TEACHER PERSPECTIVES ON MATHEMATICS CONTENT AND PEDAGOGY: DESCRIBING AND DOCUMENTING MOVEMENT

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In this paper we report on a study examining how teacher perspectives on mathematics content and pedagogy changed after the teachers’ schools participated for one year in the Mathematics Coaching Program. Teacher narrative responses to questions about student work samples provided qualitative data for analysis. Questions asked for teacher input on student thinking and instructional decisions. Responses reveal teacher change in the area of the program goals of movement toward an integrated procedural/conceptual perspective on content and a learner responsive perspective on pedagogy.

Purpose of the Study

We know that teachers bring to their classroom practice many factors that influence their pedagogy. They bring knowledge, skills, and understandings of mathematics that impact student learning (Adler & Davis, 2006; Ball, Lubienski, & Mewborn, 2001; Fennema & Franke, 1992; Hill & Ball, 2004; Hill, Rowan, & D. Ball, 2005). They also bring into their classrooms perspectives on the nature of mathematics and how it should be taught. Perspectives on mathematics include whether it is a procedural or a conceptual activity, whether it is necessary to know mathematics both conceptually and procedurally, and whether there is some combined way to know mathematics (Baroody, Feil, & Johnson, 2007; Star, 2005). Perspectives on mathematics pedagogy include whether it is better taught with a teacher-directed or more learner-responsive approach, or if one can use and apply both approaches. In this paper we report on a study exploring teacher change in terms of perspectives on mathematics and pedagogy with teacher participation in a professional development project. We examine these particular perspectives because the program serving as the context for this work has among its goals to support teacher movement along a continuum from a strict and superficial procedural perspective on content to a more richly connected and integrated procedural/conceptual perspective; and along the pedagogy continuum from a teacher-directed to a learner responsive perspective.

In the following paragraphs, we define the perspectives on mathematics and pedagogy that are central to this work, and review the literature on which those definitions are based. We then describe the methods utilized in our study, including the context of the work, the nature of the data, and data analysis procedures. Finally, we discuss findings and closing thoughts.

Theoretical Framework

Supporting teachers’ practice toward students’ mathematics learning necessitates consideration of multiple concepts brought to the teaching of mathematics. The area of teacher mathematics content knowledge already has a deep base in the literature. What we bring to the discussion are teachers’ perspectives on mathematics content and pedagogy is a distinction between perspectives and what is generally understood in the literature as ‘disposition’. Scholars define dispositions as traits that lead a person to follow certain choices or experiences (Damon, 2005) or as tendencies to exhibit frequently a pattern of behavior directed to a broad goal (Katz, 1993). For Gresalfi and Cobb (2006), the word “disposition” encompasses ideas...
about, values of, and ways of participating with a discipline, identifying with it, and how it is realized in the classroom. A dictionary definition of disposition describes it as “a person's inherent qualities of mind and character” (New Oxford American Dictionary, 2005). Similarly, a perspective is defined as “a particular attitude toward or way of regarding something; a point of view; the state of one’s ideas” (Oxford Dictionaries, 2010). Perspective, although sometimes related to attitudes and beliefs in the literature, is usually recognized as being a result of experience, and that it can change, and that it influences practice. (e.g. Ross, 1986; Ross & Smith, 1992; Zeichner & Tabachnick, 1983).

In our work we did not chose to name the mathematics and pedagogy concepts we study dispositions because definitions of dispositions as cited above suggest less the likelihood that they can change than does the meaning of perspective. As educators, we believe that the perspectives teachers bring to bear on mathematics and pedagogy are not “inherent” but have been learned through lived experiences in and out of school. Additionally, our work in professional development relies in part on the belief that professional development can result in teacher change in terms of perspectives on content and pedagogy.

Mathematics Content and Pedagogy Perspectives

Contrary to what many in the general public believe, the content of mathematics is much richer than only arithmetic or computation; and learning mathematics content with understanding is a more complex endeavor than merely knowing the “how to do” of mathematics (Heibert et al., 1997). These and other perspectives are on the opposing “sides” of what Jon Star (2005) calls the “so-called math wars” (p. 404), about which he cites Judith Sowder’s statement that “Whether developing skills with symbols leads to conceptual understanding, or whether the presence of basic understanding should precede symbolic representation and skill practice, is one of the basic disagreements” (1998, as cited in Star, 2005). Star clearly advocates for procedural understanding, but does so from the position that both procedural and conceptual understandings are viewed too simplistically: Conceptual understanding as rich and concrete and procedural as superficial and lacking connections. Star suggests a framework where both knowledges are studied for their rich, connected, and deep relationship and integrated qualities.

Baroody, Feil, and Johnson (2007) suggest a conceptualization that is consistent with Star’s “recommendation to define knowledge type independently of the degree of connectedness” (p. 123). Baroody et al. propose the following definitions of procedural and conceptual knowledge:

a) Procedural knowledge refers to “mental ‘actions or manipulations’, including rules, strategies, and algorithms, for completing a task” (de Jong and Ferguson-Hessler, p. 107 as cited in Baroody et al., p. 123);

b) Conceptual knowledge is “knowledge about facts, [generalizations], and principles” (de Jong and Ferguson-Hessler, p. 107 as cited in Baroody et al., p. 123).

Baroody et al. (2007) distinguish their conceptualization with degrees of depth/superficiality, connectedness, and mutual dependence/independence and note: “depth of understanding entails both the degree to which procedural and conceptual knowledge are interconnected and the extent to which that knowledge is otherwise complete, well structured, abstract, and accurate” (p. 123). We take our content perspectives from this literature, and assign a range of conceptualizations, from procedural to integrated procedural/conceptual, to form a continuum of perspectives on mathematics content for our study.

The NCTM, particularly by way of its standards publication (2000), puts forth a vision of school mathematics “where all have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all … Knowledgeable teachers have adequate resources …


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The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding” (NCTM, 2000, p. 3). This vision, which includes learning mathematics with understanding (Heibert et al., 1997), has been embraced by much of the mathematics education community. It is in direct contrast to what one might describe as traditional teacher-directed mathematics instruction that often includes less visible engagement in the learning process. That is not to say that students are not cognitively engaged; but it does mean that students are not communicating mathematical ideas, not drawing on reflective practices and deep understanding. Instead, the teacher directs students on what to do, on what and how to learn, playing the dominant role; and students respond to the teacher by following instructions (Eccles, Midgley, & Alder, 1984; Gmitrová & Gmitrov, 2003).

Student-centered instruction is that which is “[d]esigned to elicit and build on students’ ways of understanding mathematics” (Empson & Junk, 2004, p. 122) and is often problem-based (Ma & Zhou, 2000). Student-centeredness includes the teacher talking less and the learner talking more, with the learner doing the mathematical thinking and having opportunities to self-correct and generate knowledge through rich mathematical practices.

The project context of our work had a goal to move teachers beyond student-centered instruction to learner-responsive pedagogy (LRP). As in student-centered pedagogy, the LRP teacher makes decisions based on the learner’s interests and focused on the learner’s active engagement in the lesson. But LRP includes two additional distinctive qualities a) a shift, or expansion of each constituents’ responsibilities and roles in the learning process from teacher as authority to authority shared by teacher and learner; and b) action: a resulting and deliberate instruction based on teacher knowledge of learner thinking and understanding. On-going analysis of student thinking is a fundamental component of LRP so instructional decisions can be made in direct response to the learners’ understanding and action in making pedagogical choices based on the learner’s needs is an intentional move. These pedagogy perspectives form a continuum from teacher-directed to learner responsive perspectives in our study.

Methodology

Participants, Sampling, Context, and Data Source

Participants in this study are teachers in schools enrolled in a mathematics coaching program during the 2008-2009 academic year. All are certified or licensed teachers, and are credentialed to teach in any of grades one through eight; some also are credentialed to teach kindergarten. They teach in elementary, intermediate, or middle schools served by the coaching program. Teachers were free to choose whether or not to participate in the research, and are solicited to allow access to their data from the coaching program for research and evaluation purposes.

In the 2008-2009 project year, 143 teachers consented to participate in the research. Of the 143 consenting teachers, 100 responded to student work items in the autumn and again in the spring of the academic year, as pre- and post responses. From the set of 100 participants with both pre and post responses, we created a purposeful, random sample (Patton, 2002) of 20 participants for analysis. We based our sampling strategy on the following principles:

a) Include all responders with narrative responses on all items making the sample purposeful in its inclusion of only full sets of extended responses (Patton, 1990).

b) Randomize within the purposeful sample to assured a representative set.

c) Limit participation to 20 participants: 20 participants, each with 20 response in each of two administrations generated 800 data points for analysis, a significant number of data points.
for qualitative analysis, given the practicalities of time constraints, collaborative coding, and inter-rater reliability goals of the work (Patton, 2002).

The context of the study is the Mathematics Coaching Program, a statewide program training mathematics coaches to work in elementary, intermediate, and middle schools. Program goals include teaching movement: a) toward richly connected procedural/conceptual perspective on mathematics and away from a strict (and superficial (Star, 2005)) procedural perspective; and b) toward a learner-responsive and less teacher-directed pedagogy perspective.

The data source used for our analysis is a questionnaire generating teacher analyses of student work. Participants provided narrative responses to two questions for each of ten student work samples. One question asked for teacher interpretation of the student’s thinking and the second asked for teacher suggestion of next instructional moves. For example, one item provides the following student work: Bobby was given the problem 17 – 9 = __ and solved it as follows: 17 – 9 = 17 – 10 – 1. Teachers were asked to a) Provide a rationale to describe what Bobby was thinking; and b) provide an explanation of what to say or do to help Bobby further his thinking.

Data Analysis

Participant extended responses were coded through two lenses on the autumn (pre) and spring reviews (post), and also compared for pre and post results. Responses on student thinking were coded for content on a continuum from Procedural to Conceptual to Integrated Procedural/Conceptual perspectives. Responses about instruction were coded on a continuum from Teacher Directed to Problem/activity-based and Student Centered to Learner Responsive Pedagogy. See the abbreviated codebook of Table 1 for the list of codes. One or more content codes was assigned to each student thinking response; and one or more pedagogy codes to each instruction response. Multiple researcher reviews of sample responses, collaboration on coding assignments, and comparisons to coding by an outside reviewer (Morse, Barrett, Mayan, Olson & Spiers, 2002) helped reach inter-rater reliability goals of 85% in the analysis.

<table>
<thead>
<tr>
<th>Code</th>
<th>Mathematics Content Perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>IPC</td>
<td>Integrated procedural/conceptual perspective</td>
</tr>
<tr>
<td>C</td>
<td>Conceptual perspective</td>
</tr>
<tr>
<td>P</td>
<td>Procedural perspective</td>
</tr>
<tr>
<td>IO</td>
<td>Incorrect/other</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Code</th>
<th>Mathematics Pedagogy Perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRP</td>
<td>Learner Responsive Pedagogy</td>
</tr>
<tr>
<td>PSC</td>
<td>Problem/activity-based and Student-Centered instruction</td>
</tr>
<tr>
<td>TD</td>
<td>Teacher Directed</td>
</tr>
<tr>
<td>O</td>
<td>Other does not clearly belong to any of the other categories</td>
</tr>
</tbody>
</table>

Table 1. Abbreviated Codebook

Results: Movement in Content and Pedagogy Perspectives

Table 2. includes results of the coding analysis of participant perspectives on mathematics content, showing movement or lack thereof on a P to C to IPC continuum. Data points are best viewed as clustered around or tending toward a particular position, allowing for some variance in the content and pedagogy perspectives, while still describing a location.

As the data in Table 2 shows, 25% of the participants tended to exhibit positive movement from P to IPC. Consider an example teacher response: Jenny uses the following method to find


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28% of 60,000 mentally. 20% is 1/5 and 1/5 of 60 is 12, so 20% of 60,000 is 12,000. One percent of 60,000 is 600, and that times 8 is 4800. So the answer is 12,000 + 4,800, which is 16,800. We coded a teacher’s response of “When finding answers to problems mentally it is easier to break it down into easier chunks,” as P because the response only commented on what one would do. Later in the year, that same teacher responded “Jenny broke apart the problem into easier chunks. She understands the relationships between percents and fractions and understands that you can divide 60 by 5 to show 1/5” coded IPC because it included both procedural and conceptual component; and that conceptual and procedural components are connected.

Another problem showed a toothpick aligned with a ruler between 8” and 10.5” and a student’s response that the toothpick measured 10.5” in length. We coded a response of “I think she doesn’t understand that you start at the beginning of the ruler to measure” as a P response, while a second response of “She seems to be able to read the ruler but is struggling to understand how to measure when an object doesn't begin at 0” was coded as an IPC response because of the teacher’s analysis that notes the concept of the starting point of a measure.

On a division of fraction problem where the student used a mathematically valid alternative algorithm to find a correct answer, negative movement is exampled by a teacher’s analysis of “It seems she made common denominators.” This was coded C because it simply mentioned a conceptual component. It did not suggest a procedure, and thus could not be P or IPC. Later, that same teacher responded with “Common denominators are used for adding or subtracting fractions. She doesn’t understand that you divide fractions by multiplying the reciprocal of second fraction,” coded as P because it focused on the procedure, without explaining meaning.

<table>
<thead>
<tr>
<th>Positive Movement</th>
<th>Percent of participants</th>
<th>Negative Movement</th>
<th>Percent of participants</th>
<th>No movement</th>
<th>Percent of participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>P to C</td>
<td>5%</td>
<td>IPC to P</td>
<td>15%</td>
<td>Remain P</td>
<td>20%</td>
</tr>
<tr>
<td>P to IPC</td>
<td>25%</td>
<td></td>
<td></td>
<td>Remain IPC</td>
<td>10%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Remain P &amp; IPC</td>
<td>25%</td>
</tr>
</tbody>
</table>

Table 2. Participant Movement on Mathematics Content Continuum

Table 3 includes the proportional results of the coding analysis of the qualitative data on participant perspectives on mathematics pedagogy. These results show percentages of movement on the TD to PSC to LRP mathematics pedagogy continuum. In Table 3 25% of the participants revealed positive movement from TD to PSC, and PSC to LRP. An example of such positive movement from Bobby’s 17-9= ___ problem cited above starts with “By using his own explanation I could: 1. verify his mistake as I see it and 2. allow him to discover his own error and then he could recognize his error in the future” which is coded PSC because it does focus on helping the student realize his error. Later in the year that same teacher’s response becomes “I would have him solve both sides using pictures or models and compare his answers. Using this method Bobby could see that his process is wrong. He can visualize the need to add that 1 back in the equation.” This end of the year instructional suggestion is coded LRP because the teachers knows and helps the student discover his errors, by having the student compare and reflect upon his own work. The teacher and the student share the authority in the experience.

An example of the 15% of the participants who showed negative pedagogical movement is as follows: A teacher’s first response about Bobby’s problem was, “Bobby I like how you rounded the 9 to 10. Now we have to subtract 17-10=7 and add 1 back to get to the 9. Let me show you with our cubes what I would do.” We coded this response TD, but with expectation of movement at the post administration because of the potential in the use of manipulatives. However, at the post administration, the same teacher responded, “If Bobby explains his answer to me then I


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would correct him when he explains -1 and tell him he needs to +1 because the problem was 17-9 and show him with base 10 blocks - 9 cubes is less than 10 cubes or 1 long (10 cubes).”

Bobby’s use of manipulatives was clearly still from a teacher directed perspective, and perhaps even more TD in the language of “I would correct him,” “tell him,” and “show him.”

<table>
<thead>
<tr>
<th>Positive Movement</th>
<th>Percent of participants</th>
<th>Negative Movement</th>
<th>Percent of participants</th>
<th>No movement</th>
<th>Percent of participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD to PSC</td>
<td>20%</td>
<td>LRP to PSC</td>
<td>10%</td>
<td>PSC</td>
<td>40%</td>
</tr>
<tr>
<td>PSC to LRP</td>
<td>5%</td>
<td>More TD</td>
<td>5%</td>
<td>TD</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LRP</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 3. Participant Movement on Mathematics Pedagogy Continuum

Analysis by Mathematics Content Strands

An additional review of the data by mathematics content strand revealed interesting results for two different NCTM content standards: number and operations; and data analysis and probability. Two of the items drawn from the number and operations content standard provided student work showing unusual solutions. The first was one in which the student reduced the fractions to common denominators and divided along the numerators, using a mathematically valid approach. Seventeen out of twenty teachers did not accept this as a valid method, insisting that the student should have used the “invert and multiply” strategy. One other responded that the student “got lucky.” The second item is a three-digit subtraction problem for which the student developed his own alternate algorithm. Thirteen of the twenty teachers refused to accept this student’s mathematically valid solution as a legitimate solution. In both of these items, although the teachers who doubted the solutions did not necessarily lack content knowledge, they were unwilling to accept the alternative strategies. This suggests a reluctance to value student thinking, which would hinder the use of a learner-responsive pedagogical approach.

Two items in the sample drawn on the data analysis and probability standard also revealed interesting findings. One item included student interpretation of a graph that had no labels or numbers. The student explanation described a representation of distance against time, but every teacher in the sample of 20 viewed the graph as representing only speed against time. Hence no teacher interpreted the student’s explanation as correct. On a different problem, focusing on probability, 20% of the teachers responded with thorough explanations revealing an understanding of the mathematics; but most offered responses that were clearly incorrect or with what we might call “non-answers” circumventing the topic and suggested little to no knowledge of the relevant content. In both cases, data suggest that even those with overall PSC or LRP pedagogical perspectives did not know this particular mathematics well enough to question students through explorations or help students come to a mathematically valid understanding.

Closing Discussion

As noted earlier, the MCP context for this research study has among its goals to support teacher movement in mathematics content and mathematics pedagogy perspectives. We find that the program impacts teacher perspectives and that our coding helps document and describe that movement. That we can capture even small movement with data on only one year in the MCP suggests a useful methodology in capturing the subtleties of incremental change. As opposed to definitive, consistent, permanent positions, teachers are positioned in-between categories, tend toward a position, or contribute data that shows only slight movement toward a position. In the day-to-day work that the MCP coaches do with teachers, being able to identify subtle changes


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and the nuances of individual teacher’s perspectives is critical to the coaches’ work. That a set of teachers may fill in many different positions on the continua does not suggest more codes are needed, but, rather, that the continua represent the realities of teacher growth. They are practical and useful tools for describing fluid movement, being flexible enough to capture the teachers’ sometimes daily and often small, incremental changes in perspectives. Many teachers also are likely to be positioned differently for some content than for others, so connecting this work to teacher content knowledge can reveal additional directions for professional development.

Finally, this work has implications for equity pedagogy (Erchick, Dornoo, Joseph, and Brosnan, 2010). A teacher’s strict and superficial procedural perspective on mathematics limits students’ opportunities for the rich mathematical learning of the integrated procedural/conceptual perspective; and examples of limitations of content knowledge that emerged in this work also hinder students’ access to the mathematics. A teacher directed perspective as we define it in this study is akin to Friere’s “‘banking’ concept of education” (1973; 1989, p. 58), where the teacher transmits, deposits, and “the scope of action allowed to the students extends only as far as receiving, filling, and storing the deposits” (p. 58) and does not allow for the shared authority, and the accompanying learning, that is necessary for Learner Responsive Pedagogy.

References


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