategy (Table 1), 9% of them nevertheless used problem solving approaches to work out a correct answer to the problem. These pre-service teachers may not have recalled the specific terminology to describe their approaches but could demonstrate a solution process.

The problem solving strategies specified by pre-service teachers were grouped into four main categories. Table 4 shows these strategies and the proportion of pre-service teachers applying them effectively in their written mathematical discourse.

Table 4
Written Discourse and Effective Use of Problem Solving Heuristics

Problem solving heuristics	% of effective responses
Working backwards/reverse	21
Using a diagram/model	12
Using an equation/algebraic	1
Process of trial and error/elimination	8

Problem solving strategies are not always distinct and are often combined. For example, the strategy of working backwards can be demonstrated through using numerical approaches, algebraic equations as well as a model. Mary's solution indicates the naming of and combining of strategies to effectively solve the problem (see Figure 4). However, the majority of the cohort did not specify combined strategies even though most of their correct solutions used these combinations. Peter's solution is an example of how several of the cohort effectively used problem solving strategies but were unable to use mathematical language to describe their thinking and solution processes.



Mary's solution   Peter's solution

Figure 4. Mary and Peter's specification and use of problem solving strategies.

Sheree may have known how to solve the problem intuitively or by other means but her solution involving the manipulation of fractions indicated a creative use of mathematics but also the need for appropriate instructional intervention (see Figure 5).

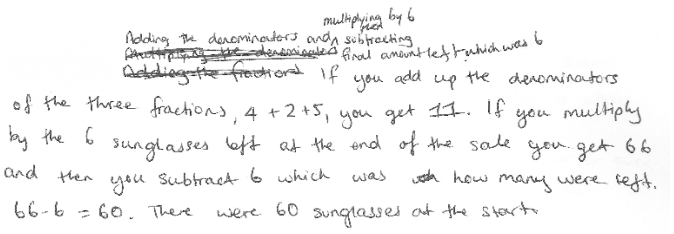


Figure 5. Sheree's manipulation of fractions.

Discussion and Conclusion

Analysis of pre-service teachers' written discourse indicated several gaps in their ability to accurately describe their thinking processes and apply fractional concepts. Being able to communicate one's actions in a common mathematical discourse is akin to thinking mathematically (Sfard, 2008) and is an important part of being a proficient problem-solver (ACARA, n. d.). Despite weekly and intensive exposure to structured problem solving tasks modelled by tutors, 60% of the cohort could not provide a correct solution to the Fashion Warehouse problem. Of the 55% of the cohort who specified the strategies they intended to use, less than half were successful.

A lack of mathematical content knowledge of future primary teachers (Sullivan, 2011), especially with fractions (Chinnappan & Forrester, 2014), present challenges for education programs. Only 19% of the cohort used either discrete or region models successfully, indicating a reliance on procedural methods by the majority. Pre-service teachers tend to have a procedural understanding of fractions and are less likely to develop conceptual knowledge for fraction problems (Tirosh, 2000). Of the 50% who used fractional manipulations to solve the problem, and may have also used models as Mary did (see Figure 4), less than half did so successfully. An additional 12% successfully applied trial and error processes involving whole number manipulations. Polya argued that trial and error is a legitimate, but often undervalued, solution method as mathematics is dependent on guessing, insight, and discovery (Schoenfeld, 2016; Wilson, Fernandez & Hadaway, 1993). However, choosing whole number methods above fractional methods by some pre-service teachers may indicate a lack of confidence when working with fractions.

Several implications can be made to build up prospective teachers' capacity for problem solving and addressing mathematical content knowledge especially with fractional concepts. More attention is needed in addressing pre-service teachers' proficiency in problem solving. Instructional approaches can improve their performance provided they have explicit instruction and practice in implementing problem solving strategies (Heller, Keith & Anderson, 1992).

One limitation of this study is that variations in instructional approaches and time taken by tutors may have affected some pre-service teachers' engagement in problem solving. The influence of the instructor on student performance (Mataka, Cobern, Grunert, Mutambuki & Akom, 2014) and perceived value of problem solving (Schoenfeld, 2016) are major considerations for future instructional models. Further, measuring growth in pre-service teachers' problem solving ability and mathematical knowledge cannot be easily achieved due to the absence of data on these two measures prior to the commencement of their course.

This paper offers a starting point for further theorisation and investigation of teachers' knowledge in problem solving and appropriate instructional models that support their learning. Teaching task-specific heuristics has been shown to effectively enable students to form problem solving plans (Wilson, Fernandez & Hadaway, 1993). Questions about how best to develop problem solving ability in students and how future teachers can be helped to become better problem-solvers and, therefore, better teachers of mathematics, are potential directions for future research.

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