We unpack the notion of mathematics teacher educators modeling teaching in mathematics teacher education courses. Specifically, we investigated what practicing teachers gained from mathematics teacher educators’ modeling by examining: (1) What do mathematics teacher educators believe they model about effective instructional practice? (2) What do practicing teachers notice about the mathematics teacher educator’s pedagogy and identify as effective mathematics teaching? (3) In what ways do these perspectives align in mathematics courses for practicing teachers? The results provide insight into what and how teachers learn from engaging in inquiry-oriented teaching. We discuss the implications for the education of mathematics teachers.

Introduction

While research has pointed to the benefit of student-centered instruction and collaborative, inquiry-oriented learning environments in both K-12 and college mathematics courses (Boaler, 1998; Bowers & Nickerson, 2001; Goos, 2004; Rasmussen, Kwon, Allen, Marrongelle & Burtch, 2006), many mathematics courses are still taught rather traditionally. Borg (2004) points to what Lortie (1975) calls the ‘apprenticeship of observation’ to explain the prevalence of traditional mathematics instruction. With respect to teaching practices, the notion of apprenticeship of observation suggests that teachers learn about teaching long before they enter the classroom. Their education as teachers starts when they themselves are students and their experiences influence the ways in which they think about the teaching and learning of mathematics (Ball, 1988). Many teachers’ experiences as learners of mathematics, from elementary through college, primarily involved their teacher in the role of the provider of information, in what has been called a factory model of education (Callahan, 1962). Since teachers bring their own experiences as learners of mathematics to their practice, it is not surprising that the way teachers teach mathematics is not often substantially different from how they learned mathematics (Hiebert & Stigler, 2000; Sowder, 2007; Thompson, 1992). The fact that teachers were educated in a traditional system has been described as “…perhaps the greatest obstacle to these reforms” (Simon, 2008, p. 17).

It has been suggested that if teachers learn from their experiences as mathematics students then mathematics teacher educators should model desired instructional practices in teacher education and professional development programs (Borasi, Fonzi, Smith, & Rose, 1999; Loughran & Berry, 2005; NCTM, 1991; Simon & Schifter, 1991; Sowder, 2007). The implication is that an important factor in helping teachers enact these shifts is to engage them as learners in inquiry-oriented mathematics communities. How might this learning occur and...
what might be learned?

We report on instructor modeling in three mathematics courses for practicing teachers. We investigated what mathematics teacher educators believed they model, what practicing teachers noticed, and the ways in which these perspectives were aligned. The mathematics courses for practicing teachers discussed in this report are a part of a university-based professional development group. As with many mathematics professional development programs, the goal is to move teachers forward in their thinking about content and student learning so teachers can work to help increase student achievement in mathematics (Nickerson, 2010, Sowder, 2007). These professional development programs are designed to provide extra preparation for teaching mathematics by providing opportunities for teachers to deepen their content knowledge and by collaboratively reflecting on their teaching and student learning.

**Theoretical Perspective**

We view learning as inherently social and seek to account for individual perspectives within evolving social practices. We are concerned with the “negotiation of meaning.” We focus on individuals’ perceptions of the mathematics teacher educator’s pedagogy to give an account of what mathematics teacher educators believe their instruction conveys, what individual teachers notice and describe the social conditions in which the teaching acts of note were situated. Neither the social processes nor the individual’s interpretations can be considered without the other.

The notion of perceptual lived experience (Loughran & Barry, 2005) suggests that some learning occurs by “living through” experiences. Loughran and Barry give an example in which student teachers develop perceptual rather than conceptual knowledge of a situation. In their example, the student teachers as students in a classroom setting experience the emotions, images, needs, values, volitions, and frustrations of individuals in the situation, which develops their perceptual knowledge of learning environments instead of their conceptual knowledge. While this kind of learning is rather passive, the learner gleans ideas about the “how to” of the activity by simply being a part of the situation.

**Intent participation** (Rogoff, Paradise, Arauz, Correa-Chávez & Angelillo, 2003), a related, more deliberate form of learning from experiences in a situation, describes learning from keen observation and listening to ongoing activities in which the learner participates or expects to participate in the future. Both perceptual lived experience and intent participation suggest that learning occurs through the (implicit) modeling of an expert, where the expert provides an example of the required performance. From these perspectives, teachers have the opportunity to learn about the practice of teaching by engaging in an inquiry mathematics classroom.

Another way teachers can learn about the practice of teaching through modeling in teacher education is that the teacher can learn as an *apprentice* to an “expert” teacher. Here the learner is not unaware of the intentional modeling of practice. Apprentices learn through methods of observation, scaffolding, and increasing independent practice (Collins, Brown & Newman, 1989; Lave & Wenger, 1991). Student teachers and observers can be viewed as apprenticing from master teachers, teacher coaches, and teacher educators. The learner can be seen as a cognitive apprentice (Collins et al, 1989; Schoenfeld, 1992) where instructional practices are learned through observation, guided experience and participation.

In the mathematics teacher education courses in this study practicing teachers participated in mathematics classes, and thus learning about instructional practice looked...
quite different. In mathematics teacher education like the courses in this study, the mathematics teacher educators are not teaching along side teachers in teachers’ classrooms. The mathematics teacher educators must make connections to K-12 classrooms through anticipation exercises, discussions of possible trajectories in classroom situations, and reporting back experiences. In these courses, the connections often took the form of commentary by the mathematics teacher educator that was related to a mathematical activity the teachers themselves were engaged in. The mathematics teacher educators prompted discussion among the teachers, asking them to anticipate how it might apply in a classroom situation. Also, the teachers discussed their own teaching as they reported on a predetermined task that all of the teachers in the class tried with their own students, called “try-ons” in this context. This anticipation and reporting back on teaching experiences with a more experienced mathematics teacher educator can provide a means of scaffolding the learning of reform teaching.

The constructs of perceptual lived experience, intent participation, and apprenticeship share a situated perspective on learning from the milieu. They all suggest that knowledge is developed and deployed in activity and is not separable from or ancillary to learning and cognition (Brown, Collins, & Duguid, 1989). Thus, learning the teaching profession stems, at least in part, from the teaching teachers see and experience as learners and the activity they engage in as professionals. Moreover, how teachers learn about practice affects how they view the practice of teaching.

Method

In this study we observed the mathematics professional development of a large urban university in the southwestern United States. Data was collected in three cohorts: a primary elementary cohort (grades k-3), an upper elementary cohort (grades 4-6), and a middle school cohort. Our data encompassed teachers’ and mathematics teacher educators’ reflections on a year of professional development and with respect to two classes. We drew on a three-step methodology (Busse & Ferri, 2003) and complementary accounts research methodology to develop a rich account from the data (Clarke, 1997). The facets of the methodologies include: observation, interview, stimulated recall and analysis of complementary perspectives.

The data set consisted of:
1) Audio recorded pre-session interviews with four mathematics teacher educators.
2) Video of the six individual classroom sessions (two consecutive classes for each of the three cohorts).
3) Stimulated recall interviews (SRI) with the mathematics teacher educators (MTEs), one for each class session.
4) Semi-structured interviews with a subset of the practicing teachers, part of which included stimulated recall interview.
5) Researcher fieldnotes of the six classroom sessions and interviews.
6) Reflection surveys, given to all 49 practicing teachers in the three cohorts at the end of the first session.
7) Quantitative surveys, given to all 49 practicing teachers in the three cohorts at the end of the second session.

The mathematics teacher educators were interviewed pre and post observation. In pre-session interviews, MTEs were asked to reflect generally on what instructional practices they believe they model as they teach. Then classroom data was collected in two consecutive classes for each


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of the three cohorts. The class sessions were videotaped and a researcher took fieldnotes. Both the MTEs and the classroom teachers were interviewed within a day or two using stimulated recall interviews (SRI). In the SRIs, it was the interviewees that pointed to noteworthy aspects. The surveys asked all the teachers to reflect on what was modeled throughout the year.

The videos of the classroom sessions were reviewed to create a descriptive timeline of classroom events to aid in analysis. Several participants were interviewed to enable the coding and subsequent creation of an integrated data set of complementary perspectives. The initial interviews with the MTEs suggested the coding categories aligned with the NCTM Professional Standards for Teaching. The transcript of the classroom sessions were analyzed in a cyclical process of coding and search for confirming and disconfirming evidence (Strauss & Corbin, 1990) to delineate the categories of modeled instructional acts. Once we developed what we thought to be an exhaustive group of codes, we coded a few episodes separately and compared codes for inter-rater reliability. The coders were in agreement 78% of the time and discussion resolved discrepancies. The primary causes of discrepancies were related to sub-codes of the categories.

The data allowed for analysis on two levels, (1) globally on modeling throughout the year of professional development and (2) locally on modeling in two consecutive class sessions. The pre-session interviews with the mathematics teacher educators, the reflection surveys and the individual interviews with eight practicing teachers provided data on perspectives on what the mathematics teacher educators modeled about practice in general. The stimulated recall interviews with the mathematics teacher educators and the practicing teachers provided data on specific events. Both levels were important to examine. Globally, with respect to the year, because though a specific instructional act may not have occurred in a particular session or one may not have noted certain aspects of instruction in a given event does not imply the action is not common or that it has not been noted elsewhere. Locally, with respect to two consecutive sessions, is important to examine because interpretations of a specific event may vary. An analysis of both levels has the capacity to highlight how implicit and explicit modeling conveys teaching practices.

Analytical Framework

An initial analysis of the classroom videos and the interviews began with the NCTM Professional Standards for Teaching Mathematics with respect to the four categories of tasks, discourse, environment and analysis. Using a cyclical process for confirming and disconfirming evidence (Strauss & Corbin, 1990) the pre-session interview data and the classroom data (fieldnotes and classroom video) informed the elaboration of the four categories derived from the NCTM Professional Standards for Teaching Mathematics. This process allowed for the emergence of parallel categories for describing the mathematics teacher educators’ pedagogy. The parallel categories characterize the mathematics teacher educators’ pedagogy as the participants are engaged as mathematics learners and as mathematics teachers. The emergent codes were used to categorize and coordinate the mathematics teacher educators and the practicing teachers’ perceptions of classroom events.

Teachers’ learning about instructional practice in mathematics teacher education is often not restricted to learning about classroom mathematics instruction. There are also opportunities for teachers to learn about other facets of the teaching profession. The current reform movement in mathematics education has a strong underlying theme of the professionalism of teaching. This view recognizes the teacher as a part of a learning


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community that continually fosters growth in knowledge, stature, and responsibility (Dufour, 2005; NCTM, 1991, Author). Reform recommendations suggest that teachers ought to collaboratively plan instruction, reflect on practice, create and reflect on new practices, and support one another’s professional growth (NCTM, 1991). Such interactions allow teachers in a school and its administrators to continuously seek and share learning and then act on what they learn. This type of communication and collaboration with a focus on inquiry about student learning are important aspects of what researchers call a “professional learning community” (Dufour & Eaker, 1998; Astuto, Clark, Read, McGree & Fernandez, 1993; Hord, 1997).

Results

An initial analysis of the pre-session interview data revealed that the mathematics teacher educators modeled more than the teaching of mathematics, they also modeled participation in the collective inquiry process of a community of mathematics teachers. The mathematics teacher educators were not explicit in their discussion about modeling the collective inquiry part of the teaching profession; however, their discussion related to developing the teachers’ pedagogical content knowledge pointed to the teachers’ engagement as professionals in the mathematics teacher education sessions. The emergent framework, summarized in table 1 (see next page), characterizes the mathematics teacher educators’ pedagogical moves with respect to the participants’ engagement as learners and teachers.

This first result, the emergent analytical framework, made it possible to characterize the mathematics teacher educators’ pedagogy as described by the mathematics teacher educators and participating teachers and observed by the researchers. These characterizations describe the mathematics teacher educators’ pedagogy as they engage teachers as learners and as professionals.

Teachers can learn about the practice of teaching from MTE modeling. In the Reflection Survey 1 and teacher interviews, the participants noted teaching actions related to the categories worthwhile tasks, discourse, tools, and learning environment in the analytical framework. Reflection Survey 1 asked the teachers to describe what they have done in their classes to support student learning drawing from what they have learned in prior mathematics content sessions. The teachers reported that they had incorporated specific activities and tools that they found useful in their own learning in the sessions. The teachers reported that they had adopted practices like engaging students in mathematical conversation (discourse), specifically, through questioning and prompting students to share out and provide justification. The teachers also reported incorporating wait-time and letting students struggle with difficulty, facets of the learning environment from the analytical framework.

While both the mathematics teacher educators’ and the participants’ utterances point to the mathematics teacher educators’ pedagogy as the participants were engaged as both learners of mathematics and as mathematics teachers, the emphasis was on participants’ engagement as learners, as opposed to their engagement as teachers. This was more so the case for the teachers than for the mathematics teacher educators. The teachers pointed to the pedagogy as they were engaged as a learner in about 86% in their reflections on the year and 88% in their reflections on the two sessions.

The teachers in the primary and upper elementary cohorts noted pedagogy related to their engagement as learners almost exclusively, while the middle school teachers reported it in only about 71% of their coded utterances. On average in the pre-session interviews where the
mathematics teacher educators reflected on the year, they emphasized mathematics pedagogy they believe they model as the teachers were engaged as learners in about 71\% of the coded utterances. In the stimulated recall where the mathematics teacher educators reflected on the two sessions noted such engagement in about 78\% of the coded utterances. Like the teachers, the mathematics teacher educators from the elementary cohorts more frequently discussed instructional acts where the participants were engaged as learners. The middle school cohort facilitator, MTE-Karla, emphasized pedagogy related to the participants’ engagement as teachers and as learners almost equally, reflecting the dual purpose of the mathematics teacher education. This points to the likelihood that the difference is due to the intended purposes of the mathematics professional development sessions and the mathematics courses.

The pedagogical moves most noticed by both the mathematics teacher educators and the teachers were characterized under the categories of tasks and discourse. The mathematics teacher educators and the teachers highlighted the way the mathematics teacher educator modeled the use of tasks, specifically noticing tasks that problematized the mathematics and prompted the teachers to make connections between mathematical ideas and among representations. These tasks were seen as promoting discussions of the mathematics. Both the mathematics teacher educators and the teachers pointed to the scope of tasks, particularly the large amount of time spent on a single task devoted to developing mathematical concepts in learning situations.

Table 1: Summary of Analytical Framework.

<table>
<thead>
<tr>
<th></th>
<th>Classroom Community of Mathematics Learners</th>
<th>Community of Mathematics Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tasks</strong></td>
<td>Worthwhile <em>Tasks</em> are the projects, questions, problems, constructions, applications, and exercises in which teachers engage intended to develop teachers’ mathematical content knowledge.</td>
<td><em>Tasks</em> are the projects, questions, problems, constructions, applications, and exercises in which teachers engage intended to develop the teachers’ understanding of the students’ mathematics.</td>
</tr>
<tr>
<td><strong>Discourse</strong></td>
<td>Classroom <em>Discourse</em> refers to the ways that mathematics teacher educators mediate discourse about mathematical ideas to focus discussion on concepts and solution paths instead of answers.</td>
<td>Classroom <em>Discourse</em> refers to the ways that mathematics teacher educators mediate discourse about practice to focus discussion on teaching, student learning and/or thinking.</td>
</tr>
<tr>
<td><strong>Tools</strong></td>
<td><em>Tools</em> are objects, tangible or intangible that the MTE and/or teacher’s use in learning situations to reason, make connections, solve problems, communicate and enhance discourse.</td>
<td><em>Tools</em> are objects, tangible or intangible that the MTE and/or teachers use to reason or enhance learning about or discussion of lesson planning, student learning, thinking and understanding, and so on.</td>
</tr>
<tr>
<td><strong>Environment</strong></td>
<td><em>Environment</em> represents the setting for which the development of each teachers mathematical power is fostered.</td>
<td><em>Environment</em> represents the setting for which the development of each teacher’s knowledge for teaching is fostered.</td>
</tr>
<tr>
<td><strong>Analysis</strong></td>
<td>Ongoing <em>Analysis</em> is the systematic reflection in which mathematics teacher examine relationships between what they and their students are doing and what students are learning.</td>
<td>Ongoing <em>Analysis</em> is the systematic reflection in which mathematics teacher educators engage in analysis to foster teachers’ participation in the Collective Inquiry Process</td>
</tr>
</tbody>
</table>

While the teachers pointed to tasks and discourse, some pointed to how the coordination of the pedagogical moves related to structuring learning environment, specifically structuring the sessions. The teachers’ discussion about discourse often pointed to discourse in task situations.


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collaborative work on tasks and sharing ideas. The teachers specifically mentioned how the mathematics teacher educator pushed for justification and asked questions as opposed to giving answers. As a part of this, the teachers did point to a facet of the learning environment category, reporting that the mathematics teacher educators allowed them to struggle with difficulty. The teachers characterized the mathematics teacher educators’ instruction as Socratic, that is, the mathematics teacher educators asked questions that helped them shape their thinking and move them forward in their thinking. The teachers reported that they try to incorporate these facets of the mathematics teacher educators’ pedagogy in their own teaching.

There are aspects of ongoing analysis that the mathematics teacher educators believe they model that are not noticed by the teachers. There is a belief among the mathematics teacher educators and the teachers that a deeper understanding of the mathematics content better prepares teachers to present challenging tasks and ask questions that help students move forward in their thinking. However, the mathematics teacher educators’ report that one of the purposes of questioning as the teachers are engaged in tasks is ongoing analysis of the teachers’ thinking. The mathematics teacher educators note that their questions are driven by the need to develop conceptual models of the teachers’ thinking. The teachers did not note developing conceptual models as a reason for being able to ask “good” questions; the teachers pointed to experience, mathematical knowledge, and being well prepared for the lesson as key to the mathematics teacher educators’ ability to ask questions that move them forward in their thinking. This perhaps points to one of the reasons for differences in the emphasis the mathematics teacher educators and the teachers put on the category of ongoing analysis.

Conclusions

In general the mathematics teacher educators and practicing teachers reported that the mathematics teacher educator modeled student-centered instruction as conveyed in the NCTM Professional Standards for Teaching Mathematics (1991), but also facets of the collaborative work of teachers outside of the classroom. This work outside of the classroom might include teachers collaboratively planning instruction, reflecting on practice, creating and reflecting on new practices, and supporting one another’s professional growth.

This research informs the body of knowledge about teaching the practice of teaching mathematics. The work described in this report makes recommendations regarding modeling by mathematics teacher educators based on an analysis of empirical data. It provides insight into what and how teachers learn from modeling reform teaching. Specifically, the study draws on an analysis of data from three classrooms to illustrate how explicit discussion about mathematics teaching and implicit modeling of instruction supported practicing teachers’ noticing of the instruction they experienced, and how engaging in facets of the teaching profession that take place outside of their interactions with students has the potential to foster the enculturation of teacher into a professional learning community of mathematics teachers.

References


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