ON ‘MATHEMATICS FOR ELEMENTARY TEACHERS’ COURSES

Rina Zazkis
Simon Fraser University
zazkis@sfu.ca

Roza Leikin
University of Haifa
rozal@edu.haifa.ac.il

Simin Jolfaee
Simon Fraser University
sjolfaee@gmail.com

A Mathematics course for elementary school teachers (MFET) is required in North America in most teacher education programs. Our study investigates the perceptions of prospective elementary school teachers with respect to the contributions of such a course to their teaching. The results show that acquiring an understanding of concepts from the elementary school curriculum is the main contribution. We conclude with two perspectives – a pessimistic one and an optimistic one – on this finding.

In many teacher education programs in North America a Mathematics course (or a sequence of Mathematics courses) for elementary teachers is either a requirement of, or a prerequisite for, entry to a teacher education program. Despite repeated calls for a “thorough rethinking of mathematics courses for prospective teachers of all grade levels” (CBMS, 2001, p.6), and an agreement about the need for integration of mathematics and pedagogy at the elementary level in order to develop profound knowledge of mathematics for teaching (Ball & Bass, 2000), in many programs there is still the traditional separation between the Mathematics-content courses and the Mathematics-methods courses for prospective elementary school teachers. This study investigates prospective teachers’ views of the contributions, both actual and potential, of the Mathematics-content course, referred to as MFET – Mathematics for Elementary Teachers – to their teaching.

On Teachers’ Knowledge

With the extensive emphasis on teacher education in recent mathematics education research, the primary foci have been on assessing the knowledge that teachers have and exploring what knowledge teachers should have (e.g. Hill, Sleep, Lewis & Ball, 2007; Davis & Simmt, 2006). Acknowledging that teachers’ knowledge is multi-faceted, different attempt were made to categorize the components of such knowledge. Shulman’s (1986) classical categories refer to subject matter knowledge (SMK), pedagogical content knowledge (PCK) and curricular knowledge. An extended categorization of teachers’ knowledge was introduced by Deborah Ball and colleagues (Hill, Ball and Schilling, 2008). It was referred to as “mathematical knowledge for teaching” (MKT) and presented as an extension of Shulman’s categorizations. The PCK refinement included Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT) and Knowledge of Curriculum. The SMK refinement contained the categories of Common Content Knowledge (CCK), Specialized Content Knowledge (SCK) and Knowledge at the Mathematical Horizon. As knowledge acquired in a Mathematics-content course is of interest in this study, we note that CCK was described as shared among individuals who use mathematics, while SCK was considered as the domain of teachers that allows them “to engage in particular teaching tasks” (ibid, p. 377).

In contrast to this categorization, Davis and Simmt (2006) argued against the traditional separation of content and pedagogy, and claimed that “mathematics-for-teaching” can be considered as a distinct branch of the discipline of mathematics. Extending the research of Davis and Simmt (2006) on what teachers need to know, Askew (2008) examined the research evidence for the mathematics discipline knowledge that primary teachers might need in order to


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teach effectively. Askew suggested that attention of mathematics educators should be shifted from what mathematics primary teachers should know to why they should know this mathematics. He also supported the view that distinction between content knowledge and pedagogical content knowledge may no longer be helpful. However, despite the repeated claims against separation of content and pedagogy in teacher education and research, this long-established separation still exists within the coursework towards teachers’ certification. Our prior research investigated the perceptions of secondary school mathematics teachers on the use of their knowledge of ‘advanced’ mathematics – knowledge they acquired during their studies at colleges and universities – in their teaching practice (Zazkis & Leikin, 2010). The results varied significantly: some teachers claimed that they have never used what they learned in their university courses, while others claimed that they used it “all the time”, but had difficulty in providing specific examples of this usage. The current focus on elementary school teachers, the mathematics they study in their University degrees and its perceived usefulness for teaching is a natural follow up.

Mathematics for Elementary Teachers (MFET) Course

While certification at the secondary level requires teachers to acquire a significant background in the subject matter, usually a degree or at least a minor, for elementary school teachers, as generalists, the mathematical subject matter requirements are limited. If such a requirement exists, it is usually for one or several mathematics-content courses designed specifically for this population. A typical MFET course – and we infer what is ‘typical’ from a variety of textbooks with a similar tables of contents (e.g., Bassarear, 2007; Billstein, Libeskind & Lott, 2009; Musser, Burger & Paterson, 2006; Sowder, Sowder & Nickerson, 2010) and a variety of course outlines or course syllabi posted on the web – provides an overview of the underlying concepts of elementary mathematics. Typical topics include number systems and algorithms, patterns and introductory number theory, measurement and geometry, probability and data analysis. Different authors and publishers, in an attempt to satisfy the market, chose different perspectives on concepts and topics, such as problem solving, mathematical reasoning, the use of manipulatives, connection to the Standards, or the use of technology. The degree to which a certain perspective is implemented depends on the instructor’s choice; however, the core topics remain the same.

The Study

Participants in this study were prospective elementary school teachers (PTs) enrolled in a one-year teacher certification program. All the PTs had some teaching experience in elementary school, having completed a ‘practicum’ of either 6 weeks or 5 months. All of the PTs had taken a MFET course, similar to a ‘typical’ course described above, as it is a required prerequisite in their teacher education program. However, they completed this course at various times, with various instructors and at various colleges or universities. As such, our study concerns the course, rather than its specific implementation. Our study attempted to address the following question: How do prospective teachers describe the contribution of their MFET course to their teaching? Or, stated differently: What have prospective teachers learnt in their MFET course that they perceive as useful for their teaching?

Data Collection and Analysis

The data collection included a written response task and a clinical interview. A written response task was administered to a group of 25 PTs. Initially, they were asked to


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provide examples of several teaching situations in which their mathematical knowledge from the MFET course could have been useful. The teaching situation could have been actual or imaginary. However, several students claimed that they “just knew things” and had difficulty identifying the source of their knowledge. Acknowledging this difficulty the task was modified. The task sought examples of usage, either actual or potential, of mathematical knowledge beyond the topic that was being taught.

The interviews were conducted with 14 PTs from a different group. The interviews initially attempted to solicit explicit examples of mathematical knowledge usage in teaching elementary school mathematics. However, the flexible structure of the interviews turned in part into a conversation about the MFET course and its contributions to teaching. The data analysis was ongoing, using a qualitative approach based on grounded theory procedures and techniques (Strauss & Corbin, 1990). In the written responses that explicitly addressed mathematical knowledge from the MFET courses we identified the mathematical topic in each example and the setting in which the example was presented. We then identified several recurring themes in the provided responses. Then, in the analysis of the interviews we identified additional emerging themes. The themes that were identified in participants’ responses serve as an organizing structure for the subsequent results and analysis.

Results and Analysis
To foreshadow our main observation, we start with an illustrative comment from Linda:

*Linda:* I likely learned about … in grade school. It was not until MFET course that I learned the reasoning behind why this works and fully understood […]. This was very helpful when teaching […] during my practicum.

Please note that we intentionally deleted the mathematical content that Linda mentioned, as similar comments were provided with respect to different topics and procedures. We return to the ideas of ‘understanding’ and “reasoning behind why this works” in our subsequent analysis. In what follows we attend to the recurring themes in the participants’ responses as well as to their particular examples of knowledge usage.

Initial Hesitation
In the clinical interviews, among the initial ‘warm-up’ questions, the participants were asked about their MFET course. The questions were of general nature, such as, where did you take the course, how long ago was it, what did you think of it? A typical response is presented below.

*Rita:* Actually when I enrolled in it I was so freaked out the first couple of days, because I’m so terrified of math that I was going to transfer out, but I really wanted my elementary school pre-req’s. And so I was in the middle of transferring out and then I just really started enjoying it and I stuck with it and I ended up getting an A, so I was like, this wasn’t so bad.

The words anxious, nervous, terrified, freaked out, intimidated, panicky and alike were common in participants’ descriptions of their entry point to the course. However, many students reported a degree of confidence and satisfaction towards completing the course, as well as some joy and excitement. While a level of confidence with the mathematical content taught is essential for teaching, we were further interested in specific examples of what PTs believe they learned that were helpful for teaching.

Examples of Usage
In this section we summarize examples of usage from the written response task. The particular
examples that were provided in the interviews appear further on, as illustrations of the recurring themes.

Out of 25 PTs who completed the written response task, 17 explicitly referred to the knowledge acquired in their MFET course, generating 42 examples of knowledge usage. Table 1 presents a distribution of the 42 examples by topics. Table 2 presents a distribution of the 42 examples by the intended usage.

<table>
<thead>
<tr>
<th>Mathematical content</th>
<th>Number of examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computation (shortcut tricks, estimation, order of operations)</td>
<td>9</td>
</tr>
<tr>
<td>Elementary Number Theory (divisibility, prime numbers)</td>
<td>8</td>
</tr>
<tr>
<td>Fractions and Decimals</td>
<td>8</td>
</tr>
<tr>
<td>Geometry (area, perimeter, angles, π)</td>
<td>8</td>
</tr>
<tr>
<td>Algebra</td>
<td>5</td>
</tr>
<tr>
<td>Other (e.g., different bases, division by zero)</td>
<td>3</td>
</tr>
<tr>
<td>TOTAL</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 1: Distribution of examples by topic

<table>
<thead>
<tr>
<th>Method of usage</th>
<th>Number of examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluating correctness of a student’s response</td>
<td>24</td>
</tr>
<tr>
<td>Helping a student, responding to a question</td>
<td>8</td>
</tr>
<tr>
<td>Creating examples/tasks/activities for students</td>
<td>5</td>
</tr>
<tr>
<td>Classroom management/grouping of students</td>
<td>5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>42</td>
</tr>
</tbody>
</table>

Table 2: Distribution of examples by the intended usage

**Computation**

The most frequent category is that of computation, however the examples that we clustered there are very different. They include computational shortcuts and tricks, such as how to multiply a 2-digit number by 11, attending to compatible numbers by performing computation mentally while students learn algorithms, evaluating the potential correctness of a student’s answer by estimation, and identifying a source of a student’s error, such as disregard of the order of operations. In most cases the knowledge from MFET course was used in order to evaluate a response from a student by a method different from the one used by the student. Awareness and appreciation of different approaches is elaborated upon further in the analysis of the interviews.

**Elementary Number Theory**

As “Elementary Number Theory” we clustered examples that referred to divisibility rules, prime numbers and prime decomposition. Familiarity with divisibility rules served teachers in recognizing an error in a student’s answer, such as when the difference of two odd numbers was not even, or when the sum of two numbers divisible by 9 was not divisible by 9. It further served when designing examples for student work, for example, when students are learning the long division algorithm, divisibility rules help the teacher to present students with exercises where division is without remainder, without carrying out the division. Moreover, some PTs reported that divisibility rules helped them when planning an activity that involved student work in small groups, that is, deciding on whether equal size groups were possible with a given student attendance.

**Fractions and Decimals**


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Included in this category were examples related to introducing students to different models of representing fractions and equivalent fractions, to converting improper fractions to mixed numbers, comparing fractions and performing operations. These examples referred mostly to designing pedagogy for student understanding and, again, evaluating student answers, achieved by applying a standard procedure, by different means, such as comparing fractions by attending to units and not to a common denominator.

**Geometry**

Examples related to geometry mentioned the sum of the angles in a triangle and in a quadrilateral, the concepts of area and perimeter, and the meaning of $\pi$. Knowledge from the MFET course was helpful in assessing students’ work, for example, when students measured all the angles of a quadrilateral and calculated their sum to 274 degrees, the teacher immediately recognized an error either in measurement or in addition. Examples in this category also referred to designing activities to introduce concepts. One such example replicated with elementary school students an activity previously experienced in the MFET course: It involved measuring the diameters and circumferences of different circular objects in order to introduce $\pi$ as a ratio and not “as a strange appearance in some formulas”.

**Algebra**

In this category PTs’ examples mentioned their knowledge of solving equations in order to obtain the answer “quickly” and evaluating the work of students performed without algebraic means.

The repeating themes identified in the written responses of PTs were those of understanding and awareness of different ways to perform a mathematical task. We elaborate on these themes in further detail, and add other themes, as we turn now to the analysis of clinical interviews.

### Understanding and Explaining

The most robust theme, which was mentioned in the written responses and which appeared in all of the interviews, was that of understanding. Only two participants mentioned that their MFET course helped with “revisiting” or “refreshing basic skills”. The majority maintained that in their MFET course they understood mathematical ideas, in some cases for the first time, and this personal understanding was ultimately related to the ability to explain, that is, to teach. While Tanya (below) makes an explicit connection between understanding and teaching, Betty elaborates further, starting with a severe criticism of her elementary school experience and her desire to find out why certain rules exist.

_Tanya:_ I always struggled in math for myself and taking this course helped me understand math better. You definitely need to understand that to be able to teach that.

_Betty:_ She explained all of the things that we just were taught in elementary school as this is the way it is, like this is the rule. She’d explain why. I remember when I was a kid, I was like why? Why is this the rule? And then they’d be like, because it is. And I’m like well, OK, that doesn’t help. So she explained why those rules were and I was just like, finally, I can understand it because it’s true, if you understand why the rules exist your application of them will be more accurate but also you can figure out other rules. You need to really understand why the rules exist in order for you to teach this.

Once the connection between understanding and teaching was mentioned, the interviewer invited specific examples of concepts or ideas that were “really understood” or “understood better” as a result of the MFET course experience. The mathematical topics mentioned in the interviews echoed those from the written response task. These included measurement, algorithms, fractions,
focusing on division of fractions, and multiplication tables. For limitations of space, we illustrate only the last topic in the following excerpt:

Lisa: Multiplication tables – I thought that has to be a skill that’s just memorization and drills and, actually, you can do a lot with understanding around that, like patterns and stuff.

**Multiple Ways, Questions and Connections**

Closely related to the theme of understanding is the theme of acquiring different ways to approach a mathematical task. Lisa makes this connection explicit, while Betty connects availability of different methods to understanding her students’ thinking:

Lisa: I have a deeper understanding for it and then I can see lots of different ways of doing it.

Betty: I think that I’ve learned, and this is sort of intimidates me, is that I have to open my mind to understand how other people think […] to not underestimate the students and to try really hard to understand how they came up with, to say show me…

The connection here is obvious: personal awareness of multiple strategies is helpful in trying to understand students’ strategies and approaches that may be different from the conventional ones (Leikin & Levav-Waynberg, 2009). Moreover, the ability to acknowledge different approaches in students’ work was also connected to the ability to deal with students’ questions.

Tanya: I also think it helps when they have questions, you’ll have other knowledge to draw from for questions they may have.

Betty: I don’t want to be the teacher who says that’s the way it is. […] And I remember she told us why that works, I remember she did bring that up and I was like, oh, well that totally makes sense […] If they ask I’d like to be able to provide an answer, at least guide them in the direction of finding the answer, rather than just saying this is the way.

Note that Betty makes explicit reference to her Mathematics instructor, “she told us why that works”, whereas she herself balances the ability “to provide an answer” with her pedagogical belief in the need to “guide them in the direction of finding the answer”.

**“Math today is different”**

The idea that “Math today is different”, that is, different from the mathematics they experienced as learners, was the second most prominent and unsolicited theme in the interviews, mentioned by nine participants. Several PTs mentioned that the course opened their minds, extended their horizons and influenced their views on what mathematics is about. This difference is contrasted with a description of prior experience, which is common among the participants.

Cara: When I was going through math, it was just the numbers, not the problems.

Anne: Math is completely different today. I asked my teacher, I don’t understand this and he’s like well can you do it, that’s how I remember most of my math being, all I learned was equations and if you could do the equation, if you could use the equation then you didn’t need to know anything else.

At first we considered the theme of mathematics being different, and the presented prior experiences of frustration, as not explicitly related to our research questions. However, on a second look, acknowledging and appreciating this difference – between mathematics of yesterday and that of today or tomorrow – can be considered as the main impact of a teacher education program in general and a MFET courses in particular. Participants implicitly or explicitly contrasted their experience of “doing” and being shown how to do, with the desired explanation and understanding, which was considered essential for teaching.


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Discussion

Acquiring understanding is a declared purpose of the MFET courses. For example, Musser, Burger and Peterson (2006), authors of one of the popular textbooks for such a course, state explicitly in their introduction:

“This book encourages prospective teachers to gain an understanding of the underlying concepts of elementary mathematics while maintaining an appropriate level of mathematical precision” (p. xi).

In the “Message to prospective and practicing teachers” on the first pages of their book, Sowder, Sowder and Nickerson (2010) mention different perspectives and contributions to teaching:

“Some mathematics may be familiar to you, but you will explore it from new perspectives. […] Though the course is about mathematics rather than about methods of teaching mathematics, you will learn a great deal that will be helpful to you when you start teaching” (p. xiv).

As such, our findings suggest that the MFET course, or at least the offerings of the course that our participants were enrolled in at different times and at different places, achieved the set goal, at least from the perspective of participants in this study. However, this personal perspective of participants needs to be investigated further. It cannot be concluded from the participants’ testimonies that they have indeed acquired a desirable level of what researchers referred to as PUFM - profound understanding of fundamental mathematics (Ma, 1999) or KDU – key developmental understanding (Simon, 2006), that is deemed as a prerequisite for MKT – mathematical knowledge for teaching (Silverman & Thompson, 2008). We further also note that the MFET course is only one step in the mathematics education of prospective teachers, and it is likely that ideas that developed in this course are reinforced and reexamined in courses that attend to “methods” of instruction, that is, to pedagogy and curriculum.

Taking a pessimistic view on our results, we note that the majority of participants entering a teacher education program for certification at the elementary level acknowledged that they did not sufficiently understand the concepts and procedures of elementary school mathematics. While this is consistent with prior research, our specific contribution, however, is in basing this finding on participants’ testimonials related to their understanding (or lack of it) within particular concepts and topics of elementary school mathematics, rather than on researchers’ observations.

Taking an optimistic perspective, we note that following a MFET course PTs reported that they “really understood it”, whereas “it” referred to various concepts, such as place value or fractions, or algorithms, such as column addition or division by a fraction. However, what does it mean to “really understand” something? Betty summarized this as “knowing the reasons behind all the things that you teach the kids”. Moreover, according to our participants, several related and further elaborated answers can be offered: For some PTs this means to know why and not only to know how, for others it means being able to provide an explanation to a student, and, furthermore, to be equipped with several different explanations. These views are in accord with the shift from what mathematics teachers should know to why they should know this mathematics, suggested by Asken (2008). The teachers’ (partial) answer to “why” is the implementation of knowledge in teaching.

We mentioned above that self-report of acquired understanding does not necessarily mean that an appropriate level of understanding was achieved. Nevertheless, the personal acknowledgement of the importance of understanding and the ability to explain mathematics to students, rather than provide rules, is a valuable contributor to a teacher’s comfort zone (Borba & Zulatto, 2010). Furthermore, the awareness that “math today is different”, that is, that the


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A desirable way of learning mathematics is different from the personal experience of participants, is an important step towards "teaching differently" or "teaching for understanding". We also suggest that the repeated reference to "different mathematics" may also signify a change in personal beliefs of what mathematics is about. That is to say, not that mathematics has changed, but rather the PTs' view of mathematics.

What can this imply for the teaching of mathematics? Anne's opinion is embedded in her question:

Anne: And there were all those things that you learn that you're like, why didn't we just learn it like that from the beginning, because it would have helped me so much more.

In our optimistic perspective we would like to conclude with the hope that Anne's students will "learn it like that from the beginning".

References


**PROSPECTIVE ELEMENTARY TEACHERS’ SUSPENSION OF SENSE-MAKING WHEN SOLVING PROBLEMATIC WORD PROBLEMS**

José N. Contreras  
Ball State University  
jncontrerasf@bsu.edu

Armando M. Martínez-Cruz  
California State University, Fullerton  
amartinez-cruz@exchange.fullerton.edu

This study investigates the extent to which pre-service elementary teachers (PETs) use their real-world knowledge to solve problems for which the result of the arithmetic operation is problematic, if one takes into consideration the reality of the context. A paper-and-pencil test was administered to 566 PETs enrolled in mathematics content courses. The test included 8 experimental items and 4 buffer items. The findings for a sample of 68 PETs are not very encouraging. The total number of realistic responses varied from 5 to 58 (out of 68 possible for each problem).  

Word arithmetic word problems play an important role in learning mathematics at the elementary school level. There are several practical and theoretical reasons of the inclusion of arithmetic word problems in the elementary curriculum. First, they provide contexts in which students can use their mathematical knowledge so they can develop their problem-solving abilities, an important goal of learning mathematics. Second, word problems provide practice so students can develop their abilities to use their knowledge in real-life situations. Third, word problems serve as motivators so students can see the relevance of the procedures and algorithms learned in school. Fourth, word problems have the potential to provide students with rich contexts and realistic activities in which to ground mathematical concepts and, thus, facilitate the learning of more complex concepts. Finally, word problems provide students with experiences to learn how to use mathematical tools to model aspects of reality, that is, to describe, analyze, and predict the behavior of systems in the real world (Burkhardt, 1994; De Corte, Greer, & Verschaffel, 1996; Verschaffel, Greer, & De Corte, 2000; Verschaffel & De Corte, 1997).

Some critiques (e.g., Gerofsky, 1996; Lave, 1992; Nesher, 1980) argue, however, that the mathematics curriculum fails to achieve these lofty goals because traditional instructional tasks tend to focus on a straightforward application of procedures and computations to solve artificial problems unrelated to the real world. As a result, students tend to approach word problems, more often than desirable, in a superficial and mindless way with little, if any, disposition, to modeling and realistic interpretation. Several pieces of research provide empirical evidence to these claims.