Longitudinal Algebra Prediction for Early versus Later Takers

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RUNNING HEAD: LONGITUDINAL ALGEBRA PREDICTION

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Abstract

Algebra I is a crucial course for middle and high school students for successful STEM related coursework. A key issue is whether students should take Algebra I in 8th versus 9th grade. Large-scale policy studies show conflicting results, and there are few (particularly longitudinal) individual difference studies. Here, 53 students were assessed in 6th grade on cognitive, numerosity, and mathematical skills, and then followed; 26 students took Algebra I in 8th grade, and the other 27 in 9th grade. Comparisons between groups at grade 6 revealed gaps in some (but not all) cognitive skills and on mathematical competencies, but not on numerosity. By Algebra I, gaps in cognitive skills diminished, but gaps in mathematical skills remained constant. Gaps in algebra skills were also apparent, despite the age difference between groups. Results suggest that the additional year of instruction was not optimally tuned to pave the way for strong Algebra I performance.

Longitudinal Algebra Prediction for Early versus Later Takers

It is widely accepted that algebra serves as a "gateway" for later mathematical achievement, as seen in position papers, meta-analyses, and empirical reports (e.g., Crisp et al., 2009; Hansen, 2014; NMAP, 2008; Rose & Betts, 2004). Most colleges require algebraic competence as a pre-requisite for more advanced mathematical coursework, and algebra competence relates to STEM persistence and careers. Although details vary, most studies argue that taking high school math coursework in general, and algebra specifically, improves academic, occupational, and financial outcomes. However, the majority of studies that focus on algebra at more than a single time point are national evaluations focused on policy issues (such as the impact of more students taking Algebra early – i.e., grade 8 versus grade 9). This work has concentrated on school, demographic, and specific mathematic factors (e.g., Domina et al., 2015; Gaertner et al., 2014; Liang, Heckman, & Abedi, 2012; Nomi & Raudenbush, 2016), rather than individual differences, which is relevant given that numerosity and cognitive factors are emphasized in models of math development (e.g., Geary, 2004; LaFevre et al., 2010; von Aster & Shalev, 2007).

Expanding on current knowledge, the present study prospectively evaluated a group of sixth-grade students on a range of cognitive, number, and mathematical outcomes (Cirino, Tolar, Fuchs, & Huston-Warren, 2016), to assess whether groups of students differed on some or all of these factors before their Algebra I course was set or known to the researchers. We did, however, choose schools that varied in the proportion of students who take Algebra I at different times. We then followed a portion of this sample through Algebra I, which was completed in grade 8 or 9. We evaluated the same cognitive and arithmetic predictors, as well as algebra outcomes (an experimental measures as well as the state Algebra exit exam administered by schools), at the end of their Algebra I course. Identifying key contributors to algebraic performance, and how these differ among early versus later takers of algebra, can deepen understanding about mathematics development and help inform the design of instruction that precedes algebra.

Current Knowledge of Algebra Predictors

Predictors of early developing mathematical skills, particularly those involving computation, are well-known. There is agreement that both cognitive and numerosity factors are important (Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, & Schatschneider, 2010; Hansen, Jordan et al., 2015), although debate surrounds the relative contribution of each. Recently, attention has been focused on later developing skills, particularly fractions performance (Cirino et al., 2016; Bailey, Siegler, & Geary, 2014; Hecht & Vagi, 2010; Jordan, Hansen, Fuchs, Siegler, Gersten, & Micklos, 2013; Siegler, Thompson & Schneider, 2011; Torbeyns, Schnieder, Xin, & Siegler, 2015), with a small but growing number of studies that address individual differences in algebra outcomes (Booth, Newton, & Twiss-Garrity, 2014; Siegler et al., 2012; Tolar, Lederberg, & Fletcher, 2009). There is also increasing awareness that these later developing mathematical skills rely on earlier developed skills (Cirino et al., 2016; Fuchs et al., 2006; National Mathematics Advisory Panel, 2008).

In fact, relatively little is known about the combination of the domain-general and domain-specific individual differences that are important for algebra. There was a spate of literature on the prediction of algebra performance prior to 1990 (e.g., Boyce, 1964; Hanna, Bligh, Lenke, & Orleans, 1969; Hanna & Sonnenschein, 1985; Grover, 1932; Seagoe, 1938; Taylor, 1976). Our search yielded only 46 quantitative studies in peer-reviewed journals since 2000, and many of these (also noted above) focus on national longitudinal data sets. To date, we identified only three studies, all recent (Booth et al., 2014; Geary et al., 2015; Matthews, Lewis, & Hubbard, 2016) that focused on individual differences in algebra performance for students at this level (grade 8 or 9). In a sample of 8^{th} graders using Cognitive Tutor, Booth et al. (2014) found that performance on their measure of non-unit fraction (e.g., 5/14) number line estimation was moderately correlated (*r*'s ~ .30 to .50) with algebraic feature knowledge, equation solving, and conceptual encoding errors. Matthews et al. (2016) emphasized the role of nonsymbolic ratio processing, which were predictive of outcomes including fractions and algebra (*r*'s = .33 to .40). Geary et al. (2015) focused on the role of the approximate number system (ANS) and its relations to algebra achievement; the ANS measure (Panamath) showed moderate correlations (*r* ~ .40) with algebraic performance and was predictive of coordinate plane placement and algebraic expression evaluation, but not equation memory, even when other cognitive factors (central executive, processing speed, reading, IQ, algebra achievement) were included in their predictive model. These studies collectively argue for the role of basic numerical processes as important for component algebra skills.

Siegler et al. (2012) focused on algebraic outcomes measured at later ages (15 to 17) with panel data from two large cohort studies; predictors were primarily earlier arithmetic skills, with fraction performance and whole-number division being most robust. Complimentary data comes from studies focused on pre-algebraic skills for students earlier in middle school (e.g., DeWolf et al., 2015; Lee, Ng, Ng, & Lim, 2004; Lee, Ng, & Ng, 2009; Rittle-Johnson et al., 2009; Star et al., 2015) or on algebra skills in university students (Smith & Michaels, 1998; Tolar et al., 2009). Tolar et al. (2009) found support for the predictive power of several domain-general factors (e.g., working memory, spatial visualization) for algebra in university students.

In sum, numerosity, mathematical, and cognitive factors are likely to predict algebraic performance. A role for numerosity specific to algebra is supported by the studies noted above, as well as by the understanding that expansion of the representation of mental number line occurs throughout development (Siegler & Lortie-Forgues, 2014). Fractions are key mathematical skills for algebra; Booth and Newton (2012) argued that if algebra is the "gatekeeper" to advanced math and STEM, then fractions are "the gatekeepers' doorman". In the context of fractions, whole number math facts are less likely than other skills such as proportional reasoning to impact algebra performance. Cognitively, working memory is important for math given the need to keep in mind the components and order of multistep computational problems (Geary et al., 2015; Swanson & Beebe-Frankenberger, 2004). Language skill is necessary to understand the meaning of mathematical symbols and their relationships (Vukovic & Lesaux, 2013). Both working memory and language skills are also required for reading comprehension (e.g., Cain, Oakhill, Barnes, & Bryant, 2001; Fuchs, Fuchs, Compton, Hamlett, & Wang, 2015; Sesma, Mahone, Levine, Eason, & Cutting, 2009), and reading demands are often present for math problem solving tasks. Linear and nonlinear equations represent patterns of relations and so visual-spatial skills may also be invoked, particularly where graphs are utilized.

When to Take Algebra?

Although there is little debate that courses such as Algebra I are important, not all high school students take and receive credit for Algebra I (e.g., 69% in 2009; NCES, 2015). A further issue is at which grade the course should be taken (usually grade 8 or 9) and in corollary, how to determine which students should take it at what time. As noted, these issues are typically considered at the level of policy – for example, the Algebra for All initiative (American Diploma

Project, 2004; College Board, 2000; NCLB, 2001). Stein, Kaurman, Sherman, and Hillen (2011) reviewed when Algebra I is taken, and how its timing impacts achievement in both "selective" (where schools choose which students to take Algebra I earlier) and "universal" (where all students take Algebra I at a given grade) contexts. Stein et al. (2011) noted that approximately 30% of students who take Algebra I do so in grade 8 and 50% do so in grade 9; they also noted that eighth-grade enrollment is proportionately lower for students who are Black, Hispanic, or of low SES.

Some studies find that taking Algebra I in grade 8 improves later course taking and achievement (Heppen et al., 2012; Rickles, 2013; Smith, 1996), although in some contexts, students with low math achievement do not benefit (Clotfelter, Ladd, & Vigdor, 2012; Gamoran & Hannigan, 2000). Domina et al. (2015) found that large increases in grade 8 Algebra I has more negative effects in larger districts. Loveless (2008) reported that many grade 8 algebra students may be misplaced, a concern echoed by Domina (2014) who noted a need for balance between boosting general achievement against ensuring all students are adequately prepared. These conclusions were also reached in the review by Stein et al. (2011), and similar issues have been identified in contexts where policy pushes for all students to take Algebra I in grade 9 rather than grade 8 (i.e., in Chicago; Allensworth, Nomi, Montgomery, & Lee, 2009; Nomi, 2012; Nomi & Raudenbush, 2016).

Beginning Algebra I early makes sense because the sequence of math courses is more or less fixed (Domina et al., 2015). To take Calculus in High School, Algebra I typically needs to be taken in eighth grade (with Geometry, Algebra II/Trignometry, and Precalculus taken between Algebra I and Calculus). On the other hand, if students are not prepared for the demands of Algebra I in grade 8, then beginning the sequence later makes sense. Indeed, Liang et al. (2012)

compared two groups of students: (a) those who took and passed regular math in grade 8, followed by Algebra I in Grade 9; and (b) students who took but did not pass Algebra I in grade 8. The former group was more likely to pass the ninth-grade California Algebra test. Although students with stronger math skills tend to take more advanced math classes and to take them earlier, deciding which students to enroll in grade 8 Algebra is not always a straightforward decision. The study of Doughtrey et al. (2015) is an exception; they used a regression equation incorporating all available prior courses for each student in Wake County, North Carolina to systematically predict passing performance on the state end-of-course Algebra exam for students in grades 6, 7, and 8 (admitting such students to a more advanced mathematical trajectory). This policy resulted in increased accelerated participation, including girls and Black and Hispanic students (though for these latter groups, still not at a level proportionate to their overall district enrollment). However, algebra performance per se was not examined in that study.

A different (although related) issue from how students are slotted to take Algebra in grade 8 versus 9, is their level of *readiness* to take algebra whenever it begins, and whether this differs by trajectory group. An underlying assumption seems to be that the additional year of instruction allows time to prepare students for the course. A further assumption might be that students upon exit from Algebra I in grade 9 do so with a similar level of *proficiency* as those taking it in Grade 8 (albeit one year later). If this latter assumption is correct, then delaying Algebra I is a more logistic than substantive issue (i.e., students end in the same place, just at different times). However, if students taking Algebra I later have weaker algebra skills upon its completion, then the delay may in some ways compound the problem, given that math skill builds in hierarchy (e.g., NMAP, 2008). One of the advantages of assessing the level and trajectory of individual differences in cognitive competencies related to algebra is that they may help explain readiness from the perspective of capacity to deal with abstract quantities. Such understanding may allow for a better juxtaposition of taught skills relative to the cognitive capacities for integrating those skills. In other words, certain levels of cognitive development may be necessary to build a strong conceptual understanding of fractions and related skills that in turn could engender algebraic understanding. Given what is known about the contribution of various cognitive, numerosity-based, and arithmetic skills to early and intermediate math skills (Cirino et al., 2016), it is important to consider if these building block skills differ among students selected for, or who elect to take, algebra early versus later, and whether they co-evolve to impact future math attainment.

Summary and Rationale

There are few individual differences studies as they relate to algebra, but extant studies, and studies from the broader mathematical literature suggest a range of key factors. Separately there is debate about the optimal timing for Algebra I. Finally, there is little data on how individual differences relate to such timing issues. The key rationale for and contribution of this study is its potential to begin to inform the above issues. If later takers attain the same level of algebra proficiency (albeit a year later), then the delay may be effective in establishing the needed background for algebra. But if differences persist, then the delay may not be productive, and the question should move toward deepening understanding about why later takers remain weaker in certain areas (and how to address this). For example, if *fraction* differences exist early and remain large years later and if fraction gaps track algebra differences, this would further support the importance of fractions for screening students into early algebra or for diagnostically target eighth-grade instruction to better prepare them for algebra in grade 9. If *cognitive* differences exist early and remain large, then instruction may need to incorporate more cognitive supports (e.g., for language and/or working memory), to minimize the impact of such weaknesses on skills that are foundational to algebra or on actual algebra competencies.

Hypotheses

The prospective-retrospective cohort design of the present study allowed us to address and hypothesize about three issues. First, we examined individual difference predictors of algebra performance (in cognitive, numerosity, and mathematical domains) in grade 6 as well as at the start of Algebra I (either grade 8 or 9). We expected measures from all three domains to predict algebra outcomes at both earlier and later time points. We expected cognitive and numerosity predictors to have moderate relations with algebra performance, but expected stronger zero-order correlations for mathematical predictors, in particular sixth-grade fractions and proportional reasoning.

Second, we asked whether differences between sixth-grade students who take Algebra I at different times diminish, remain stable, or increase over a two to three year period. Because cognitive abilities such as working memory and language are expected to improve with age, any differences between early and later takers apparent at grade 6 should be smaller during Algebra I (since later takers are afforded an extra year of development). For foundational number skills, we expected differences to be minimal. To the extent that the mental number line expands with exposure and development (von Aster & Shalev, 2007; Siegler & Lortie-Forgues, 2014), this could suggest an advantage for later takers, though the accelerated academic pacing for early takers that may ameliorate some of that advantage. For arithmetic, we expected sixth-grade performance to favor early relative to later takers. There is insufficient data to predict whether an extra year with lower level math solidifies these later takers' arithmetic skills or whether the gap continues to expand, particularly if there is a mismatch between the timing of cognitive development and exposure to the concepts underlying fractions and proportional reasoning. Whatever differences do manifest, we expect these to be wider for fractions than for math facts.

Third, we evaluated whether any above differences track with algebraic outcomes, which is relevant given that by the end of Algebra I, students have all been exposed to approximately the same material. We expected that, to the extent that skills such as fractions and proportional reasoning are tightly linked to algebra understanding, then algebra performance differences should be as large as mathematical differences.

Method

Participants

Fifty-three students were selected from two larger samples: a group of 162 students assessed in grade 6 (Cirino et al., 2016), and a sample of 532 students assessed during the course of their Algebra I year. These 53 students were the only ones who received these measures at both grade levels and therefore comprised the only subgroup that could specifically address the questions raised above. Of the 53 students, 51% took Algebra I in grade 8, 43% were female, and 83% received lunch assistance (9% did not; data were missing for 8%); they attended seven schools. The majority (79%) were Hispanic, with 13% African American, and 6% Caucasian. A small minority (4%) were designated limited English proficient. Grade 8 and grade 9 subgroups did not differ in sex or ethnicity, but all five students who did not receive lunch subsidy took Algebra in grade 8, and the two students with limited English were in grade 9.

Students attended a range of schools; specifically, one of the three grade 6 schools was a public charter whose campus was contiguous from grade 6 to 12, where grade 8 algebra is common; in our sample, most of the students who took Algebra I in grade 8 (78%) attended this school. For the public district, a smaller proportion of students take Algebra I in grade 8; in this district, students attend separate schools for grades 5 and 6, versus grades 7 and 8, versus grade 9, making them more difficult to follow. We attempted to track and retain all students. Most attrition was due to students moving out of district. Further, for students who moved across schools within districts, separate permission was required for each new school, and this was not practicable where the number of students was very small, especially given other goals of the parent study that required efficient ascertainment of large numbers of students taking Algebra I. All procedures were approved by the relevant institutional IRB.

Measures

Cognitive. Automated Symmetry Span (Kane et al., 2004; Unsworth et al., 2009) was used to evaluate working memory. This measure has a processing component (is the figure symmetrical?) and a recall component (of a spatial location). Twelve trials are randomly ordered across span lengths of 2, 3, 4, and 5. The maximum score is 42, with points allocated according to span length ("partial credit-load" scoring; Conway et al., 2005). In this sample, α was .79. We assessed language with the Wechsler Abbreviated Scales of Intelligence (WASI) Vocabulary subtest (Wechsler, 1999), requiring definitions of words presented visually and orally; the measure is widely used and has strong reliability (α = .86 to .90; Wechsler, 1999). The raw total score was utilized. The Visual-Auditory Learning subtest of the WJ-III (Woodcock, McGrew, & Mather, 2001) is a language-related task that requires the association of a symbol with a word, followed by a string of symbols that students "read" to demonstrate that an association was made. Reliability among 11 to 15 year-olds is good (range of alpha, .76 to .90; McGrew & Woodcock, 2001). The Rasch-based W score was utilized. The Revised Vandenberg & Kuse Visuospatial Rotations Test (Peters, 1995) is a redrawn version of the Vandenberg and Kuse (1978) figures, and is a familiar measure of mental rotation, with good reliability (in this sample, α = .76). A target figure and 4 related stimuli are shown; an item is correct if both of the stimuli that are rotated versions of the target, are identified. Participants were allocated 6 minutes, and the maximum raw score (used for analyses) was 24. This measure loaded highly on a spatial visualization latent factor (.83) and correlated with measures of algebra procedural skill (.28 to .30) among college students (Tolar et al., 2009).

Numerosity. Panamath (Halberda, Mazzocco, & Feigenson, 2008; www.panamath.com), is commonly used as an assessment of the approximate number system (Jordan et al., 2013, Libertus, Odic, & Halberda, 2012). The version used for this study employed 200 trials that varied the ratio of dots (yellow on the left; blue on the right); each pair of dots are displayed on the computer for 600 ms, followed by a backward mask. Participants choose one of two keys to indicate as quickly as possible which side was more numerous; dot size, area, and the ratio between the dot set-sizes systematically vary (1.1 to 1.2; 1.2 to 1.3; 1.33 to 1.44; 1.5 to 2.0; and 2.3 to 2.5). The primary measure was the Weber fraction (W), with lower values indicating better acuity. For Symbolic Comparison, participants view 91 pairs of Arabic numerals (single or double digit), and must decide, as quickly and accurately as possible, which number is larger. The ratio between each pair were chosen to maximally overlap with those of Panamath. The primary measure was response time for correct trials (accuracy was routinely above 95%). In this sample, α = .98 for RT. Magnitude placement was assessed with Number Line Estimation (Booth & Siegler, 2006; Siegler & Booth, 2004). An Arabic numeral sits above a line labeled with 0 and 1000 on its ends; the task is to place "where on the line" the Arabic numeral should go. The primary variable used was the mean deviation from the number's actual line position (Booth & Siegler, 2006; Siegler & Opfer, 2003; Siegler & Booth, 2004). Reliability in this sample was α = .88.

Mathematics. Two brief fact measures were utilized, Single Digit Addition (all possible combinations of addends 0 through 9) and Single Digit Multiplication (multiplicands and multipliers ranged from 0 to 9). Participants completed as many of 99 problems across 17 rows with 6 items per row (the last row had 3 items) in 1 min, and were told to work as quickly but as accurately as possible; total correct was the primary measure. These two measures were averaged to form a composite; they correlated $r = .53$. To assess fractions, the Brown and Quinn Fraction Competency Test (Brown & Quinn, 2007) was used; this is a 25-item measure of fractions meant to assess conceptual knowledge and procedures, with presentation primarily computational and only a few word problems. Performance accuracy correlated $r = .58$ with algebra course final exam (Brown & Quinn, 2007). For the present study, students were given 10 minutes to complete as many items as possible; reliability in this sample was $\alpha = .69$, and total score was used for analyses. Finally, the Diagnostic Assessment of Proportional Reasoning (Misailidou & Williams, 2003) has 13 items developed via item response theory in a sample of 303 students aged 10 to 14 years. Item selection, development, and other psychometric properties are described in some detail in the original publication (Misailidou & Williams, 2003). For the present study, 10 of the 13 items were used, with difficulty parameters ranging from -2.72 to +2.01. Students were given 10 minutes to complete as many items as possible, and the total score was used for analyses. Reliability in this sample was $\alpha = .69$.

Algebra. We employed two measures. The first was developed for the parent study to include items that align with the Algebra I curriculums of the two states involved in the parent project (State 1, 2002-2005, 2009; State 2, 2008), but also includes algebraic reasoning items similar to those assessed by the SAT (The College Board, 2005). Students have 45 minutes to complete 40 items (maximum score = 40) assessing single-variable equations (e.g., Solve: 2x + x

= 9), systems of equations (e.g., Solve: 3p+4q = 9; 3p + 2q = 3), procedural skill and conceptual understanding of linear functions (e.g., calculate slope from two points, identifying linear functions vs non-linear functions from different scenarios/representational formats), isolation of variables (e.g., Solve for x: xy + yz = 2y), and algebraic reasoning (e.g., If x + a = 9, then what does a + x = ?). The test purposefully de-emphasizes computational requirements (e.g., manipulation of fractions is not required; only single-digit whole number computations are required). For purposes of this work, the measure given at the end of Algebra 1 was used in the analysis reported here. In the parent study, alpha was .90 (*N* = 284). The other measure was the state test for Algebra end-of-course exam (State 2, 2015a), which requires knowledge of factoring, exponent properties, linear and quadratic equations, in a multiple choice format. Needed formulas are provided. There are 50 items, and students have up to three hours to complete. Released items are available (State 2, 2015a). It should be noted that Algebra I is required for graduation in this state, and that all students receive the same end of course exam, which is produced in accordance with the standards of the state (State 2, 2015b).

Analyses

We used raw scores in all analyses. We first examined variable distributions for all relevant variables (11 in each of Grade 6, and Grade 8/9, and 2 additional algebra variables in Grade 8/9). One variable (number line accuracy) was log transformed due to skewness, and 13 values (of the 1272 examined, or ~1%) were trimmed due to outlier status. To examine the impact of demographic differences, we included sex, ethnicity, English proficiency, and free lunch status in preliminary regressions, predicting the two algebra outcomes. We did not include age or school, as these are redundant with the grade that algebra is taken (and although age differed within Grade 9 students, this did not carry over to differences on performance measures). Sex and ethnicity were unrelated to these outcomes, and for the state algebra exit exam, English proficiency and free lunch status were also unrelated. For the researcher developed algebra measure, those two variables were impactful. Therefore, any differences below were run with and without students (n = 7) who were either limited in English, or who did not receive free lunch, to evaluate whether this resulted in any undue influence; however, these are not reported, because they did not substantively effect results in any case.

To address the first purpose (predictors of algebra), the present study utilizes a relatively small sample with a large set of predictors. Therefore, we explicitly did not employ complex multiple or other regression-based analyses (such as path analyses) to differentiate the relative importance of the different types of predictors. Instead, zero-order correlations were examined for effect sizes, and compared to extant literature. These differences were compared with t-tests after z-score transformation (e.g., strength of early versus later measured skill), using the method of Weaver and Weunsch (2013) for dependent and independent correlations, as needed. To address the second and third purposes (differences between early and later algebra takers on cognitive, mathematical, numerosity predictors; and on algebraic outcomes), we compared mean performances for early versus later takers across these measures, via repeated measures ANOVA (with time as a within subjects effect, and group as between); diminished group differences with time would appear as interactions of time and group. We also evaluated group differences on number and math variables with regard to vocabulary performance, and for algebra, with regard to both vocabulary and math performance. Statistical significance was corroborated by effect size indices, in order to balance potential Type I and Type II errors.

Results

Preliminary Results: Sample Representativeness

Before addressing hypotheses, we evaluated our sample to determine their representativeness compared to (a) other grade 6 students who were not followed, and (b) other grade 9 students who had not been earlier tested; we also compared subgroups who received some measures at the beginning versus at the end of grade 9.

We first compared 53 students who were followed to 109 students who were not, on their grade 6 characteristics. The two groups did not differ on free lunch status, ethnicity, or sex, but did differ on English language status (4% followed were limited versus 22% of those not followed). Followed students outperformed those who were not, on visuospatial rotation, arithmetic facts, fractions, proportional reasoning, working memory, and symbolic comparison, consistent with the fact that the present study had a balanced number of early versus later takers, whereas the distribution is typically skewed (it is more common to take Algebra I in grade 9 than grade 8), with early math performance and school policy related to the decision. Performance differences above tracked school performance differences on similar variables.

Students who were followed were also compared to their same within-site grade cohort, as part of the parent study. That is, 27 grade 8 students who were followed from grade 6 were compared to 108 grade 8 students from the same group of schools, who were not earlier seen; and the 26 grade 9 students who were followed were compared with 68 grade 9 students who were not earlier seen. At grade 8, followed students did not differ from their same-grade peers in race/ethnicity, free lunch status, or English language status, but the followed group was more likely to be male. Followed students outperformed non-followed students on measures of visual-auditory learning and visuospatial rotation; the groups did not differ on the other 10 compared cognitive, mathematical, or algebraic variables. At grade 9, followed students did not differ from their same-grade peers in race/ethnicity, free lunch status, sex, or English proficiency status.

Followed students outperformed non-followed students on measures of visual-auditory learning and symbolic comparison; the groups did not differ on the other 10 compared variables.

Finally, of the 26 students followed from Grade 6 to Grade 9, 12 of these were ascertained at the end of their Grade 9 year. While this does not impact end-of-year algebra performances, those assessed later received arithmetic and cognitive variables 6 months later than other students. Beyond the necessary age differences, these subgroups did not differ on sex, free lunch status, English language status, or ethnicity. Of the 12 psychometric measures, they differed on only visuospatial rotation (with later ascertained students performing lower).

In summary, 12 of the 62 comparisons between subgroups were significant, and any systematic differences appeared to be due to the fact that the sample was balanced in terms of early and later takers, rather than skewed to the more typical pattern (that is, with many more students taking Algebra I in grade 9 then grade 8).

Hypothesis I: Prediction

The first goal was to evaluate how well potential predictors correlate with algebra performance. We expected the strongest correlations to be with fractions and proportional reasoning, at both time points (see Table 1 for correlations for grade 6 predictors; Table 2 for grade 8/9 predictors). Consistent with hypotheses, measures from each domain (cognitive, numerosity, mathematical) were significantly related to algebraic outcomes of each time point. The strongest correlations were with mathematical measures, median $r = .48$ and .55 (as assessed in grade 6 and 8/9, respectively; six correlations each), with all $p < .05$. In general, language variables across grades were modestly related to algebra outcomes (median $r = .31$; eight correlations, five of these, $p < .05$). Other cognitive variables (working memory and visuospatial rotation) were more variable in their relations (range r = .01 to .39, four correlations); this pattern was also evident for numerosity variables (range r = |.02 to .43|, six correlations).

(insert Tables 1 and 2 about here)

Some correlations with the two algebra measures appeared stronger for predictors assessed closer in time (e.g, language measures, median r = .28 early versus median r = .36 later; visuospatial rotation r = .11 early versus r = .27 later; number line, r = .18 early versus r = .32 later. Other correlations appeared weaker for predictors assessed closer in time (e.g., working memory r = .35 early versus r = .05 later; symbolic comparison, r = -.22 early versus r = -.02 later). However, only one of these individual dependent correlations was significantly different: The grade 6 correlation between working memory and the State Algebra Exit exam was stronger than the grade 8/9 correlation, r(G6) = .39 versus r(G8/9) = .01, t(48) = 2.41, p = .020. As an example contrast where there appeared to be a difference but there was not, fractions measured in grade 6 showed r = .29 with the State Algebra Exit exam, and fractions measured in grade 8/9 showed r = .44, though this difference was not significant, t(48) = -1.53, p = .133.

Across the whole sample, the two measures of algebra (one researcher-generated and the other school-based), showed a similar pattern of relations with the set of predictors. The exceptions were that the experimental algebra measure correlated more strongly than the State Exit exam with grade 6 Visual-Auditory Learning, t(48) = 3.22, p = .002, but less strongly with grade 6 math facts, t(48) = -2.36, p = .022, and with grade 8/9 math facts, t(48) = -2.32, p = .024. The other 15 differences were not significant, all p > .05.

We conducted some post-hoc analyses on variables that showed an unexpected pattern (i.e., stronger relations when predictors were assessed earlier in time versus later) – namely working memory and symbolic comparison. For grade 6 working memory, relations with both algebra measures within group (early versus later takers) was consistent with their relations across group (and were of moderate size). For grade 8/9 working memory, the relation with the State test was also consistent within and across group (suggesting no relation); however, the relation with the experimental algebra measure was quite different within group. Specifically, there was a weak *negative* relation for early takers, r = -0.27, but it was moderate and *positive* for later takers, r = .44, and this difference was significant, z = -2.48, p = .013. For symbolic comparison, relations with the State test were similar between and within groups (showing moderate relations for grade 6 symbolic comparison, and no relation for grade 8/9 symbolic comparison). Again, however, the relation with the experimental algebra measure was quite different within group. Specifically, there were weak positive relations for early takers, r = 0.23 and r = .15 when symbolic comparison was measured in grade 6 and grade 8/9, respectively, but strong and negative for later takers, r = -.60 and r = -.65; these differences were significant, z = 3.10, p = .002, and z = 3.03, p = .003, respectively. Thus, most correlations showed the expected directional relations, but for early takers, working memory and symbolic comparison correlated much more weakly (and in the opposite direction than expected) with the experimental algebra measure.

Hypotheses 2 and 3: Group Differences

To address our second hypothesis, we compared early versus later takers on the set of predictor measures, and expected larger differences between these groups on cognitive variables early on versus later (with the exception of language), and potentially similar results (though smaller in size) for numerosity variables. By contrast, we expected similarly large differences between groups on mathematical measures. Group means (and effect sizes) for both time points are displayed in Table 3.

(insert Table 3 about here)

For cognitive variables, there were no time (grade 6 vs. grade 8/9) by group (early vs. later takers) interactive effects (all $p > .05$). For the two language variables, however, there were overall group differences ($p < .010$ and .025, respectively), with early takers outperforming later takers; there was also change over time ($p < .001$ for both), showing improvement. Neither working memory nor visuospatial rotation showed between-group differences ($p = .144$ and $p = .262$, respectively); scores did increase over time for visuospatial rotation ($p < .001$), but not for working memory ($p = .144$). Individual effect sizes indicated vocabulary and working memory differences between groups at grade 6 ($d = +0.80$ and $d = +0.63$, respectively), and to a lesser degree for visual-auditory learning ($d = +0.45$), favoring those who took Algebra I earlier, though other grade 6 effect sizes were small. None of the numerosity variables showed interactive or between groups effects (all $p > .05$), but both symbolic and nonsymbolic comparison showed improvement ($p < .001$ and $p = .018$, respectively). When vocabulary (at the time of Algebra I) was included as a covariate, effects were similar, except that the growth in symbolic comparison was now not significant, though growth in nonsymbolic comparison remained. Effect sizes between earlier and later takers across cognitive and numerosity variables at grade 8/9 were moderate for visual-auditory learning and symbolic comparison ($d = +0.50$ and $d = +0.44$), but smaller for others (range $d = +0.09$ to $+0.35$).

The pattern across mathematical variables was consistent. Group and time effects were present, but interactions were not significant. The group and time effects were stronger for fractions and proportional reasoning (all $p < .001$) than for math facts (between groups, $p = .001$; over time, $p = .033$), always favoring students who took Algebra I earlier, both in grade 6 and during the Algebra I year. When vocabulary was included as a covariate, the within group effects (change over time) were now not significant; however, all between group differences remained. The effects sizes between groups were all large (see Table 3; range d = +0.90 to +1.77, median d = +1.16); fractions performance showed the widest difference.

Finally, we evaluated algebraic differences (shown in Table 3). We expected these to be large if these skills track mathematical variables (and as noted above, these measures showed the strongest correlations with algebra, and largest group differences). Algebra I variables showed large differences among early (grade 8) and later (grade 9) takers assessed toward the end of the course (both $p < .001$). When vocabulary was included as a covariate, all group differences remained significant. When grade 6 math performances were included as covariates, group differences for the researcher-designed measure remained significant, but group differences for the State Exit Exam did not. Effect sizes were much larger for the researcher-designed measure (d = +1.58; +1.41 with math covariates), relative to the State Exit Exam (d = +0.79; +0.40 with math covariates). As with other variables, differences again favored the group taking Algebra I early.

Discussion

We evaluated cognitive, number, and arithmetic influences on algebraic outcomes for students who took Algebra I earlier (grade 8) or later (grade 9), relying on a unique sample who were followed for 2 to 3 years. As such, it provides novel insight into the role of these predictors over time, for this outcome, and in a way that also considers the timing of Algebra I. In doing so, this study contributes to an understanding of how different educational decisions impact eventual algebraic performance. Our expectations regarding the relations of these sets of variables to algebraic outcomes were partially supported, with particularly strong relations between mathematical and algebraic measures measured either in grade 6 or during Algebra I. Comparing early versus later takers, large differences were noted (always favoring students who took Algebra I earlier) on measures of intermediate math as well as for algebra; fewer and smaller differences were observed on cognitive and numerosity measures, assessed at either time point.

Hypothesis I: Predictive Correlations

The correlational data presented here support and extends prior studies. For example, studies with younger populations and for math outcomes including whole number computation, math problem solving, and fractions generally show contributions of both domain-specific and domain-general predictors (Cirino, 2011; Cirino et al., 2016; Fuchs et al., 2010a; 2010b; Jordan et al., 2013; Szucs, Devine, Soltesz, Nobes, & Gabriel, 2014; Vukovic et al., 2014) and often show a pattern whereby domain-general and numerosity influences are subsumed by more proximal arithmetic predictors, when all are assessed. The present study contributes to this growing knowledge base but focuses specifically on Algebra I.

That measures of math fact retrieval, fractions, and proportional reasoning were strongly related to algebra (median r = .53), is highly consistent with previous investigations (Booth et al., 2014; Geary et al., 2015; Siegler et al., 2012), particularly when considered at the zero-order correlation level. That these correlations were similar whether measured in the same year or two to three years earlier lends support to the hierarchical relation between earlier and later developing math skills. It remains unclear whether differences on these measures in middle school are due to weaknesses in more basic arithmetic versus weakness in cognitive or number abilities. However, speaking to this, the results of Cirino et al. (2016) suggest that cognitive and number abilities influence complex mathematics through math facts and procedural computations. This is important because it may allow researchers and teachers to identify (a) who is likely to struggle at later grade levels, and importantly for instruction, (b) which variables are likely most foundational to future learning. If foundational skills are weak, a focus on only the grade-level skill is unlikely to be sufficient; instead, it may be necessary to allocate additional resources to bring deficient skills to mastery before (or at least while) higher-order concepts and procedures are being introduced.

The present study only partially supports prior work showing the importance of numerosity to math performance. Interestingly, several recent studies show measures of magnitude to be effective predictors of algebra (e.g., Matthews et al., 2016; Booth et al., 2014), even in the context of other relevant cognitive and arithmetic predictors (e.g., Geary et al., 2015). However, with the exception of concurrent whole number line, our measures tapping symbolic or nonsymbolic magnitude comparison were weak predictors, even at the zero order correlational level. Even so, number line relations may have been larger if an alternate measure was used; for example, Booth et al. and Matthews et al. each found that whole number line was a less-strong predictor of algebra relative to fractional or decimal number lines. Our results for ANS acuity differ from Geary et al. (2015), who found a correlation of ~.40 with several algebraic outcomes. The present sample also showed a lower Weber fraction (W score, .20 versus .35) than in Geary et al., (2015), suggesting greater acuity for our sample, though the number and types of trials also differed in the two studies. However, our results are in line with the meta-analytic correlation of ANS with math skill (e.g., Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014).

Among cognitive skills (language and working memory), the pattern of modest to moderate relations with algebra range (r = |.25 to .40|) is consistent with what is known about the contribution of language and working memory to basic math outcomes in younger populations of students. The present study contributes to this knowledge base, while broadening that literature by including an experimental researcher-developed measure of algebraic procedures and concepts, as well as a State Exit Exam for Algebra I. Consistency in the size of these relations demonstrates that algebra is a highly complex cognitive activity, requiring a significant investiture of cognitive resources. However, it is likely that the way these cognitive resources are deployed depends to a strong extent on the mastery of lower level math skills. The post-hoc analyses suggest that at least some skills may operate differentially depending on level of skill. For example, one of the more interesting correlational patterns was that working memory assessed in G6 was predictive of later Algebra I performance, but when both are assessed during Algebra I, the variables were not significantly correlated. The exploratory follow-up analyses suggested this may be only partially true – only among early takers. For later takers, the expected effect was found, though on the experimental algebra measure, which unlike the State Test did not supply reference formulas. These results, which are exploratory in light of the present sample sizes, must be taken with caution. Further work is necessary to identify the consistency and robustness of these general patterns.

Hypotheses 2 and 3: Group Differences

At the time of the grade 6 measurement occasion, the researchers were unaware of which students would take Algebra I when, although because decisions about grade 8 versus grade 9 placement were presumably based at least in part on mathematical skill, we were most confident in hypothesizing large grade 6 arithmetic differences. This expectation was supported, as early takers clearly had stronger fractions performance (d = +1.77; Table 3). Although this finding is not new conceptually, the size of the difference represents an important contribution to the literature, and emphasizes the need to develop fractions competence well prior to when algebra placement decisions are formulated (perhaps prior to grade 6). Differences on proportional reasoning and even math facts also showed large effect sizes, suggesting that while fraction performance appears paramount, it does not exist in a vacuum. Rather, they are supported by a range of related mathematical skills.

The fact that early versus later takers also differed, even in grade 6, on language and working memory (e.g., range $d = +0.45$ to 0.80) is not unexpected, given that such processes have demonstrated relations with math performance across the age range and thus contribute (directly or indirectly) to the math skill differences observed in grade 6. The fact that visuospatial rotation was not different in early versus later takers is perhaps consistent with weaker prior evidence of relations for these skills with the types of math skills emphasized in grade 6.

There were fewer group differences for numerosity specific to grade 6, suggesting that numerosity's influence is superseded by more proximal measures. This is consistent with Booth et al.'s (2014) comments regarding their measure of number line performance, in which unit fraction number line performance was predictive of pre-algebra but not algebra skills. It is also possible that the numerosity measures we employed were insufficiently sensitive to individual differences in numerosity-based competencies.

In the context of these differences (or lack thereof) in grade 6, we expected that group differences in numerosity would remain stable from grade 6 to 9, and this was the case. Analyses showed only improvement over time, and Table 3's effect sizes shows that group differences on these measures were small. In fact, there were no interactive effects of group and time for any of the variables we examined. Generally speaking, whatever differences were present at grade 6 were also present at the time of Algebra I; and effect sizes were still pertinent when covarying for other relevant variables (e.g., vocabulary and earlier math skill). This was true despite that most performances showed raw score gains over time and despite that later takers had an extra year of development (and schooling). This pattern of results is striking, if disappointing (as the extra year did not yield any “catch up”). However, if this general pattern holds in other larger samples, it certainly highlights the influence and relevancy of these skills for algebraic learning. Given the recent emphasis on fractions in the research literature, if this emphasis is concomitant with a similar emphasis in educational practice, it is also possible that a future longitudinal study might show diminished differences.

We know of few prior data in this regard. The large-scale policy studies that examined broad implementation of Algebra initiatives reviewed in the introduction did not generally compare final algebra performance across early versus later takers, but those studies clearly offer insight. For example, using ECLS-K data, Domina (2014) found that that as early as kindergarten, those destined to enroll in grade 8 Algebra I were approximately 0.70 standard deviations (*SD*) higher their peers who enrolled in regular grade 8 math, and this kindergarten gap was even wider at grade 5 (~0.85 *SD*). That study did, however, also find that including kindergarten or fifth-grade math and reading performance, produced a pseudo-R^2 for grade 8 course taking of only 8%. Similarly, the graphical displays in Ma and Wilkins (2007) indicate that early and later Algebra I takers maintain a similar gap from one another on NAEP items, whether assessed in grade 8 or 9 or later. In contrast, however, Liang et al. (2012), as noted in the Introduction, found that students who passed their grade 8 Math exam, and then took Algebra I in grade 9, did better on the Algebra exit exam relative to students who took Algebra I in grade 8 and who, due to poor performance were required to re-take the Algebra exam a second time in grade 9. Such results imply that for some students, delaying Algebra I can in fact provide an advantage for achieving algebraic competence. Domina (2014) had also highlighted this balance between an earlier “opportunity to learn” versus a potential mismatch of coursework with preparedness. The results of the present study offer more individual data to supplement this larger context.

For our students, not only did later takers have a delayed math course trajectory, but whatever gaps in basic mathematic skills were evident in grade 6 remained 2 to 3 years later. Although longitudinal improvement was noted, this improvement was not enough to scaffold algebraic development. Findings suggest that targeting fractions and proportional reasoning intervention, which also requires mastery of lower level math skills, is critical to prepare these students for Algebra I. Some encouragement is, however, warranted in that controlling for math skill, differences on the State Test were not significant (d = +0.40), implying that if such skills improve, between group differences would be smaller. On the other hand, on material that was less directly aligned, and where computation was de-emphasized, gaps between early and later takers were much more evident, even when controlling for math skill (d = +1.41). In addition to scaffolding mathematical skills, it may also be that introducing algebraic concepts earlier may help promote and support the transition from arithmetic to algebra (Carraher, Schliemann, Brizuela, & Earnest, 2006).

Along these lines, it would be particularly interesting to know if students with low fractions performance were to be randomized to remedial intervention that occurs prior to grade 8, would they outperform control peers in terms of later course taking, behavior, and achievement. A number of recent fraction intervention studies are now available (e.g., Fuchs et al., 2013); however, students were not followed through Algebra I, where late effects of such interventions might be observed. Some research suggests that the effects of early interventions fade (Bailey et al., 2016). Yet, other evidence demonstrates that early intervention on math problem solving which focuses on schema development related to problem structure and transfer have delayed effects (2 years after intervention) on arithmetical problem solving and transfer to algebra (6 years after intervention), even when controlling for the types of cognitive and arithmetical abilities that differentiate early and late algebra takers in this study (Tolar, Cirino, & Fuchs, in revision). This evidence suggests that early, research validated math interventions may have effects that are not fully realized until cognitive resources have more fully developed to leverage earlier formed knowledge structures for complex and abstract (i.e., algebraic) problem solving. Similar types of early interventions related to conceptual knowledge of fractions and proportions may reap similar benefits.

Limitations

The results of the present study should be considered in the context of three important limitations. The most obvious is sample size, particularly regarding the interactive effects or the within group correlations. In this vein, it is nevertheless important to note that differences between early and later takers were clear and large; that many of the obtained correlations were in line with previous findings; and that these students were generally comparable to the larger cohorts from which they were obtained, suggesting these students were representative of their school populations. Further, the types of results reported here are novel and longitudinal, which may balance some of sample size considerations. Nonetheless, it will be important to examine the general trends in our database (e.g., correlational results; differences between early and later takers) in the context of the larger parent data (for which longitudinal analyses are not possible). A second limitation is that more robust relations may have been obtained with more comprehensive coverage of the cognitive and numerosity domains, for example by including both verbal and nonverbal working measures and by examining more nuanced measures of estimation and magnitude. Finally, it is difficult to know how well the present sample generalizes to the larger population of students taking Algebra I in grade 8 versus grade 9, particularly in settings with no universal policy on the grade at which Algebra I is completed. To have adequately sampled and retained a large enough grade 6 sample with wide variability in school ascertainment would have been prohibitive in terms of resources, particularly given the other aims of the parent study. However, the present results do provide a basis for formulating more cogent questions about the patterns of relations that might be found in other samples.

Conclusions

The present study is among the few to examine individual difference predictors of Algebra I performance longitudinally. This complements a host of individual difference studies in younger samples with respect to other mathematics outcomes. It also complements policy-based studies that evaluate the implications of state and district decisions regarding when to take Algebra I, which focus dominantly on systemic factors or math performance and course taking behavior alone. We found that some measures of cognitive, numerosity, and mathematical performance predicted Algebra I performance, for early and late takers, although these relations were clearly most robust for fractions and proportional reasoning. We also found that differences between early and later takers apparent in G6 remained through the end of Algebra I, even though later takers have an extra year of development and math instruction. Addressing performance gaps that pre-date entry to the Algebra I course may identify strategies for diminishing the long-term achievement gaps between early and late takers.

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Table 1. Correlations of Grade 6 Predictors with One Another and Grade 8/9 Algebra.

	VOC	VAL	WM	VR	NL	SC	PM	FACT	FRAC	PROP	ALG	STATE
WASI Vocabulary[a] (VOC)	1.00											
WJ-III Visual Auditory Learning[b] (VAL)	0.24	1.00										
Symmetry Span[c] (WM)	-0.15	0.12	1.00									
Visuospatial Rotation[c] (VR)	-0.09	0.09	0.20	1.00								
Number Line Estimation[c] (NL)	-0.24	-0.06	-0.17	-0.20	1.00							
Symbolic Comp[d] (SC)	-0.20	0.08	-0.11	-0.28	0.15	1.00						
Panamath[e] (PM)	-0.06	0.07	-0.32	-0.07	0.20	0.18	1.00					
Math Facts[c] (FACT)	0.33	-0.10	0.35	0.07	-0.44	-0.38	-0.17	1.00				
Fractions[c] (FRAC)	0.32	0.05	0.21	0.08	-0.37	-0.19	-0.04	0.56	1.00			
Prop. Reasoning[c] (PROP)	0.28	0.06	0.35	0.08	-0.36	-0.19	-0.24	0.47	0.49	1.00		
Experimental Algebra[c] (ALG)	0.31	0.30	0.33	0.13	-0.12	-0.18	-0.12	0.35	0.41	0.58	1.00	
State Algebra[c] (STATE)	0.25	-0.05	0.39	0.08	-0.24	-0.26	-0.21	0.59	0.29	0.56	0.64	1.00
Mean	40.72	498.50	23.85	5.43	4.20	732.44	0.23	66.58	6.51	3.91	19.98	33.65
SD	6.94	6.47	7.00	3.13	0.47	145.66	0.07	17.44	3.23	2.01	6.91	9.00

Note. Correlations greater than $r = |0.28|$ are significant $p < .05$. N range 47 to 53. WASI = Wechsler Abbreviated Scales of Intelligence; WJ-III = Woodcock-Johnson III Tests of Cognitive Ability.

[a]T-score

[b]Rasch-based W score

[c]Raw Score (Symmetry Span raw based on partial credit-load, see Methods; Number Line Estimation is deviation score from target number, in cm; other measures are total scores within given time limit).

[d]Response Time for Correct Responses (in msecs.)

[e]Weber fraction

Table 2. Correlations of Grade 8/9 Predictors with One Another and Grade 8/9 Algebra.

	VOC	VAL	WM	VR	NL	SC	PM	FACT	FRAC	PROP	ALG	STATE
WASI Vocabulary[a] (VOC)	1.00											
WJ-III Visual Auditory Learning[b] (VAL)	0.23	1.00										
Symmetry Span[c] (WM)	0.00	0.03	1.00									
Visuospatial Rotation[c] (VR)	0.14	0.11	0.31	1.00								
Number Line Estimation[c] (NL)	-0.04	-0.01	0.15	-0.05	1.00							
Symbolic Comp.[d] (SC)	0.03	0.00	-0.18	-0.06	0.16	1.00						
Panamath[e] (PM)	0.30	-0.26	-0.09	-0.03	0.20	-0.04	1.00					
Math Facts[c] (FACT)	0.16	0.01	0.17	0.15	-0.28	-0.32	-0.12	1.00				
Fractions[c] (FRAC)	0.11	0.19	0.04	0.14	-0.19	-0.10	-0.06	0.49	1.00			
Prop. Reasoning[c] (PROP)	0.30	0.27	-0.11	0.29	-0.39	-0.10	-0.13	0.43	0.46	1.00		
Experimental Algebra[c] (ALG)	0.44	0.41	0.09	0.27	-0.22	0.02	-0.05	0.41	0.50	0.60	1.00	
State Algebra[c] (STATE)	0.26	0.31	0.01	0.27	-0.43	-0.06	-0.10	0.63	0.44	0.64	0.64	1.00
Mean	45.54	501.80	25.52	7.04	4.05	614.21	0.20	69.94	8.09	5.11	19.98	33.65
SD	6.71	7.27	7.25	3.72	0.34	87.37	0.06	20.73	3.57	2.49	6.91	9.00

Note. Correlations greater than $r = |0.28|$ are significant $p < .05$. N range 47 to 53. For other notes and abbreviations, see Table 1.

Table 3. Group Differences Between Early versus Later Takers

	G8 N	*Mean*	*SD*	*G9 N*	*Mean*	*SD*	*d*	*95% Conf. Interval*
G6 WASI Vocabulary	27	43.29	5.370	26	38.038	7.459	+0.80	+0.24 to +1.36
G6 WJ-III Visual Auditory Learning	27	499.92	5.452	26	497.02	7.18/9	+0.45	-0.10 to +0.99
G6 Symmetry Span	27	25.96	6.346	26	21.654	7.076	+0.63	+0.08 to +1.18
G6 Visuospatial Rotation	27	5.556	3.142	26	5.308	3.185	+0.08	-0.46 to +0.62
G6 Number Line Estimation	27	4.164	0.480	26	4.237	0.467	-0.15	-0.69 to +0.39
G6 Symbolic Comparison	27	727.337	133.24	26	737.73	160.03	-0.07	-0.61 to +0.47
G6 Panamath	27	0.229	0.059	26	0.222	0.083	+0.09	-0.45 to +0.63
G6 Math Facts	27	73.741	15.823	26	59.153	16.111	+0.90	+0.33 to +1.47
G6 Fractions	27	8.62962	2.6768	26	4.3076	2.0740	+1.77	+1.14 to +2.41
G6 Proportional Reasoning	27	4.92592	1.7525	26	2.8461	1.7133	+1.18	+0.60 to +1.77
G8/9 WASI Vocabulary	27	46.8148	6.6912	25	44.16	6.5744	+0.39	-0.16 to +0.94
G8/9 WJ-III Visual Auditory Learning	27	503.537	6.6776	25	499.92	7.5498	+0.50	-0.05 to +1.05
G8/9 Symmetry Span	27	25.9629	7.6132	25	25.04	6.9550	+0.12	-0.42 to +0.67
G8/9 Visuospatial Rotation	27	7.66666	3.7622	25	6.36	3.6157	+0.35	-0.20 to +0.90
G8/9 Number Line Estimation	27	4.018/95	0.3701	24	4.0933	0.3130	-0.21	-0.76 to +0.34
G8/9 Symbolic Comparison	27	632.614	102.43	25	594.33	63.735	+0.44	-0.11 to +0.99
G8/9 Panamath	26	0.20146	0.0565	25	0.1955	0.0728	+0.09	-0.46 to +0.64
G8/9 Math Facts	27	78.4444	18.411	26	61.115	19.541	+0.90	+0.33 to +1.46
G8/9 Fractions	27	10.4074	3.1896	26	5.6923	2.0546	+1.72	+1.09 to +2.35
G8/9 Proportional Reasoning	27	6.33333	2.3534	26	3.8461	1.9736	+1.13	+0.55 to +1.71
G8/9 Experimental Algebra	26	24.2307	5.3164	25	15.56	5.4778	+1.58	+0.95 to +2.21
G8/9 State Algebra	26	37.0384	7.1076	26	30.269	9.5312	+0.79	+0.23 to +1.36

Note. Within each grade, measures are grouped by domain (cognitive, numerosity, mathematical, respectively).