WHERE'S THE PROOF? PROOF IN U.S. HIGH SCHOOL GEOMETRY CONTENT STANDARDS

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This study examined High School Geometry standards from recent individual states' and the Common Core Standards (CCSSM) featuring reasoning and proof. We examined specificity of content and use of proof or reasoning as a process in the language of standards. Results indicated standards with proof as its primary process are more likely to have higher specificity of content, and vice versa. Further, the CCSSM were found to have higher content specificity and more consistent use of proof as a primary process than the vast majority of states' standards.

Background and Objectives

Since the publication of *A Nation at Risk* almost 30 years ago (NCEE, 1983), an ever increasing emphasis has been given to assessment and content standards. The publication of the *Curriculum Standards for School Mathematics* by the National Council of Teachers of Mathematics (1989) helped initiate development of content standards written by individual states (Porter, 1994). The NCTM Standards (1989) did not have a specific standard strand for proof but included it within the Reasoning Standard. Some noted this absence, noting that proof "is the backbone of mathematics" (Wu, 1996 p. 1534), and after many "mathematicians objected to what they considered…an under emphasis on proof" (Kilpatrick, 2003, p. 1), the following *Principles and Standards for School Mathematics* (NCTM, 2000) included a more prominent role to mathematical proof across the grades through the more explicit Standard on "Reasoning and Proof." In spite of this greater emphasis on proof, some reports on individual states' mathematics content standards have shown either a lack of emphasis on proof and reasoning or vague descriptions of its use by students (Klein et al., 2005; Raimi & Braden, 1998; Reys et al., 2006). In 2010 yet a new set of standards is being put forth in the form of the *Common Core State Standards for Mathematics* (CCSSM) (CCSI, 2010b). These Standards uphold all of the NCTM Process Standards including Reasoning and Proof; they also underscore constructing and critiquing arguments as a mathematical practice for students to learn. Given past reports concerning the role of proof and reasoning in state content standards, and considering the implementation of this new set of standards across the nation, we argue that it is useful to examine both the emphasis and specification of proof and reasoning in the various states' content standards, as well as those of the CCSSM. We have chosen to focus on standards concerning High School Geometry both for the historical importance proof has taken in American Geometry curriculum (Herbst, 2002) and for the sake of clarity in comparisons of both proof and reasoning in standards across the U.S.

Theoretical Perspective

Content standards have been developed in the U.S. for at least a century (Porter, 1994), and since the publication of *A Nation at Risk* (NCEE, 1983), the pressure for standards has increased dramatically. Porter (1994) notes that the publication of the NCTMs' *Curriculum Standards for School Mathematics* (NCTM, 1989) spurred states into writing their own content standards In addition to NCTM, various organizations have focused on developing and examining standards,

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including the National Center on Education and the Economy (NCEE), Council for Basic Education (CBE), the Mid-continent Regional Laboratory (McRel), the Council for Chief State School Officers (CCSSO), the Thomas B. Fordham Foundation, and the American Federation of Teachers (AFT). These and other organizations provided specific standards, guidelines and/or evaluations of standards as various states continued to publish and edit their documents (Porter, 1994). Of particular interest for the current investigation are the evaluations that have been conducted of different state mathematics standards, as these will provide a frame of reference to both the historical development and evolution of such evaluations as well as the development of reasoning and proof in mathematics content standards, with specific focus on secondary Geometry.

The first evaluation of state standards was conducted by the AFT (Gandal, 1996): The report examined whether states were developing standards, how such standards were assessed and implemented, and the detail provided within the different standards. That report concluded that 30 states had sufficiently clear and specific mathematics standards. The organization's most recent report in 2001 concluded that 44 states had clear and specific mathematics standards (AFT, 2001). The CBE produced a similar finding in their 1998 report *Great Expectations?* (Joftus & Berman, 1998), suggesting that only three states (Alaska, Montana, and Nebraska) had a low level of rigor in their mathematics content standards. All of these reports have provided general descriptions of the mathematics standards and not details regarding specific topics, in particular, not specific accounts of the role of proof or mathematical reasoning in state standards.

In contrast, the Fordham Foundation sponsored a report on state mathematics standards that did provide detail on specific mathematics content. The first of these reports (Raimi & Braden, 1998) examined the standards of 46 states and the District of Columbia. Contrary to reports by AFT and CBE (Gandal, 1996; Joftus & Berman, 1998), Raimi and Braden (1998) found that many states were not providing clear or specific standards. Specifically when focusing on mathematical reasoning, the report cited only five states as having a sufficient degree of clarity and focus. Further, the report notes that only 17 states explicitly indicate that proof should be taught in mathematics. A more recent report by Klein et al. (2005), suggests that this trend has not changed much in that "the majority of states fail to incorporate mathematical reasoning directly into their content standards…many state documents do not ask students to know proofs of anything in particular. Few states expect students to see a proof of the Pythagorean Theorem or any other theorem or any collection of theorems" (p. 21).

While not focusing on High School Geometry, Reys et al. (2006) provide a similar and concurrent finding to Klein et al. (2005), stating that "reasoning is not well articulated or integrated across K-8 standards documents…most state standards fail to address reasoning aspects in a thorough and comprehensive manner across grade levels and content strands" (Reys et al., 2006, p. 9). Given that Reys et al. examined specific elements of mathematical content in a comparable manner to Klein et al., it appears that many state standards do not provide clear or specific standards on mathematical reasoning.

The reports thus far referenced provide two types of reports, those that paid specific attention to the way mathematics was described in standards (e.g., Klein et al., 2005; Reys et al., 2006) and those that did not (e.g., Gandal, 1996; Joftus & Berman, 1998). This breakdown generally describes the reports that appear most prominent in the literature. Further, there tends to be a general depiction of standards as 'mostly good' in earlier years, and 'mostly in need of improvement' in recent years. These criticisms added to the lack of commonalities have led to the development of the Common Core Standards.

The Common Core Standards

In June 2009, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) announced a new initiative to develop a common set of standards for mathematics and language arts. This initiative was called the Common Core Standards Initiative (CCSI), and included 46 states, the District of Columbia, and two U.S. territories (CCSI, 2010a). As of June 2, 2010, this total included two additional states and a working draft of the standards had been developed. The standards were developed to be "fewer, clearer, and higher" (CCSI, 2010b, p. 1) as compared to previous standards in many states. Among the criteria used in designing the standards are the NCTM process standards and the strands of mathematical proficiency from the National Research Council's report *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001; see also CCSI, 2010b). Yet, numerous citations to various reports, assessments, and standards are cited as works consulted (see CCSI, 2010b for a full description). A review of the documents and descriptions of the Common Core Standards by CCSI suggest a specific focus on a number of topics that have surfaced in mathematics education, including mathematical modeling and quantitative literacy. Continued reference to reform-oriented documents and sources also convey the notion that the Common Core Standards are reform-oriented. Additionally, some organizations, such as NCTM "diligently monitored the development of CCSSM and advised NGA and CCSSO throughout the process" (p. ix, NCTM, 2010). Yet, CCSI (2010c) specifically states that the Common Core Standards are a set of expected learning goals for students and do not constitute a curriculum nor do they advocate specific materials or teaching practices to be used. This appears to be a different goal for designing content standards than some states have used, where content standards were written as documents intended to reform teaching more so than specify content (see Thompson, 2001 for a relevant description).

As is customary with newly published standards documents, a large amount of skepticism and criticism has been levied against CCSI. For example, Zhao (2009) stated that "the Common Core Standards Initiative or any such movement to create national standards risks America's future by destroying its traditional strengths in cultivating a diverse and creative citizenry" (p. 52). Tienken (2009) expressed concerns that the Common Core Standards would only serve to increase an 'achievement test drive mentality' in the U.S. Providing a more even-handed critique, Porter, McMaken, Hwang, and Yang (2010) compared the Common Core Standards to the state standards in place and found that "the common core does represent considerable change…" but are not "…more focused as some might have hoped" (p. 8). However, Porter et al.'s description of their analysis does not provide content-specific focus in regards to reasoning or proof. Therefore, it appears there is a need for a transparent and clear examination of the way that reasoning and proof are represented in the Common Core Standards as compared to the existing state standards.

A main objective of the CCSSM (2010b) is to provide mathematics standards that are clear, specific, and rigorous. According to some reports (Klein et al., 2005; Raimi & Braden, 1998) many of the states' standards the CCSSM will be replacing, however, have not provided clear or specific standards regarding proof or reasoning in High School Geometry. Yet, since Klein et al.'s (2005) depiction of proof and reasoning in various states' standards, many states have published revisions of their standards, and therefore may have provided more sufficiently clear and specific expectations of students regarding reasoning and proof. Additionally, there is currently no available examination of reasoning and proof in the High School Geometry standards in CCSSM. In this paper we provide a current and comparative account of the depiction of reasoning and proof in the CCSSM and various states for High School Geometry. In order to accomplish this goal, the following research questions will be examined:

1. To what extent do different state content standards describe reasoning or proof in association with specific mathematical content?
2. To what extent do different state content standards characterize reasoning or proof as processes engaged by students?
3. Is there a relationship between the extent to which reasoning or proof are characterized as processes that students engage in and the extent to which such reasoning and proof standards refer to specific mathematical content?

Methods

Standards documents from all 50 states were collected and coded for analysis. These documents were collected from the websites of each state's department of education. However, various states have different formats for organizing their standards in High School mathematics, such as by course, by grade band or benchmark, or by specific grade level. Given our focus on High School Geometry, it was decided that standards in the format of a course would have the Geometry course standards examined. For those states with benchmarks or grade level formats, the standards for the Geometry strand (or equivalent) was used for the full range of High School standards provided by each state. In some cases, advanced level standards were provided. To ensure a fair comparison, we examined only those standards expected of every student. In addition to taking into account varying format and structure, different states may represent the same content with a different number of standards. In other words, one state may say in one standard what another state says in three. This phenomenon was taken into account by recording the number of High School Geometry standards each state had, and using this number to calculate mean emphasis on reasoning and on proof.

As our focus was on the presence and emphasis of standards outlining mathematical reasoning and proof in Geometry, we examined the standards documents for the presence of standards that referred to these mathematical processes. For proof, standards were identified through the use of words such as proof(s), proving, or prove present in a standard's statement. Explicit use of these words was necessary for the standard to be counted as a "proof" standard, as this ensured that such a standard would be interpreted by those using the document in practice as referring to proof. A similar process was used for coding reasoning, by using words referring to logic as indicators of such standards. Words such as inductive, deductive, conjecture, logic, reason, and their cognates were used as indicators. Once standards were identified as being a proof and/or reasoning standard, they were coded for their specificity to which they referred to the mathematics concepts involved.

As noted by certain reports (e.g., Klein et al., 2005; Reys et al., 2006) many standards lacked a degree of specificity in descriptions of mathematical reasoning and proof. Rather, a state may provide a number of standards requiring that proof be taught, but may not specify what mathematical content should be associated with proof. Therefore, we developed three categorizations to distinguish the degree of content specificity in a standard. The first, low specification, is a standard that makes either no reference to a particular mathematical concept, or where such specification is vague, e.g., New York standard "*Write a proof arguing from a given hypothesis to a given conclusion.*" A standard with medium specification is a standard that makes reference to a set of mathematical concepts, but still lacks a degree of specificity, e.g., CCSSM standard "*Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.*" While this example indicates the action of proving

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relationships in the area of triangle congruence and similarity, the standard does not specify any theorems or specific properties. Standards coded as having high specification were standards that make reference to a set of mathematical concepts with specific identification of such concepts, e.g., Michigan standard "*Know a proof of the Pythagorean Theorem, and use the Pythagorean Theorem and its converse to solve multi-step problems.*" This example makes reference to a specific concept, the Pythagorean Theorem, rather than a category or set of concepts.

In addition to coding for concept specification, each standard was examined for transitivity. In Systemic Functional Linguistics (Halliday & Matthiessen, 2004) transitivity is a system of linguistic resources with which language supports the ideational metafunction (i.e., the way language represents the world). Transitivity permits to represent the world as sequences of processes involving participants and circumstances. Canonically, processes are realized by verbs and participants by nominal groups. To illustrate our analysis of transitivity note, for example, Delaware standard "*Reason deductively to justify a conclusion or to create a counter-example;*" this contains three verbs (reason, justify, create), but *reason* represents the transitive process associated with this standard. Similarly, in Delaware standard "*Use appropriate technologies to model geometric figures and to develop conjectures about them*" the verb *use* represents the transitive process for the standard. What interests us here is the difference between these two examples in the way that reasoning is expressed as an expectation of students. Whereas *to reason* is the actual process students are expected to engage in the first standard, the second standard expects students *to use* technologies in specific ways, one of which is reasoning. Reasoning in the second standard acts as what Halliday and Matthiessen (2004) would refer to as a participant in the clause. As can be noted in the two examples given, other verbs can represent transitive processes for clauses nested within the main clause (clause complex). To account for such use of proof and reasoning terms as transitive processes and/or participants, we ranked such usages of identified words and terms in regards to the degree to which they were a primary process.

The Pennsylvania standard "*Write formal* ***proofs*** *to validate* ***conjectures*** *or arguments*" contains the primary process of *to write*, which we coded as having a distance of zero from the primary process. *Proofs* is a participant in this clause, as is *to validate conjectures*. However, *proofs*, while not a primary process, is relatively closer to the primary process than *conjectures*. Therefore, we coded *proofs* as being a distance of one away from the primary process, and *conjectures* as a distance of two away from the primary process. In this way, we accounted for the semantic emphasis placed on proof and reasoning in various standards.

Results

The number of standards for High School Geometry varied dramatically from state to state, with some states having as few as 5 and others having as many as 74. The Common Core Standards included 42 individual standards for High School Geometry while the mean for states was 21.98 (SD = 15.47). This means that the Common Core Standards do indeed have more standards for Geometry than most states, although some states do have more.

Approximately 24% of the CCSSM Geometry standards (n = 10) featured proof while the average for states was 11.7% (SD = .118), indicating that CCSSM provided proportionally more standards featuring proof than the typical state document. The CCSSM had an average specificity score of 2.90 for proof standards while the mean specificity score for states' proof standards was 1.88 (SD = .59). This indicates that the CCSSM standards featuring proof tended to be more specific in regards to mathematics content than the typical state document. The CCSSM proof standards had an average transitivity distance of 0.10, while the average transitivity distance for the various states' standards featuring proof was 1.24 (SD = .60). This indicates that the CCSSM proof standards tended to treat proof as the primary transitive process while various states did not do so as consistently. Taken altogether, the CCSSM provide a greater emphasis on proof in their Geometry standards than the most current state standards, and also provides more specific and clear standards with a strong emphasis on the action of proof. While some states showed stronger tendencies on at least one of the three criteria (overall emphasis, specificity of content, transitivity distance), no state showed higher scores on more than one indicator. Some states did appear somewhat comparable to the CCSSM proof standards: California, Florida, Indiana, and Michigan. With the exception of these states, the CCSSM in proof for High School Geometry show clear and definitive improvement in specification and increase in focus.

While the CCSSM show an increased focus on and specification of proof in High School Geometry, this trend was not as evident when examining other forms of mathematical reasoning. The CCSSM emphasized logical reasoning in mathematics in 5% of their standards. While the mean for states was a 14% emphasis, there was a large degree of variance (SD = .14). However, the CCSSM Geometry standards that featured reasoning showed a transitivity distance of 1.00, while the average for varying states was 1.26 (SD = .42). Further, only two states had lower averages than the CCSSM regarding transitivity distance, indicating that the CCSSM was similar to many states in regards to specifying reasoning as the primary process in its standards. Additionally, the CCSSM appear to provide more specificity (M = 3.00) than did the typical state (M = 1.68, SD = .81) regarding mathematics content in standards featuring reasoning for High School Geometry. Overall, the descriptive statistics for High School Geometry standards featuring reasoning indicates that CCSSM provides fewer relative standards, but with more specificity and a slightly stronger emphasis on the act of reasoning.

Correlation analysis between standards' content specificity and transitivity distance indicated a strong and negative relationship for standards featuring proof ($\rho = -.42$, $p < .05$). This indicates that the closer the distance between proof and the transitive process, the more specific a standard tended to be regarding mathematics content. However, when examining the same relationship for standards featuring reasoning, virtually no relationship was found ($\rho = .00$, $p = .98$). This indicates that an excessive amount of variance is present in many of the states' content standards regarding the specification of mathematics content and the use of reasoning as a transitive process. Practically speaking, the correlation found for proof standards indicates that standards which describe proof as the primary process students are expected to engage in tend to also provide more specific connections to mathematics content, and vice versa. However, the near zero correlation found for the same relationship in reasoning standards indicates little consistency in the way reasoning is described in various High School Geometry standards across the nation.

Discussion

While many concerns have been expressed regarding the CCSSM (e.g., Porter et al., 2010; Tienken, 2009; Zao, 2009), the results presented here may give cause for some mindful consideration of potential benefits of the CCSSM in regards to reasoning and proof, at least as it concerns High School Geometry. The results presented here indicate that the CCSSM provide a greater emphasis on proof with more specificity to the content and a clearer representation of the process than the vast majority of states' standards. Further, while the CCSSM does not have, proportionally, as many High School Geometry standards featuring forms of reasoning other than proof, the standards tend to be more specific to the content and have a clearer representation of the process than the majority of states. Therefore, in regards to proof and reasoning in High School Geometry standards, the CCSSM appear to show an improvement over state standards in general.

By articulating specific aspects of mathematics content, the CCSSM provide a clearer message to what teachers are expected to have High School students prove and reason about. For example, the CCSSM provides a list of theorems for students to prove regarding triangles (e.g., base angels of isosceles triangles are congruent; the medians of a triangle meet at a point). The CCSSM do not indicate how teachers are expected to engage students in such proofs or in what format students should write those proofs. Teachers are provided not with a blanket expectation of having students "construct proofs about triangles" that might be met with token proof exercises whose conclusions are not memorable (Herbst, 2002); rather they are provided clear guidelines that tie proof practices to important content that students need to learn. As such, we consider this a general improvement over many of the state content standards currently in use.

Another aspect of the CCSSM High School Geometry standards featuring proof that we consider beneficial to both teachers and students is the predominate use of *prove* as the primary transitive process. With the exception of one out of ten standards featuring proof, the CCSSM are consistent in emphasizing proof as the primary process. The benefit of writing "*prove theorems about lines and angles*" rather than "*write proofs about lines and angles*" is the activity conveyed by one wording over the other. Many states use words such as *write*, *construct*, *know*, or *use* as the primary process for standards featuring proof, and this leads to potentially different interpretations of what students are expected to do. Do students memorize proofs when the primary process is *know*? Does their engagement in proof come in the form of writing when the primary process is *write*? While these questions are rhetorical, they demonstrate the bias that can come when using words other than *prove* as the primary process for standards concerning proof. As such, the CCSSM does not convey bias in the way students are expected to engage in proof, but only in the content they are expected to prove.

The results of the present study provide evidence in support of the adoption of the CCSSM in regards to High School Geometry standards featuring reasoning and proof. However, the present analysis is narrow in focus, and several other mathematical strands and grade levels might need to be examined in a similar manner to properly understand both the potential benefits and drawbacks of implementing CCSSM. For example, while the present results concerning proof standards were generally positive, the results concerning reasoning standards indicate a need to use reasoning as the primary transitive process in CCSSM.

The results of the current study do support the implementation of CCSSM in regards to High School Geometry standards featuring proof and reasoning, but our discussion and conclusions should not be taken as a wholesale endorsement of the CCSSM document. Rather, as specified, further examination of CCSSM is needed. Further, by examining specific aspects of the CCSSM document, rather than an overarching depiction of the document as a whole, we are better able to either validate or make recommendations for improvement out of a solid base of knowledge.

Author Note

The work reported in this paper was supported by NSF grant DRL-0918425 to Patricio Herbst. All opinions are those of the authors and do not necessarily represent the views of the Foundation.

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