Sample Size and Measurement Occasion Planning for Latent Change Score Models through

Monte Carlo Simulation

Zhiyong Zhang and Haiyan Liu

Department of Psychology, University of Notre Dame

2018/09/21

Author Note

Citation: Zhang, Z., & Liu, H. (2018). Sample Size and Measurement Occasion Planning for Latent Change Score Models through Monte Carlo Simulation. In E. Ferrer, S. M. Boker, and K. J. Grimm (Eds.) Advances in Longitudinal Models for Multivariate Psychology: A Festschrift for Jack McArdle.

Abstract

Latent change score models (LCSMs) proposed by McArdle (McArdle, 2000, 2009; McArdle & Nesselroade, 1994) offer a powerful tool for longitudinal data analysis. They are becoming increasingly popular in social and behavioral research (e.g., Gerstorf et al., 2007; Ghisletta & Lindenberger, 2005; King et al., 2006; Raz et al., 2008). Although conducting both univariate and multivariate latent change score analysis is not a difficult task any more (e.g., Ghisletta & McArdle, 2012; Zhang et al., 2015), there is little discussion on the design issues such as sample size planning for LCSMs. To fill the gap, this study proposes a Monte Carlo based method to determine the required sample size and the number of measurement occasions for both univariate and bivariate LCSMs. The method can obtain the power for testing each individual parameter of the models such as the change rate and coupling parameters. The Monte Carlo procedure is implemented and provided in a free R package RAMpath (Zhang et al., 2015). Examples for sample size and measurement occasion planning for both univariate and bivariate LCSMs are provided.

Sample Size and Measurement Occasion Planning for Latent Change Score Models through

Monte Carlo Simulation

Introduction

Longitudinal data collection and data analysis are becoming a norm for psychological

research (e.g., Grimm et al., 2016; McArdle & Nesselroade, 2014). A longitudinal design often

involves data collection on multiple variables from multiple participants at multiple times.

Despite of the increased cost and complexity, there are many advantages to collect longitudinal

data. For example, a longitudinal design naturally enables a researcher to study change and

related phenomena. In addition, inter-individual differences in change can also be investigated.

Growth curve models are probably the most widely used technique for analyzing

longitudinal data benefiting from the fact that a growth curve model can be fitted from a structural

equation modeling (SEM) framework (e.g., McArdle, 1986; McArdle & Epstein, 1987; McArdle

& Anderson, 1990; McArdle & Hamagami, 1992; McArdle, 1998; McArdle & Bell, 1998;

McArdle & Nesselroade, 2014). With the increasing use of longitudinal design, it is not

surprising that more and more complex models and methods have been developed. For example,

in order to deal with missing data, full information maximum likelihood methods, multiple

imputation, and Bayesian methods have been developed and used (e.g., Enders, 2011; Lu et al.,

2013). To deal with non-normal data, robust methods have been proposed (e.g., Yuan & Zhang,

2012; Zhang, 2013; Zhang et al., 2013).

A difficult issue in longitudinal research is to model the nonlinear trajectory of data. With

more data collection, a linear growth curve model is often not sufficient. When moving to

nonlinear models, issues such as computational difficulty can arise (e.g., Grimm et al., 2011;

Wang & McArdle, 2008). Linearizing a nonlinear model provides an efficient way to deal with

such difficulties. Although the method based on Taylor expansion is well known (e.g., Browne,

1993; Neale & McArdle, 2000), it is less known that the latent change score models (LCSMs)

provide a potentially more efficient way to model nonlinear trajectories.

Proposed by McArdle and colleagues, LCSMs combine difference equations with growth

curves to study change in longitudinal studies (e.g., McArdle, 2000; McArdle & Hamagami, 2001; Hamagami & McArdle, 2007a; Hamagami et al., 2010). In such models, change is directly modeled, which is often the focus of a longitudinal study. As we will show shortly, the models allow to easily accommodate certain nonlinear growth trajectories. In addition to the univariate LCSMs, bivariate LCSMs have also been proposed to model the inter-relationship between two growth processes (e.g., McArdle & Hamagami, 2001).

Fitting a LCSM in the SEM framework is easy to understand but can be tedious. It can be done in almost any SEM software. Recently, Ghisletta & McArdle (2012) showed how to estimate a univariate LCSM using different R packages, including Lavaan (Rosseel, 2012), OpenMx (Boker et al., 2011) and sem (Fox, 2006). More recently, Zhang et al. (2015) automated the estimating procedure for the typical univariate and multivariate LCSMs through an R package RAMpath that is developed based on RAM notations (Boker et al., 2002; McArdle & Boker, 1990).

The importance of conducting statistical power analysis at the beginning of a study is universally accepted (e.g., Cohen, 1988; Hedges & Rhoads, 2010). Without adequate statistical power, the validity of statistical conclusions from all kinds of research is endangered (e.g., Cohen, 1988; Hedges & Rhoads, 2010; Myors & Wolach, 2014; Shadish et al., 2002). For example, without a carefully planned sample size, a study can easily fail to detect an existing effect by chance, which in turn creates problems for replication or cross-validation. Although there are studies on sample size planning and power calculation for growth curve analysis (e.g., Zhang & Wang, 2009), we are not aware of any discussion on such design issues for LCSMs.

To fill the gap, this study proposes a Monte Carlo based method to determine the required sample size and/or the number of measurement occasions for both univariate and bivariate LCSMs. The method can obtain the power for testing each individual parameter of the models such as the change rate and coupling parameters. We also implement the Monte Carlo procedure in a free R package RAMpath (Zhang et al., 2015).

In the rest of the chapter, we first present the univariate and bivariate LCSMs. Then, we

introduce the Monte Carlo based method for power analysis. After that, we show how to conduct

power analysis for LCSMs through several examples using our developed software. We conclude

the chapter with discussion and future directions.

A Univariate Latent Change Score Model

Let $Y[t]_n$ denote the data from the nth ($n = 1, \ldots, N$) participant at time t ($t = 1, \ldots, T$)

of a sample consisting of N participants measured for T times. The first part of a LCSM is a

measurement error model where an observed score $Y[t]_n$ is the sum of the latent true score $y[t]_n$

and the measurement error/uniqueness score $ey[t]_n$:

$$Y[t]_n = y[t]_n + ey[t]_n.$$

It is generally assumed that the error follows a normal distribution with mean 0 and variance

$varey$. The second part of the model builds the relationship between consecutive latent true

scores so that the current score at time t is equal to the sum of the true score at the previous time

$t - 1$ and the change score, $dy[t]_n$, from time $t - 1$ to time t:

$$y[t]_n = y[t - 1]_n + dy[t]_n.$$

This effectively defines the change score as

$$dy[t]_n = y[t]_n - y[t - 1]_n.$$

Note that in the classic LCSM, the relationship between consecutive latent true scores is

deterministic although it is not required to be so. The third part of the model concerns the

modeling of the difference scores. One way is to model the difference score at time t as the sum

of a linear constant effect ys and the proportional change from time $t - 1$ such that

$$dy[t]_n = ys_n + \beta_y \times y[t - 1]_n,$$

where β_y is a compound rate of change.

Given the three part of the model, we can model the observed score as

$$Y[t]_n = y[t]_n + ey[t]_n$$

$$= y[t-1]_n + dy[t]_n + ey[t]_n$$

$$= (1 + \beta_y)y[t-1]_n + ys_n + ey[t]_n.$$

Successively expressing the above equation will lead to

$$Y[t]_n = (1 + \beta_y)y[t-1]_n + ys_n + ey[t]_n$$

$$= (1 + \beta_y)(y[t-2]_n + dy[t-1]_n) + ys_n + ey[t]_n$$

$$= (1 + \beta_y)^2 y[t-2]_n + (1 + \beta_y)ys_n + ys_n + ey[t]_n$$

$$= (1 + \beta_y)^{t-1}y[1]_n + [1 + (1 + \beta_y) + \ldots + (1 + \beta_y)^{t-2}]ys_n + ey[t]_n$$

$$= (1 + \beta_y)^{t-1}y0_n + [1 + (1 + \beta_y) + \ldots + (1 + \beta_y)^{t-2}]ys_n + ey[t]_n$$

where $y0_n$ is the initial latent score and note that the latent score at time t follows

$$y[t]_n = (1 + \beta_y)^{t-1}y0_n + [1 + (1 + \beta_y) + \ldots + (1 + \beta_y)^{t-2}]ys_n.$$

Clearly, the observed and latent scores behave as a nonlinear function of time and therefore can capture the nonlinear trajectory except when $\beta_y = 0$. To visually show this, we plot the latent scores with different values for β_y in Figure 1. The basic LCSM can only handle this specific type of nonlinearity with exponential changes. For other types of nonlinearity, more complex LCSMs are needed.

The initial latent score and the linear constant change score can be correlated. In the model, they are assumed to have a bivariate normal distribution

$$\begin{pmatrix} y0_n \\ ys_n \end{pmatrix} \sim MN \left[\begin{pmatrix} my0 \\ mys \end{pmatrix}, \begin{pmatrix} vary0 & vary0ys \\ vary0ys & varys \end{pmatrix} \right]$$

with MN denoting a multivariate, here bivariate, normal distribution. Therefore, the initial latent score follows a normal distribution with mean $my0$ and variance $vary0$ and the constant change

score also follows a normal distribution with mean mys and variance $varys$. The covariance between them is $vary0ys$ with the correlation expressed as

$$\rho_{y0ys} = \frac{vary0ys}{\sqrt{vary0 \times varys}}.$$

Using a path diagram, this model is portrayed in Figure 2. In the path diagram, squares represent observed variables, while circles represent latent variables. A single-headed arrow is for deterministic parameters such as regression coefficients, or factor loadings, while a double-headed arrow represents stochastic parameters such as variance and covariance. A triangle represents a constant. Any arrow originating from the triangle represents an intercept or mean of variables pointed by the arrow. We matched the notations in the formulas and in the path diagram. For simplification, we removed the brackets and the subscripts for the variables in the path diagram.

A Bivariate Latent Change Score Model

A bivariate LCSM is first a combination of two univariate LCSMs. Above and beyond that, it allows the two processes represented by the LCSMs to interact with each other. Let $Y[t]_n$ and $X[t]_n$ denote the observed data on the two variables X and Y, respectively, from the nth $(n = 1, \ldots, N)$ participant at time t $(t = 1, \ldots, T)$ of a sample consisting of N participants measured for T times. For the measurement error part of the model, we have

$$Y[t]_n = y[t]_n + ey[t]_n$$

$$X[t]_n = x[t]_n + ex[t]_n,$$

where $ey[t]_n$ follows a normal distribution with mean 0 and variance $varey$ and $ex[t]_n$ follows a normal distribution with mean 0 and variance $varex$. For the latent score from time $t - 1$ to time t, we have

$$y[t]_n = y[t - 1]_n + dy[t]_n$$

$$x[t]_n = x[t - 1]_n + dx[t]_n,$$

with $dy[t]_n$ and $dx[t]_n$ denoting the latent change scores for the two variables, respectively.

The innovative part of the bivariate LCSM is to allow the latent score of one variable to influence the change score of another variable. Specifically, we model the change scores as

$$dy[t]_n = ys_n + \beta_y \times y[t-1]_n + \gamma_y x[t-1]_n$$

$$dx[t]_n = xs_n + \beta_x \times x[t-1]_n + \gamma_x y[t-1]_n$$

where γ_y and γ_x are called coupling parameters. γ_y represents the effect of x on the change score of y and γ_x represents the effect of y on the change score of x. We let $x0$ be the initial latent score and xs be the constant change for x. A multivariate normal distribution is assumed for the initial latent scores and constant changes for the two variables such that

$$\begin{pmatrix} y0_n \\ ys_n \\ x0_n \\ xs_n \end{pmatrix} \sim MN \left[\begin{pmatrix} my0 \\ mys \\ mx0 \\ mxs \end{pmatrix}, \begin{pmatrix} vary0 & vary0ys & varx0y0 & vary0xs \\ vary0ys & varys & varx0ys & varxsys \\ varx0y0 & varx0ys & varx0 & varx0xs \\ vary0xs & varxsys & varx0xs & varxs \end{pmatrix} \right].$$

Using a path diagram, a bivariate LCSM is portrayed in Figure 3.

Statistical Power Analysis Based on Monte Carlo Simulation

Statistical power analysis concerns the power of a test to detect an effect different from the null. For a model with a set of parameters $\boldsymbol{\theta}$, one can conduct power analysis for one or a subset of parameters, denoted by $\boldsymbol{\tau}$, to investigate whether they are equal to 0 or known values $\boldsymbol{\tau}_0$. Therefore, the null and alternative hypotheses of interest are

$$H_0 : \boldsymbol{\tau} = \boldsymbol{\tau}_0 \text{ vs. } H_1 : \boldsymbol{\tau} \neq \boldsymbol{\tau}_0.$$

Existing procedures for power evaluation are mostly based on the Wald test or the likelihood ratio test. The Wald test statistic is defined as

$$T = (\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}_0)' \hat{\boldsymbol{\Phi}}^{-1} (\hat{\boldsymbol{\tau}} - \boldsymbol{\tau}_0) \tag{1}$$

where $\hat{\boldsymbol{\tau}}$ is the parameter estimates in $\hat{\boldsymbol{\theta}}$ corresponding to $\boldsymbol{\tau}$ and $\hat{\boldsymbol{\Phi}}$ is the covariance matrix of $\hat{\boldsymbol{\tau}}$. The Wald test statistic can be compared to a critical value C_α under the null hypothesis. If

$T > C_\alpha$, the null hypothesis is rejected. Under the null hypothesis and the typical normality assumption, the Wald statistic asymptotically follows a chi-square distribution (χ_q^2) with the degrees of freedom q, where q is the number of parameters in $\boldsymbol{\tau}$. Then, the critical value at the significance level α is $C_\alpha = \chi_q^2(1 - \alpha)$. Note that when working with a single parameter, the Wald test is the square of a Z test.

The likelihood ratio test works in the similar manner. In the likelihood ratio test, one first estimates the model under the alternative hypothesis to get the value of the likelihood function at L_1. Then, one can estimate the model under the null hypothesis by fixing the parameters in $\boldsymbol{\gamma}$ to be $\boldsymbol{\gamma}_0$ to get the value of the likelihood function at L_0. The likelihood ratio test statistic is defined as

$$T = -2 \ln \frac{L_0}{L_1}. \tag{2}$$

The likelihood ratio test statistic is also compared to a critical value C_α to decide whether a null hypothesis should be rejected. If T follows a chi-square distribution with degrees of freedom q, the critical value is $\chi_q^2(1 - \alpha)$. If $T > C_\alpha$, the null hypothesis is rejected.

By its definition, the statistical power is

$$\begin{aligned}
\pi &= \Pr(\text{reject } H_0 | H_1 \text{ is true}) \\
&= \Pr(T > C_\alpha | H_1 \text{ is true}),
\end{aligned}$$

where T can be the Wald statistic or the likelihood ratio test statistic. For simple statistical analysis such as a t-test, one can obtain an analytical form for π and therefore power and sample size planning can be conducted easily. However, for LCSMs, both the Wald and the likelihood ratio test statistics are complex functions of sample size and effect size. Therefore, the statistical power π is also a complex function of these factors. Generally speaking, it is difficult or impossible to get a tractable form of π so that the relationship between statistical power and sample size can be easily evaluated.

To deal with the difficulty in power analysis for LCSMs, we use a Monte Carlo simulation based method to approximate the power using the relative frequency to reject the null hypothesis given the alternative hypothesis is true. Specifically, the following procedure can be used.

1. Decide the significance level. Usually, the default 0.05 can be used. Based on that, get the critical value C_α. If only one parameter is tested, the C_α based on the normal distribution is 1.96 for a z-test and 3.84 for the chi-squared distribution for the Wald test.

2. Specify a LCSM M_1 under H_1 with the hypothesized population parameter values ($\boldsymbol{\theta}$).

3. Generate a set of data with the sample size N and the number of measurement occasions T from the model using random number generation techniques.

4. Fit Models M_1 and M_0, the model by setting $\boldsymbol{\tau} = \boldsymbol{\tau}_0$, to the generated data and obtain the Wald statistic using Equation (1) and/or the likelihood ratio statistic using Equation (2).

5. If the test statistic $T > C_\alpha$, the null hypothesis H_0 is rejected.

6. Repeat Steps (2)–(5) for a total of $R(R \geq 1000)$ times.

7. Suppose out of the R replications, the null hypothesis H_0 is rejected r times. Then the statistical power with the sample size n is estimated by $\hat{\pi} = \frac{r}{R}$.

8. For sample size planning, if $\hat{\pi}$ is smaller than the desired power, say 0.8, one can increase the sample size or the number of measurement occasions to repeat Steps 2 and 7 to recalculate the power. Otherwise, the sample size or the number of measurement occasions can be set to a smaller value.

The above Monte Carlo simulation based method for statistical power analysis has been widely used in the literature for mediation analysis and SEM (e.g., Muthén & Muthén, 2002; Thoemmes et al., 2010; Zhang & Wang, 2009; Zhang, 2014). This procedure is especially effective for complex models. For example, Muthén & Muthén (2002) illustrated how to use Mplus to conduct statistical power analysis for structural equation models using such a procedure. Zhang & Wang (2009) focused on how to conduct statistical power analysis for growth curve models with and without missing data. Thoemmes et al. (2010) discussed how to apply the procedure in mediation analysis. Zhang (2014) extended Theommes et al. for the analysis of missing data and non-normal data.

For a typical power analysis for LCSM, a single parameter is often of interest. For example, one may be only interested in the parameter β in the model. In this case, the power can be

calculated using the above procedure based on the Z test. Since Monte Carlo simulation is conducted, when estimating the power for one parameter, power for all the other parameters can also be calculated without much extra effort. Therefore, in our software, we output the power for all parameters in a LCSM model as we will show in our examples.

Software for Power Analysis for Latent Change Score Models

Although the idea of Monte Carlo simulation based power analysis is straightforward, it would still need the software to implement it to make it useful. Recent, Zhang et al. (2015) developed the R package RAMpath that can estimate both univariate and bivariate LCSMs. We expanded RAMpath so that it can carry out power analysis for LCSMs. To further simplify power analysis for researchers who might not be familiar with R, we also developed online software based on RAMpath.

R package

The R package RAMpath is now on CRAN and therefore it can be installed directly within R as a typical package. For example, to install it, use the R code `install.packages("RAMpath")`. To use the package within R, use `library("RAMpath")`. There are three functions in the package for power analysis: `powerLCS`, `powerBLCS`, and `plot`.

The function `powerLCS` is used to conduct power analysis for univariate LCSMs. The basic usage of the function is given below:

```
powerLCS(N = 100, T = 5, R = 1000, betay = 0, my0 = 0, mys = 0,
   varey = 1, vary0 = 1, varys = 1, vary0ys = 0,alpha = 0.05,
   ...)
```

In the function, `N` is the sample size and `T` is the number of measurement occasions. Both of them can be a single value or a vector. For example, using `N=c(100,200,500)` will calculate power for the three provided sample sizes. `R` is the number of Monte Carlo simulation used to

estimate the power. A larger `R` will provide more accurate power estimation but also take more computing time. As a rule of thumb, at least 1,000 should be used. `alpha` is the significance level for testing the hypothesis of the model parameters. The default value is 0.05.

To obtain statistical power, the population parameter values have to be provided. Such values can be decided based on literature review, pilot study, expert opinions, etc. By default, all the mean, intercept and covariance parameters are set at 0 and all the variance parameters are set at 1. Those values typically have to be changed in real power analysis. Note that the name of each parameter corresponds to that used in the path diagram in Figure 2. In addition to the basic input, for advanced users, other information can be provided to control the parameter and standard error estimation methods. For example, the options used in Lavaan to control model estimation can be used directly within the function. More information can be found in the help document of the R package.

The output of the R function includes four main pieces of information for each parameter in the model. The first is the Monte Carlo estimate (`mc.est`). It is calculated as the mean of the `R` sets of parameter estimates from the simulated data. Note that the Monte Carlo estimates should be close to the population parameter values used in the model. The second is the Monte Carlo standard deviation (`mc.sd`), which is calculated as the standard deviation of the `R` sets of parameter estimates. The third is the Monte Carlo standard error (`mc.se`), which is obtained as the average of the `R` sets of standard error estimates of the parameter estimates. Lastly, `mc.power` is the statistical power for each parameter.

The function `powerBLCS` is used to conduct power analysis for bivariate LCSMs. The basic usage of the function is given below. It is the same as for the univariate LCSMs.

```
powerBLCS(N=100, T=5, R=1000, betay=0, my0=0, mys=0, varey=1,
    vary0=1, varys=1, vary0ys=0, betax=0, mx0=0, mxs=0, varex=1,
    varx0=1, varxs=1, varx0xs=0, varx0y0=0, varx0ys=0, vary0xs=0,
    varxsys=0, gammax=0, gammay=0, alpha=0.05, ...)
```

The function `plot` is used to generate a power curve, which has the form `plot(x,`

parameter, ...). The first input of the function, x, is the output from either `powerLCS` or `powerBLCS`. In the input of the function for power analysis, either the sample size `N` or the number of occasions `T` should be a vector. The second input is the name of a parameter to plot its power curve. Since there are multiple parameters in a LCSM, one can generate a plot for each model parameter. The name of a parameter should match the one in `powerLCS` or `powerBLCS`. This function will generate one or multiple line plots in which power is shown on the y-axis and sample size or the number of occasions is shown on the x-axis.

Online interface

In order to help researchers who are not familiar with R, we also provide a Web-based interface for power analysis for LCSMs. The URL for the univariate LCSMs is `http://psychstat.org/lcsm` and for the bivariate LCSMs is `http://psychstat.org/blcsm`.

The Web interface for the univariate LCSMs is shown in Figure 4. Since the interface is built on the R function shown earlier, it requires the same input information and gives the same output. For both sample size and number of occasions, multiple values can be provided in two ways to calculate power for each given value. We discuss this using the sample size as an example since the same method is used for the number of occasions. First, multiple sample sizes can be provided and separated by spaces. For example, inputting 100 150 200 will calculate power for the three sample sizes 100, 150, and 200. Second, a sequence of sample sizes can be generated using the method *s:e:i* with *s* denoting the starting sample size, *e* as the ending sample size, and *i* as the interval. Note that the values are separated by a colon ":". For example, 100:150:10 will generate a sequence of sample sizes: 100 110 120 130 140 150.

The interface for the bivariate LCSMs is similar and is not provided here for the sake of space.

Examples

In this section, we show how to carry out power analysis for both univariate and bivariate LCSMs through several examples.

Example 1. Type I error rate investigation for a univariate LCSM

Note that if the null hypothesis is true, the Monte Carlo procedure will yield the type I error rate. For example, suppose the parameter $\beta_y = 0$ in the population. Then the estimated power for it should be the same as the significance level, typically 0.05. For illustration, we set the population parameter values to those shown in the second column of Table 1. Therefore, if we conduct a power analysis based on those parameter values, we will obtain the type I error rates for `betay`, `my0`, `mys`, and `vary0ys`. If our Monte Carlo procedure performs well, we expect the type I error rates to be close to the alpha level used.

The R code for conducting the analysis is shown in Code 1. Note that the significance level is set at 0.05 and therefore, we expect the estimated values in the power column are close to 0.05.

Code 1: R input script for Example 1.

```
powerLCS(N = 100, T = 5, R = 1000, betay = 0, my0 = 0, mys = 0,
    varey = 1, vary0 = 1, varys = 1, vary0ys = 0,alpha = 0.05)
```

The output of the R code is given in Code 2. First, the estimate for each parameter is very close to the true population parameter values as can be seen in the column named `mc.est`. This indicates that the power calculation procedure runs well. Second, the Monte Carlo standard errors are close to the corresponding Monte Carlo standard deviations, another indicator that the power calculation is trustworthy. Third, as expected, the power for `betay`, `my0`, `mys`, and `vary0ys` is close to 0.05, the nominal type I error rate. Overall, this suggests that the Monte Carlo based method can provide well-controlled type I error rate.

Code 2: Type I error rate and power for parameters in Example 1.

```
        pop.par mc.est mc.sd mc.se mc.power   N T
```

```
betay          0  0.001 0.056 0.056    0.046 100 5

my0            0  0.001 0.129 0.126    0.056 100 5

mys            0  0.002 0.105 0.105    0.044 100 5

varey          1  0.994 0.083 0.081    1.000 100 5

vary0          1  0.990 0.236 0.230    1.000 100 5

vary0ys        0 -0.005 0.136 0.136    0.044 100 5

varys          1  1.006 0.227 0.227    1.000 100 5
```

Example 2. Power analysis for a univariate LCSM

To conduct a power analysis, one has to specify the population parameter values for the model. Zhang et al. (2015) included an example on using a univariate LCSM model to analyze the WISC data (see McArdle & Nesselroade, 2014). In order to plan a future study with the sample size 100 and 5 measurement occasions, we use the estimates as our population parameter values. Column 3 in Table 1 shows the roundup parameter estimates being used in our example.

The R code for conducting the analysis is shown in Code 3 and the output of the R code is given in Code 4. From the output, we can see that the power to detect the parameter `betay` to be significant with a sample size 100 and a number of measurement occasions 5 is about 0.664. The power for another parameter, the constant change `mys`, is 0.274. Since oftentimes one hopes to get a power at least 0.8, a larger sample size is needed for this study. In addition, for studying power for different parameters, different sample sizes are often required.

Code 3: R input script for Example 2.

```
powerLCS(N = 100, T = 5, R = 1000, betay = 0.1, my0 = 20, mys =
   1.5, varey = 9, vary0 = 2.5, varys = .05, vary0ys = 0, alpha =
    0.05)
```

Code 4: Power for parameters in Example 2.

```
        pop.par mc.est mc.sd mc.se mc.power   N T
```

```
betay        0.10   0.103 0.043 0.044     0.664 100 5

my0         20.00 19.999 0.324 0.319     1.000 100 5

mys          1.50   1.418 1.106 1.120     0.274 100 5

varey        9.00   8.961 0.724 0.732     1.000 100 5

vary0        2.50   2.463 1.151 1.139     0.583 100 5

vary0ys      0.00  -0.004 0.408 0.403     0.048 100 5

varys        0.05   0.053 0.173 0.175     0.050 100 5
```

Example 3. Generate a power curve for different sample sizes for a univariate LCSM

Example 2 above showed that a larger sample size was needed in order to get sufficient power for parameters `betay` and `mys`. Although one can try a difference sample size greater than 100, for convenience, we can generate a power curve with different sample sizes. For example, Figure 5 shows the power curves for the two parameters `betay` and `mys` with sample sizes ranging from 100 to 200 with an interval 10. From the plot, we can easily see that to get a power 0.8 for the parameter `betay`, a sample size about 150 is needed. On the other hand, a sample size larger than 200 is needed for the parameter `mys` to have a power 0.8, with the exact number undecided based on the plot.

The R code for generating the power curve is shown in Code 5. Note that in the plot function, we refer to a specific parameter directly using its name. In the input, `seq(100, 200, 10)` generate a sequence of sample sizes and in the output, power for each sample size is provided. Code 6 shows the output when the sample sizes are 100 and 200 only to save space.

Code 5: R input script for power curve in Example 3.

```
res <- powerLCS(N = seq(100, 200, 10), T = 5, R = 1000, betay =
   0.1, my0 = 20, mys = 1.5, varey = 9, vary0 = 2.5, varys = .05,
    vary0ys = 0, alpha = 0.05)
plot(res, 'betay')
plot(res, 'mys')
```

Code 6: Output for generating power curves in Example 3.

```
$`N100-T5`

         pop.par mc.est mc.sd mc.se mc.power   N T
betay       0.10   0.100 0.044 0.044     0.627 100 5

my0        20.00 20.002 0.331 0.319     1.000 100 5

mys         1.50   1.505 1.136 1.119     0.287 100 5

varey       9.00   8.970 0.744 0.732     1.000 100 5

vary0       2.50   2.489 1.218 1.146     0.599 100 5

vary0ys     0.00 -0.009 0.413 0.403     0.059 100 5

varys       0.05   0.054 0.176 0.175     0.050 100 5



....



$`N200-T5`

         pop.par mc.est mc.sd mc.se mc.power   N T
betay       0.10   0.100 0.031 0.031     0.915 200 5

my0        20.00 20.002 0.225 0.226     1.000 200 5

mys         1.50   1.505 0.790 0.791     0.487 200 5

varey       9.00   8.971 0.532 0.518     1.000 200 5

vary0       2.50   2.480 0.803 0.808     0.904 200 5

vary0ys     0.00   0.005 0.283 0.283     0.049 200 5

varys       0.05   0.051 0.125 0.122     0.054 200 5
```

Example 4. Generate a power curve for different number of occasions for a univariate LCSM

For LCSMs, power is not only related to the sample size but also the number of measurement occasions. With the increase of the number of occasions, one would expect the

increase of power. For example, Figure 6 shows the power curves for the two parameters `betay` and `mys` with the number of occasions ranging from 4 to 10 with an interval 1 and with the fixed sample size 100. From the plot, we can easily see that the power increases along with the number of measurement occasions. For example, for the same sample size 100, the power is less than 0.2 with 4 occasions of data but increases to more than 0.8 with 7 occasions of data for the parameter `mys`. The R code for generating the power curve is shown in Code 7.

Code 7: R input script for power curve with the number of occasions in Example 4.

```
res <- powerLCS(N = 100, T = 4:10, R = 1000, betay = 0.1, my0 =
    20, mys = 1.5, varey = 9, vary0 = 2.5, varys = .05, vary0ys =
    0, alpha = 0.05)
plot(res, 'betay')
plot(res, 'mys')
```

Example 5. Power analysis for a bivariate LCSM

Power analysis can be similarly conducted for bivariate LCSMs. As an example, we use the parameter estimates from a bivariate latent change score model in Zhang et al. (2015) with some modification as population parameter values (see Table 2).

The script in Code 8 shows the R code for power analysis for the bivariate LCSM with the sample size 100. From the output in Code 9, we can see that the parameter estimates are not very accurate. This is because the bivariate LCSM requires a much larger sample size to provide accurate parameter estimates. In this case, the statistical power obtained might not be accurate either.

Code 8: R input script for power analysis for bivariate latent change score model in Example 5.

```
powerBLCS(N=100, T=5, R=1000, betay=0.08, my0=20, mys=1.5, varey
    =9, vary0=3, varys=1, vary0ys=0, alpha=0.05, betax=0.2, mx0
    =20, mxs=5, varex=9, varx0=3, varxs=1, varx0xs=0, varx0y0=1,
```

```
    varx0ys=0, vary0xs=0, varxsys=0, gammax=0, gammay=-.1)
```

Code 9: Output for power analysis in Example 5.

	pop.par	mc.est	mc.sd	mc.se	mc.power	N	T
betax	0.20	0.230	0.260	0.187	0.241	100	5
betay	0.08	0.164	0.572	0.435	0.081	100	5
gammax	0.00	-0.033	0.234	0.178	0.112	100	5
gammay	-0.10	-0.175	0.641	0.458	0.075	100	5
mx0	20.00	20.004	0.336	0.326	1.000	100	5
mxs	5.00	5.933	7.848	5.615	0.167	100	5
my0	20.00	20.019	0.346	0.326	1.000	100	5
mys	1.50	0.451	6.933	5.321	0.156	100	5
varex	9.00	8.941	0.744	0.732	1.000	100	5
varey	9.00	8.939	0.749	0.720	1.000	100	5
varx0	3.00	3.029	1.243	1.222	0.739	100	5
varx0xs	0.00	-0.210	0.768	0.767	0.030	100	5
varx0y0	1.00	1.052	0.840	0.835	0.226	100	5
varx0ys	0.00	-0.012	0.668	0.601	0.017	100	5
varxs	0.60	2.343	6.805	2.687	0.090	100	5
varxsys	0.00	0.072	3.559	1.740	0.019	100	5
vary0	3.00	2.951	1.423	1.245	0.684	100	5
vary0xs	0.00	0.198	2.263	1.629	0.031	100	5
vary0ys	0.00	-0.371	1.970	1.511	0.106	100	5
varys	0.05	1.415	3.730	2.096	0.024	100	5

Increasing the sample size will lead to more accurate results as shown in Code 10 where the sample size is 500. In planning the sample size for LCSM models, one should pay attention to the parameter estimates to make sure they are accurate enough for power calculation. Specifically for

the coupling parameters `gammax` and `gammay`, the power and type I error are 0.057 and 0.271, respectively.

Code 10: Output for power analysis in Example 5 when the sample size is 500.

	pop.par	mc.est	mc.sd	mc.se	mc.power	N	T
betax	0.20	0.2009	0.031	0.031	1.000	500	5
betay	0.08	0.0830	0.070	0.068	0.199	500	5
gammax	0.00	-0.0014	0.030	0.029	0.057	500	5
gammay	-0.10	-0.1022	0.072	0.073	0.271	500	5
mx0	20.00	19.9911	0.145	0.145	1.000	500	5
mxs	5.00	5.0308	0.939	0.942	1.000	500	5
my0	20.00	19.9999	0.143	0.146	1.000	500	5
mys	1.50	1.4684	0.889	0.885	0.420	500	5
varex	9.00	8.9836	0.340	0.328	1.000	500	5
varey	9.00	8.9961	0.341	0.328	1.000	500	5
varx0	3.00	3.0052	0.524	0.523	1.000	500	5
varx0xs	0.00	-0.0144	0.222	0.230	0.047	500	5
varx0y0	1.00	1.0064	0.360	0.360	0.808	500	5
varx0ys	0.00	-0.0012	0.199	0.201	0.051	500	5
varxs	1.00	1.0312	0.180	0.189	1.000	500	5
varxsys	0.00	0.0028	0.161	0.163	0.045	500	5
vary0	3.00	2.9777	0.519	0.547	1.000	500	5
vary0xs	0.00	0.0072	0.286	0.294	0.035	500	5
vary0ys	0.00	-0.0135	0.252	0.257	0.043	500	5
varys	1.00	1.0246	0.260	0.253	0.999	500	5

Discussion and Future Directions

To complement the research on LCSMs, in this chapter, we discuss how to plan the sample size and the number of measurement occasions for both univariate and bivariate LCSMs. Specifically, we illustrate how to calculate power for each individual model parameter of interest. Since the analytical solution to power is intractable, we used a Monte Carlo based method. We also provided an R package RAMpath and an online interface to carry out the power analysis procedure.

In calculating power, we need the information on the population parameter values. Each value can be viewed as the unstandardized effect size for the parameter of interest. We did not define the standardized effect size such as Cohen's d (Cohen, 1988) for several reasons. First, given the complexity of LCSMs, it is difficult to define a standardized effect size. Second, in general, it is easier to specify the unstandardized effect size because when conducting a literature review, one can simply adopt the parameter estimates directly from the published results. Third, if a researcher is interested in standardized measures, he/she can use the standardized coefficients as the population parameter values in conducting power analysis.

One way to streamline the specification of the population parameters is to use the R-squared. For example, in a bivariate LCSM, the variance of the change score is from three sources - the constant change, the own level score, and the level score of the other variable. By changing the parameter values, one can quantify the portion of variance explained by each source. On the other hand, depending on the expected variance explained, one can set the parameter values. Using this method, one can take advantage of the existing effect size cutoffs for R-squared, namely, 0.02 for small, 0.13 for medium, and 0.26 for large effect sizes.

The current study can be improved in many ways in the future. First, in this chapter, we have focused on the power analysis of a single model parameter as this is the most common situation. If a researcher wants to test multiple parameters simultaneously, a procedure based on the likelihood ratio test can be developed as for the growth curve analysis in Zhang & Wang (2009).

Second, in the current study, we have focused on the basic univariate and bivariate LCSMs. Since their invention, the basic univariate and bivariate LCSMs have been extended in many ways. For example, Hamagami & McArdle (2007b) expanded the traditional specifications of univariate and bivariate LCSMs to the parallel process change score model and the second-order LCSMs. Grimm et al. (2012) extended latent difference scores to allow for testing hypotheses where recent changes, as opposed to recent levels, are a primary predictor of subsequent changes. The Monte Carlo procedure used in this study can be flexibly extended to the more advanced models.

Third, the current study has assumed that the collected data will be complete. However, in practice, missing data are almost not avoidable in longitudinal studies. For example, Puma et al. (2009) found that student achievement outcomes are often missing for 10-20% in studies funded by the National Center for Education Evaluation and Regional Assistance. Missing data reduce power and without careful consideration, a well-planned study can become under-powered. Taking into consideration of missing data in power calculation requires the specification of missing data generating mechanism that can be used in the data generation step in our Monte Carlo method.

Fourth, in our Monte Carlo method, we have assumed that our data are normally distributed. However, practical data often deviate from a normal distribution. For example, Micceri (1989) evaluated 440 distributions of large-sample achievement and psychometric measures and found that all of them were nonnormal. More recently, Blanca et al. (2013) evaluated nonnormality using the skewness and kurtosis of 693 small samples and found that 94.5 % of them violated the normality assumption. In addition, Cain et al. (2016) reviewed 254 multivariate distributions of data used in Psychological Science and the American Education Research Journal and found that 68 % multivariate distributions deviated from normal distributions. Therefore, in the future, the influence of non-normal data should be considered when estimating power.

Finally, the Monte Carlo based method can be very computationally intensive because of the involvement of the Monte Carlo simulation. For example, it took about 10 minutes on a modern desktop to complete the power analysis in Example 4. At the same time, the Monte Carlo

method can be easily parallelized to take advantage of modern hardware such as multi-core

processors (e.g., Zhang, 2014). In the future, the R package RAMpath can be improved with the

capacity of parallelization.

References

Blanca, M. J., Arnau, J., López-Montiel, D., Bono, R., & Bendayan, R. (2013). Skewness and
   kurtosis in real data samples. *Methodology*.

Boker, S. M., McArdle, J. J., & Neale, M. C. (2002). An algorithm for the hierarchical
   organization of path diagrams and calculation of components of covariance between variables.
   Structural Equation Modeling, *9*(2), 174–194.

Boker, S. M., Neale, M., Maes, H., Wilde, M., Spiegel, M., Brick, T., ... others (2011). Openmx:
   an open source extended structural equation modeling framework. *Psychometrika*, *76*(2),
   306–317.

Browne, M. W. (1993). Structured latent curve models. In C. M. Cuadras & C. R. Rao (Eds.),
   Multivariate analysis: Future directions 2 (pp. 171–198). Amsterdam: North-Holland.

Cain, M. K., Zhang, Z., & Yuan, K.-H. (2016). Univariate and multivariate skewness and kurtosis
   for measuring nonnormality: Prevalence, influence and estimation. *Behavior Research
   Methods*, 1–20.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). Hillsdale, NJ:
   Lawrence Ehrlbaum Associates.

Enders, C. K. (2011). Missing not at random models for latent growth curve analyses.
   Psychological methods, *16*(1), 1–16.

Fox, J. (2006). Structural equation modeling with the sem package in r. *Structural Equation
   Modeling*, *13*, 465–486.

Gerstorf, D., Lövdén, M., Röcke, C., Smith, J., & Lindenberger, U. (2007). Well-being affects
   changes in perceptual speed in advanced old age: longitudinal evidence for a dynamic link.
   Developmental psychology, *43*(3), 705.

Ghisletta, P., & Lindenberger, U. (2005). Exploring the structural dynamics of the link between sensory and cognitive functioning in old age: Longitudinal evidence from the berlin aging study. *Intelligence*, *33*, 555–587.

Ghisletta, P., & McArdle, J. J. (2012). Latent curve models and latent change score models estimated in r. *Structural equation modeling: a multidisciplinary journal*, *19*(4), 651–682.

Grimm, K. J., An, Y., McArdle, J. J., Zonderman, A. B., & Resnick, S. M. (2012). Recent changes leading to subsequent changes: Extensions of multivariate latent difference score models. *Structural equation modeling: a multidisciplinary journal*, *19*(2), 268–292.

Grimm, K. J., Ram, N., & Estabrook, R. (2016). *Growth modeling: Structural equation and multilevel modeling approaches*. Guilford Publications.

Grimm, K. J., Ram, N., & Hamagami, F. (2011). Nonlinear growth curves in developmental research. *Child Development*, *82*(5), 1357–1371.

Hamagami, F., & McArdle, J. J. (2007a). Dynamic extensions of latent difference score models. In S. M. Boker & M. J. Wenger (Eds.), *Data analytic techniques for dynamical systems* (pp. 47–86). Mahwah, NJ: Erlbaum.

Hamagami, F., & McArdle, J. J. (2007b). Dynamic extensions of latent difference score models. In S. M. Boker & Wenger (Eds.), *Data analytic techniques for dynamical systems* (pp. 47–85). Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Hamagami, F., Zhang, Z., & McArdle, J. J. (2010). Bayesian discrete dynamic system by latent difference score structural equations models for multivariate repeated measures data. In S.-M. Chow, E. Ferrer, & F. Hsieh (Eds.), *Statistical methods for modeling human dynamics: An interdisciplinary dialogue* (pp. 319–348). New York, NY: RoutledgeTaylor & Francis Group.

Hedges, L. V., & Rhoads, C. (2010). Statistical power analysis in education research. ncser

    2010-3006. *National Center for Special Education Research*.

King, L. A., King, D. W., McArdle, J. J., Saxe, G. N., Doron-LaMarca, S., & Orazem, R. J.

    (2006). Latent difference score approach to longitudinal trauma research. *Journal of traumatic*

    stress, *19*(6), 771–785.

Lu, Z. L., Zhang, Z., & Cohen, A. (2013). Bayesian methods and model selection for latent

    growth curve models with missing data. In *New developments in quantitative psychology* (pp.

    275–304). Springer.

McArdle, J. J. (1986). Latent variable growth within behavior genetic models. *Behavior*

    Genetics, *16*, 163-200.

McArdle, J. J. (1998). Modeling longitudinal data by latent growth curve methods. In

    G. Marcoulides (Ed.), *Modern methods for business research* (pp. 359–406). Mahwah, NJ:

    Lawrence Erlbaum Associates.

McArdle, J. J. (2000). A latent difference score approach to longitudinal dynamic structural

    analyses. In R. Cudeck, S. du Toit, & D. Sôrbom (Eds.), *Structural equation modeling: Present*

    and future (pp. 342–380). Lincolnwood, IL: Scientific Software International.

McArdle, J. J. (2009). Latent variable modeling of differences and changes with longitudinal

    data. *Annual review of psychology*, *60*, 577–605.

McArdle, J. J., & Anderson, E. (1990). Latent variable growth models for research on aging. In

    J. E. Birren & K. W. Schaie (Eds.), *Handbook of the psychology aging* (p. 21-44). New York:

    Academic Press.

McArdle, J. J., & Bell, R. Q. (1998). Recent trends in modeling longitudinal data by latent

    growth curve methods. In T. D. Little, K. U. Schnabel, & J. Baumert (Eds.), *Modeling*

longitudinal and multiple-group data: Practical issues, applied approaches, and scientific

examples (p. 69-107). Mahwah, NJ: Erlbaum.

McArdle, J. J., & Boker, S. M. (1990). *Rampath*. Hillsdale, NJ: Lawrence Erlbaum.

McArdle, J. J., & Epstein, D. (1987). Latent growth curves within developmental structural

equation models. *Child Psychology*, *58*, 110-133.

McArdle, J. J., & Hamagami, F. (1992). Modeling incomplete longitudinal and cross–sectional

data using latent growth structural models. *Experimental Aging Research*, *18*(3), 145–166.

McArdle, J. J., & Hamagami, F. (2001). Latent difference score structural models for linear

dynamic analyses with incomplete longitudinal data. In L. M. Collins & A. G. Sayer (Eds.),

New methods for the analysis of change (pp. 139–175). Washington, DC: American

Psychological Association.

McArdle, J. J., & Nesselroade, J. R. (1994). Using multivariate data to structure developmental

change. In S. H. Cohen & H. W. Reese (Eds.), *Life-span developmental psychology:*

Methodological contributions (pp. 223–267). Hillsdale, NJ: Lawrence Erlbaum Associates.

McArdle, J. J., & Nesselroade, J. R. (2014). *Longitudinal data analysis using structural equation*

models. American Psychological Association.

Micceri, T. (1989). The unicorn, the normal curve, and other improbable creatures.

Psychological Bulletin, *105*(1), 156–166.

Muthén, L. K., & Muthén, B. O. (2002). How to use a monte carlo study to decide on sample size

and determine power. *Structural Equation Modeling*, *9*(4), 599–620.

Myors, B., & Wolach, A. (2014). *Statistical power analysis: A simple and general model for*

traditional and modern hypothesis tests. Routledge.

Neale, M. C., & McArdle, J. J. (2000). Structured latent growth curves for twin data. *Twin*

Research, *3*, 165–177.

Puma, M. J., Olsen, R. B., Bell, S. H., & Price, C. (2009). What to do when data are missing in group randomized controlled trials. ncee 2009-0049. *National Center for Education Evaluation and Regional Assistance*.

Raz, N., Lindenberger, U., Ghisletta, P., Rodrigue, K. M., Kennedy, K. M., & Acker, J. D. (2008). Neuroanatomical correlates of fluid intelligence in healthy adults and persons with vascular risk factors. *Cerebral Cortex*, *18*(3), 718–726.

Rosseel, Y. (2012). lavaan: An R package for structural equation modeling. *Journal of Statistical Software*, *48*, 1–36.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference*. Wadsworth Cengage learning.

Thoemmes, F., MacKinnon, D. P., & Reiser, M. R. (2010). Power analysis for complex mediational designs using monte carlo methods. *Structural Equation Modeling*, *17*, 510–534.

Wang, L., & McArdle, J. J. (2008). A simulation study comparison of Bayesian estimation with conventional methods for estimating unknown change points. *Structural Equation Modeling*, *15*, 52–74.

Yuan, K.-H., & Zhang, Z. (2012). Robust structural equation modeling with missing data and auxiliary variables. *Psychometrika*, *77*, 803–826.

Zhang, Z. (2013). Bayesian growth curve models with the generalized error distribution. *Journal of Applied Statistics*, *40*, 1779–1795.

Zhang, Z. (2014). Monte carlo based statistical power analysis for mediation models: Methods and software. *Behavior research methods*, *46*(4), 1184–1198.

Zhang, Z., Hamagami, F., Grimm, K. J., & McArdle, J. J. (2015). Using r package rampath for tracing sem path diagrams and conducting complex longitudinal data analysis. *Structural Equation Modeling: A Multidisciplinary Journal*, *22*(1), 132–147.

Zhang, Z., Lai, K., Lu, Z., & Tong, X. (2013). Bayesian inference and application of robust growth curve models using student's t distribution. *Structural equation modeling*, *20*, 47–78.

Zhang, Z., & Wang, L. (2009). Statistical power analysis for growth curve models using sas. *Behavior Research Methods*, *41*, 1083–1094.

Table 1

Population parameter values used in Examples 1-4

	Example 1	Example 2
betay	0	0.1
my0	0	20
mys	0	1.5
varey	1	9
vary0	1	2.5
varys	1	0.05
vary0ys	0	0

Table 2

Population parameter values used in Example 5

Parameter	value	Parameter	value
betay	0.08	betax	0.2
gammax	0	gammay	−0.1
my0	20	mx0	20
Mys	1.5	mxs	5
varey	9	varex	9
vary0	3	varx0	3
varys	0.05	varxs	0.6
vary0ys	0	varx0xs	0
varx0y0	1	@x M	@x M
vayx0ys	0	@x M	@x M
vary0xs	0	@x M	@x M
varxsys	0	@x M	@x M
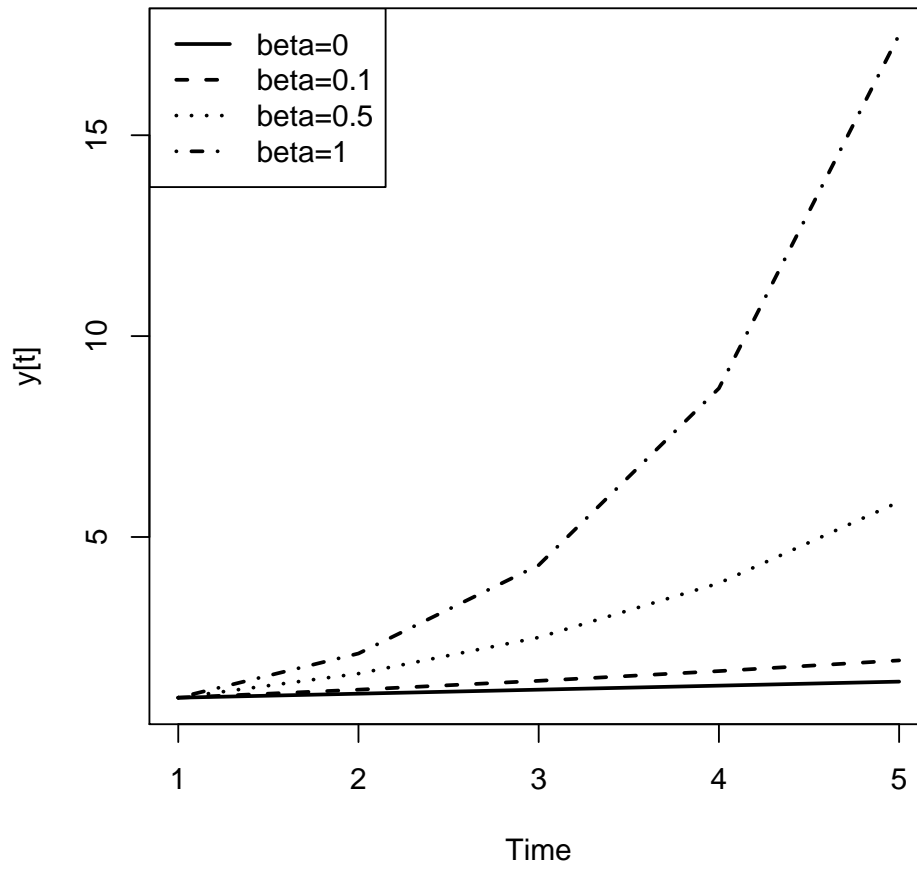
Figure 1. The trajectory plot of latent scores $y[t]$ from time 1 to time 5 with different β_y values
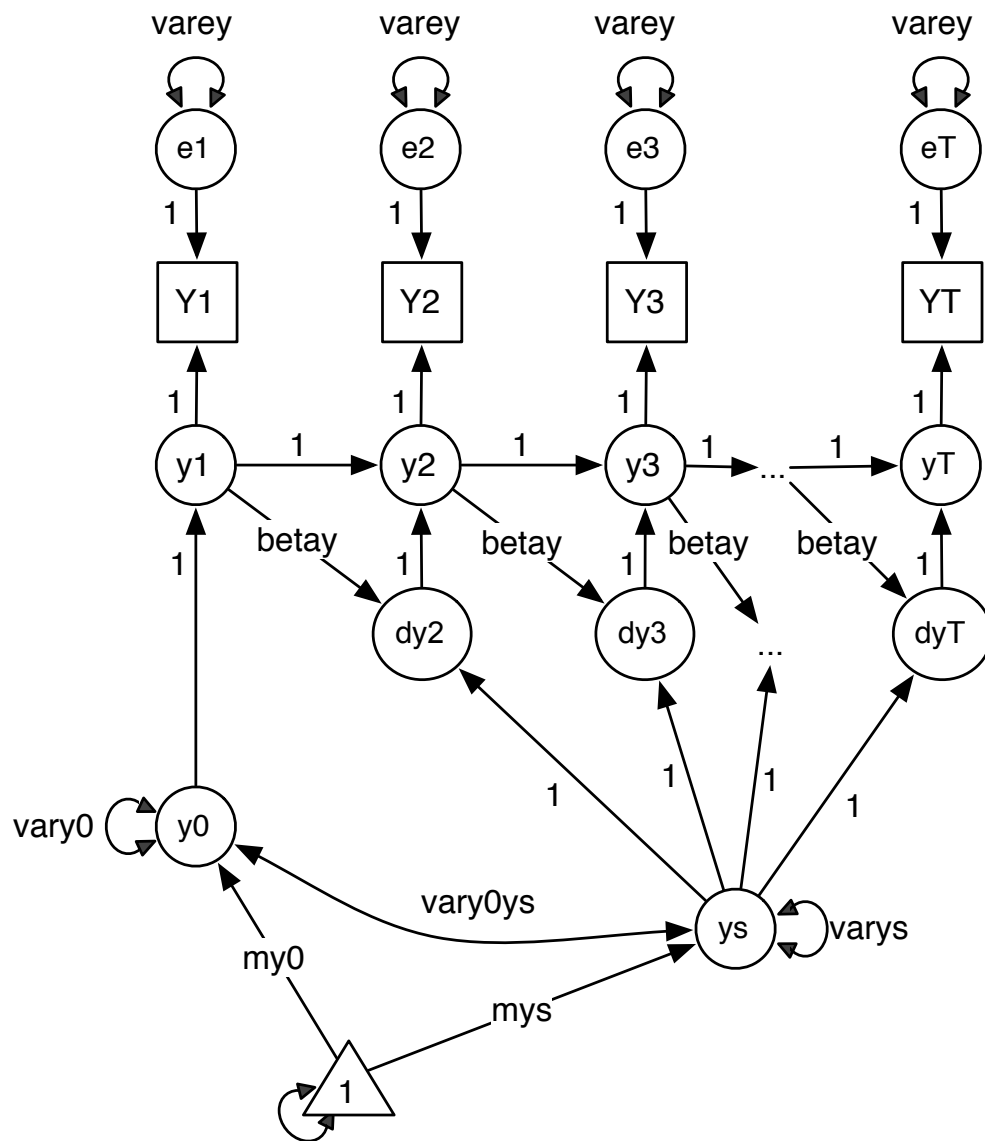
Figure 2. Path diagram for a univariate latent change score models
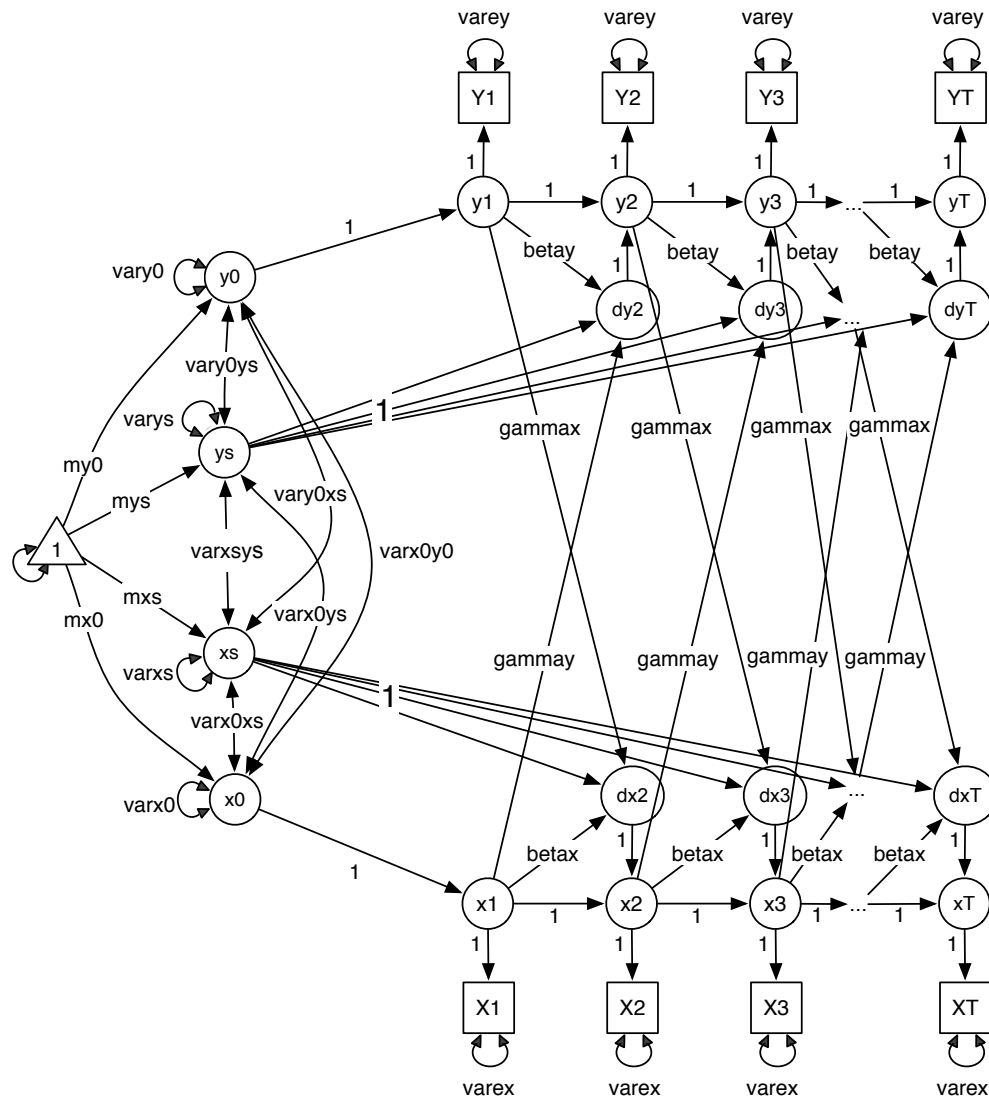
Figure 3. The path diagram for a bivariate latent change score model.

Univariate Latent Change Score Model

Parameters (Help)	
Sample size	100
Number of occasions	5
Number of replications	1000
betay	0
my0	0
mys	0
varey	1
vary0	1
varys	1
vary0ys	0
Significance level	0.05
Power	
Power curve	No power curve ▾
Note	Univariate Latent Change S

Calculate

Figure 4. The online interface for power analysis for univariate latent change score models.
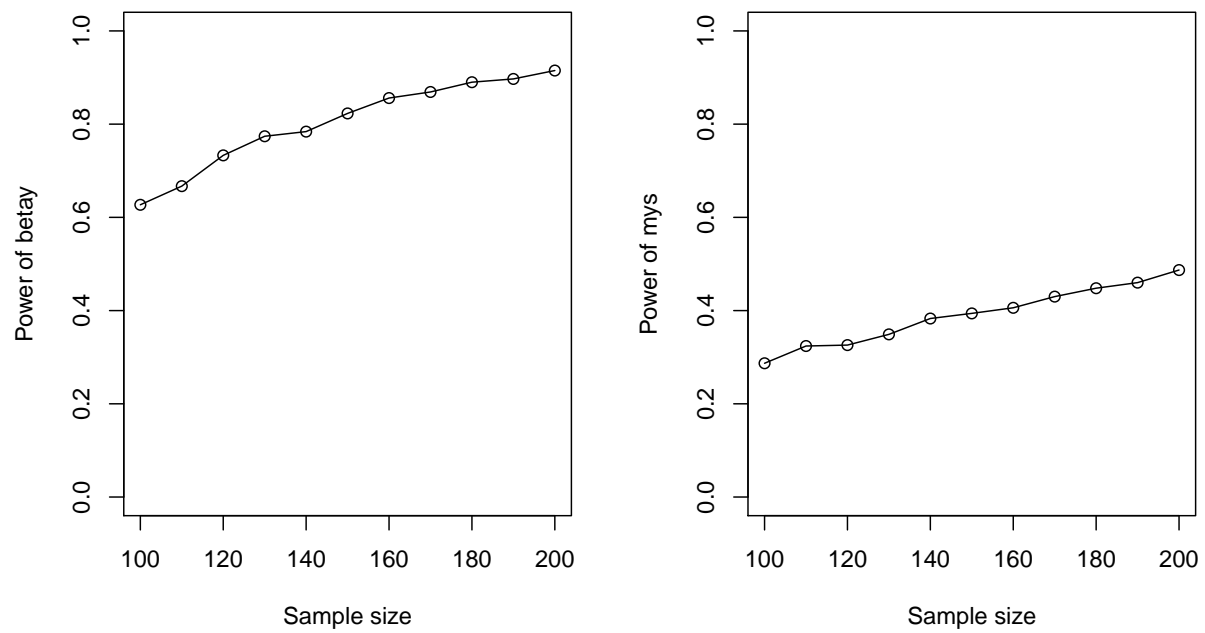
Figure 5. Power curve for `betay` (left plot) and `mys` (right plot) along with the sample size in the univariate latent change score model
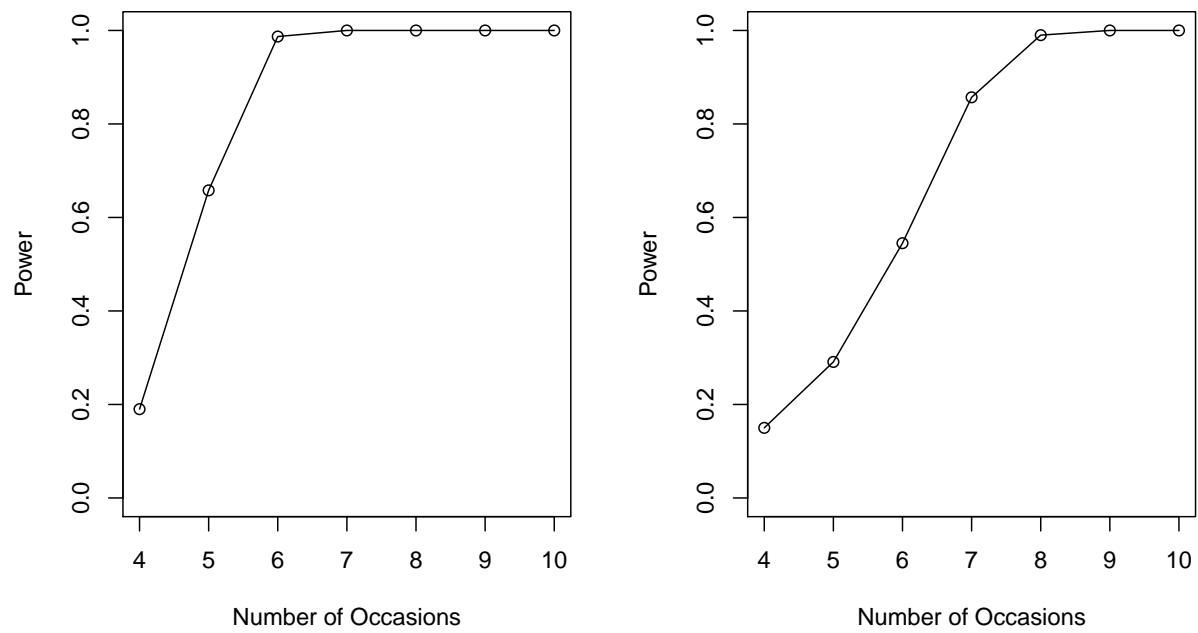
Figure 6. Power curve for `betay` (left plot) and `mys` (right plot) along with the number of measurement occasions in the univariate latent change score model