Teacher Educator’s questioning strategies are essential in maintaining the level of cognitive demand of tasks for preservice teachers’ learning of mathematics for teaching. Classroom discourse has been known to maintain the level of cognitive demand of tasks. However, qualitative differences exist in the use of discourse. This study focuses on comparing instructors’ questioning strategies with two groups of preservice elementary teachers using the same cognitively challenging task on surface area, where in one group the level of the cognitive demand of the tasks is maintained while in the other it is lowered. We highlight effective questioning strategies during whole group discussion.

Introduction

The purpose of this study is to investigate the role of instructor’s questioning strategies in facilitating discourse in mathematics methods classes for preservice elementary teachers. We analyze the role of questions used by teacher educators in setting up a classroom community that emphasizes justification and explanation (evidence and warrants). Our analysis targets specific types of discourse that teachers use and how they affect the flow of the discourse within the whole class discussion. While classroom norms have been found to affect the quality of discourse (Cobb, 1999), the role of the teachers is key to orchestrating the whole class discussion. What and how the teachers ask questions affect the course of the discussion, whether building towards key mathematical understandings as opposed to merely sharing ideas. This paper contributes to the literature in extending our understanding in supporting teacher educators to facilitate whole-class mathematical discussions with preservice elementary teachers. We share an analysis of two instructors who were using the same cognitively demanding task designed to support preservice elementary teachers’ understanding of surface area and generalized across prisms and cylinders.

Theoretical Framework

The theoretical framework of this study is derived from a sociocultural perspective, whereby mathematics teaching and learning is inherently social and embedded in active participation in communicative reasoning process (Lerman, 2001). Lerman (2001) deems that learning is a consequence of social interactions, which “along with physical and textual interactions can cause disequilibrium in the individual, leading to conceptual reorganization” (p. 55). He argues that “consciousness is constituted through discourse” (p.88) and associates speaking mathematically with learning mathematics and learning to think mathematically. Communication in mathematics classrooms provides opportunities for students to engage with ideas, refine understandings, and share insights and strategies. Walshaw and Anthony’s (2008) review of current literature on discourse indicates that “students’ active engagement with mathematical ideas will lead to the development of specific student competencies and identities” (p. 516). Rich mathematics discourse is at the center of constructing and connecting knowledge in mathematics. In classrooms where students explain and defend their ideas, analyze and evaluate the ideas of Wiest, L. R., & Lamberg, T. (Eds.). (2011). Proceedings of the 33rd Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Reno, NV: University of Nevada, Reno.
others, justify solutions, and explore multiple perspectives through conversations with peers, students deepen their own conceptual understandings and further their learning in mathematics. Classroom discourse has the potential to develop and deepen students’ conceptual understanding of mathematics. The quality and type of discourse are crucial to helping students think conceptually about mathematics (Kazemi & Stipek, 2001; Lampert & Blunk, 1998; Nathan & Knuth, 2003; van Oers, 2002; Van Zoest & Enyart, 1998).

Teacher discourse, i.e. what teachers say and how they say it, has a significant influence on how and what students learn (Knott et al., 2008). Teacher discourse can influence both the amount and the quality of learning that takes place, and may often inadvertently lower the level of the mathematical task from cognitively demanding to one of rote application of procedures (Stein et al., 2000). Particularly during whole class discussion, in the collective act of abstraction that occurs for students during the course of the lesson, the teacher facilitates and choreographs the mathematical discourse through the use of a wide range of meta-mathematical discourse moves (Knott et al., 2008). They are meta-mathematical in a sense that “they do not directly supply mathematical content but rather they are about mathematics, and employ the language or ‘register’ of mathematics” (Knott et al., 2008, p. 95). For example, teachers use moves such as steering, probing, redirecting, clarifying, validating, prompting, rephrasing, re-voicing, and generalizing during the interaction with students engaging in the mathematical tasks. Although these discourse moves are not directly mathematical content discourse, they are important in making mathematics learning happen in the classroom, by encouraging students to participate in shaping their own learning.

The use of discourse with preservice teachers poses additional challenges to mathematics educators. Specifically, preservice teachers have experienced mathematics lessons throughout their schooling and often enter their preparation program with an expectation of learning to become better at what they think mathematics teachers do (Nichol, 1999). One recommendation to rectify this problem is to focus on using problems and dilemmas of practice “as springboards for investigation of mathematics teaching and learning” (Nichol, 1999, p. 48). Morrone et al. (2004) found that instructors who consistently pressed for understanding and used scaffolded discourse to facilitate preservice elementary teachers’ learning when using a series of challenging tasks were able to generate mathematical knowledge about content and teaching.

In this study we analyze how these discourse types might be associated with the development of preservice elementary teachers’ mathematical explanation and justification. We attend to instructors’ tactical moves that encourage preservice elementary teachers to attend to the content by tracking teacher educators’ questioning strategies and noting the kinds of opportunities to engage with content that are offered to students during whole class discussion (Gresalfi & Williams, 2009).

Methods

Participants and Setting

This study is part of a larger project, the Elementary Preservice Teachers Mathematics Project (EMP), developing a series of cognitively demanding mathematical tasks (Stein et al., 2000) for preservice elementary teachers. The tasks were developed across several mathematical topics (Number Theory, Fractions, Ratios and Proportions, Geometry, and Geometric Measurement) where the questions were appropriately scaffolded and supported by classroom discourse in both small group and whole class discussion. In the pilot study, participants consisted of 32 undergraduate students at a major New England University, majoring in Mathematics.


Articles published in the Proceedings are copyrighted by the authors.
elementary education, special education, or deaf studies. The participants were in two classes taught by two different instructors, who were doctoral candidates in mathematics education in their third year of the program. Both instructors strove to develop their students’ understanding but used different methods. Preservice teachers in the two classes first worked on the mathematical tasks in small groups followed by whole class discussions. However, the two sections differed during the whole class discussion. In one section, the instructor only went over answers whereas in the other section, the instructor also included a discussion about the big ideas of the key concepts and procedures.

The study was conducted midway through the second semester of a two-course sequence for elementary education majors. Both sections had previously experienced using discourse and sense-making to discuss mathematical problems in small groups and to work toward justification. The task being used in this study was part of the EMP geometric measurement strand. It focused on surface area and covered two class periods. The preservice teachers examined the lateral and total surface area of prisms on the first day and examined similar ideas around cylinders with an emphasis on drawing connections between prisms and cylinders on the second. Specifically, the participants were introduced to a different way of finding the surface area of prisms and cylinders by considering the lateral surfaces and bases separately. They discovered the perimeter of the base of a prism or cylinder multiplied by the height of the figure is equivalent to the area of the lateral surface area. This study focuses on a question (Figure 1) in which the preservice teachers were asked to determine which of three figures (two prisms and cylinder) had the largest surface area. By this point, they should have noted that the lateral surfaces areas had the same dimensions and so the base areas were the only difference. Prior to the whole class discussion, the preservice teachers had explored and discussed the questions in their small groups both with and without the instructor.

All lessons were videotaped and transcribed. The tasks were coded using the Instructional Quality Assessment Academic Rigor (IQA-AR) Rubric for Potential of the Task (Boston & Smith, 2009) to assess the levels of cognitive demand of the mathematical tasks as they were intended to identify questions at a high level. The transcripts were coded using the Instructional Quality Assessment Academic Rigor (IQA-AR) Rubric Implementation of the Task (Boston & Smith, 2009) by pairs of coders to ensure inter-rater reliability.

Using the rubrics, we identified questions that have high levels of intended cognitive demand. We then analyzed the transcripts for instances when the level of cognitive demand was maintained or dropped during the implementation. Question 15’s level of cognitive demand was high but then was dropped in the first group and maintained in the second group.

In order to analyze and compare instructors’ discourse moves, we adapted existing analytical

Figure 1. Question 15 from the EMP Task on Surface Area

All lessons were videotaped and transcribed. The tasks were coded using the Instructional Quality Assessment Academic Rigor (IQA-AR) Rubric for Potential of the Task (Boston & Smith, 2009) to assess the levels of cognitive demand of the mathematical tasks as they were intended to identify questions at a high level. The transcripts were coded using the Instructional Quality Assessment Academic Rigor (IQA-AR) Rubric Implementation of the Task (Boston & Smith, 2009) by pairs of coders to ensure inter-rater reliability.

Using the rubrics, we identified questions that have high levels of intended cognitive demand. We then analyzed the transcripts for instances when the level of cognitive demand was maintained or dropped during the implementation. Question 15’s level of cognitive demand was high but then was dropped in the first group and maintained in the second group.

In order to analyze and compare instructors’ discourse moves, we adapted existing analytical
frameworks (Fraivillig, Murphy, & Fuson, 1999; Knott, Sriraman, & Jacobs, 2008) to attend to teachers’ discourse moves, especially the use of questioning strategies. We deemed it necessary to adapt these frameworks because there are differences in context between K-12 and preservice education settings. Table 1 shows the analytical framework used in our study.

Table 1. Analysis Framework for Teacher Educator’s Discourse Moves

<table>
<thead>
<tr>
<th>Purpose of Move</th>
<th>Questioning Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eliciting</td>
<td>Probing</td>
<td>To elicit students’ prior knowledge</td>
</tr>
<tr>
<td></td>
<td>Rephrasing/Re-voicing</td>
<td>To validate students’ mathematical thinking</td>
</tr>
<tr>
<td></td>
<td>Prompting</td>
<td>To provide background knowledge</td>
</tr>
<tr>
<td>Supporting</td>
<td>Steering</td>
<td>To move the discourse in a particular direction based on students’ engagement in and demonstrated level of understanding of the task</td>
</tr>
<tr>
<td></td>
<td>Re-directing</td>
<td>To steer the discourse back when the discourse is moving towards irrelevant or incorrect mathematical assumptions</td>
</tr>
<tr>
<td>Extending</td>
<td>Challenging</td>
<td>To demand explanation and justification of students’ claim</td>
</tr>
<tr>
<td></td>
<td>Generalizing</td>
<td>To push students to move beyond the particular to the general case</td>
</tr>
</tbody>
</table>

Findings

Comparing the transcripts from the two instructors orchestrating the whole class discussions around question 15, we identified differences in their use of tactical moves in framing the whole class discussion, orchestrating discourse, and connecting and extending the preservice teachers’ thinking. Table 2 shows the frequencies of each of the discourse strategies used by each instructor during their whole class discussion around question 15. Both instructors used moves that elicit and support preservice teachers’ thinking, but the second instructor included moves that extend preservice teachers’ understanding (Challenging and Generalizing) to think beyond what was explicitly asked in the question, and pushed them to explain and justify their mathematical thinking.

Table 2. Frequencies of whole class discourse moves for question 15

<table>
<thead>
<tr>
<th>Discourse Moves</th>
<th>Instructor 1</th>
<th>Instructor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probing</td>
<td>13</td>
<td>17</td>
</tr>
<tr>
<td>Rephrasing/Re-voicing</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Prompting</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Steering</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Re-directing</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Challenging</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>Generalizing</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Framing of the Whole Class Discussion

The way an instructor frames a whole class discussion affects how the discourse that follows


Articles published in the Proceedings are copyrighted by the authors.
will unfold. The first instructor (I1) in this study immediately steered the preservice teachers’ attention to the shapes of the bases and dismissed the lateral surface area to be unimportant. Instructor 2 (I2) begins the discussion with an open question, “What did you guys discover in question number 15?” setting up a tone of inquiry, followed by a probing question, eliciting preservice teachers’ knowledge of the three lateral surface rectangles.

<table>
<thead>
<tr>
<th>Instructor 1</th>
<th>Instructor 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>On question 15 let’s start and just talk about how we would go ahead and find the bases of those 2 shapes. Before we do that, we were asked which has the greater surface area. Was it necessary for us to find the lateral surface area of each of those shapes?</td>
<td>Let’s talk about question number 15, where maybe I think the difference in calculating the bases comes out. What did you guys discover in question number 15? We had 3 lateral surface rectangles. What was the first one? What kind of prism was the first one?</td>
</tr>
</tbody>
</table>

**Orchestrating the Whole Group Discussion**

Both instructors led preservice teachers in a whole class discussion as they validated solutions for finding the area of the bases in question 15. They used discourse to help preservice teachers synthesize the information. Although both instructors were eliciting preservice teachers’ solution methods and leading similar discussion, Instructor 2 used tactical moves that established important norms for discussion in the classroom, which maintained the level of cognitive demand of the task and pushed for clarity. Instructor 2 employed the Challenging and Generalizing tactical moves which led preservice teachers to make claims and provide justifications. Instructor 1 relied more on Probing and Steering moves that led to brief preservice teacher responses, often without justification.

In one episode, Instructor 1 asked a closed question that led to an answer, without justification, and steered the discussion back to question 15 and asked, “So which one of our three shapes is going to have the largest surface area?” The preservice teachers responded to his question with a one-word answer, “cylinder,” followed by the instructor providing them with an explanation, in which he was relating and connecting the mathematics ideas that emerged from the problem.

Instructor 2 led the preservice teachers to step back and think about the purpose of the task by asking, “What’s the big idea here? Why did I have you calculate the area of these bases? What do you notice?” The emphasis on the “big idea” and the push for explanations encouraged the preservice teachers to consolidate their ideas.

**I2:** So, what’s the big idea here? Why did I have you calculate the area of these bases? What do you notice?  
**Generalizing**

**S9:** In order to find the area of each of the bases, you have to find out which one has the greatest surface area or the greatest lengths of total surface area.  
**I2:** Oh, why is that?  
**Challenging**

**S9:** Because the prisms and cylinders that we’re making out of paper, they all have the same lateral surface area. And so the only differences are the area of the bases.  
**I2:** Can someone just repeat what S9 just said? She just made a really good point.  
**Re-voicing**

---


Articles published in the Proceedings are copyrighted by the authors.
S10: You only need to find the area of the bases because all of the prisms have the same lateral surfaces rectangle, so that measurement is going to be the same. So by finding the measure of both of the bases or just one of the bases, you can find out which one has the biggest surface area.

Connecting/Extending Preservice teacher Thinking

We found that the instructors’ questioning strategies played an important role in extending preservice teachers’ thinking and leading them to make connections. In both of these classroom exchanges, the instructors encouraged preservice teachers to make connections to their past experiences in working with the idea of determining the figure with the greatest area with fixed perimeter. Instructor 1 steered the preservice teachers to make the connection, and when a preservice teacher was unsure of her connection to the geoboard to explain why, with a fixed perimeter, a circle would have the largest area, there was no push for clarity. There was no clear indication that the preservice teachers were able to generalize that the circle would have the greatest area given that the perimeter was fixed. When the preservice teacher (S2) suggested that a square would have the largest area, the instructor steered the conversation to address that the figure should not be limited to rectangles (including squares), but could extend to circles.

I1: If we have a fixed perimeter, which rectangle has the largest area?  
S2: The one that’s closest to a square.  
I1: The one that’s closest to square if we are using tiles. If we are not using tiles then we would want to find the perfect square. So in a similar case here we are looking for the most ideal shape here, which is a circle. …if I’m taking my piece of paper, pretend this is a 6 by 32 piece of paper for a second [folds paper], we want to find the smallest surface area possible, we could just fold this in half and have the surface area on the top and bottom have zero right here. As we think about it and get further and further away from this line we are going to a circle, we are getting a larger and larger surface area as we move forward from that direction.

However, another preservice teacher’s (S11’s) choice of using a geoboard for the explanation, presented a challenge to explain why the circle would have the greatest area, but the instructor’s choice of discourse move failed to provide support to push for clarity in the preservice teacher’s explanation and justification.

S11: One way I thought about it was like if you were using a geoboard, the closer it got…I don’t know it’s hard for me to explain, but you could tell with something that has the same perimeter as a circle it’s not going to be as large of an area because it won’t fill up the space as much. I don’t know, maybe that doesn’t make any sense at all.

I1: So you’re saying that a circle wouldn’t be as large?  
S11: No, a triangle wouldn’t be as large because it wouldn’t fill up the same…I don’t know. I mean it makes sense in my head.

In contrast, the preservice teachers in Instructor 2’s class connected their past experiences and suggested several strategies (using strings, physically fitting prisms and cylinder, and using drawings) to explain why the circle would have the greatest area.

S5 I was thinking the day we were doing strings, for all of them we


Articles published in the Proceedings are copyrighted by the authors.
have to form the base, we have the same length of string and we form it in 3 different shapes. So it’s cool that even though the perimeter is always going to be the same, the circle is always going to have the greatest area according to like the way we set it up…

I2: Interesting. What do you guys think about what S5 just said? Do you think she’s correct?

S1: We took the cylinder and put the rectangular prism in it…it should fit inside it and there should be the square with little arcs, pieces missing from it, showing that it would have the same perimeter or circumference, but with a bigger area for the circle because the square can fit inside of it.

The instructor further challenged them to generalize the idea by introducing a hexagonal prism and pressed the preservice teachers to provide justification.

I2: S3 has the circle drawn around the square, and the square is drawn around the triangle. So what does this imply? What if I gave you guys the same lateral surfaces rectangle here, another 6 by 32, and I said make me hexagonal prisms? Where do you think its total surface area would fall amongst these 3?

S3: Between the square and the circle.

I2: Why must it be this way?

S4: We discussed that it must go between the square and the circle because the more sides you add to the figure, the closer it gets to a circle. So whereas the square has 4 sides, the hexagon has 6, and it just keeps breaking down the paper into closer of a circle.

S2: So if you would fit the hexagon into the cylinder, like the hexagonal prism into a cylinder, there would be less area left over on the outside than the square.

Finally, the instructor steered back the conversation to highlight the connections that the preservice teachers had made.

I2: Okay … S5 mentioned something that kind of relates back to a task we did in the beginning with area and perimeter, do you remember with string? How does this connect to that, some of the big ideas we did in that task with the string? S2 had mentioned it and I want to make sure everyone kind of makes that connection because making that connection is really important.

S4: Is the connection just that we’re dealing with a fixed length [makes string with hands], like with a fixed length of string, and forming it into different shapes? The same thing with the fixed length of the piece of paper?

I2: S5 what do you think, you brought it up? Is what S4 saying what you were trying to say?

S5: Yeah. I mean it’s a general idea. They kind of brought up the same idea with fitting the paper. It’s interesting that clearly constructing the figure with the exact same shape and as we said the lateral surface area is the same but…and the perimeter is the same but we’re using one of the lengths from the lateral surface, but it creates
a completely different area for each figure. These interactions led preservice teachers to make claims and warrants about their solutions. Big ideas and connections were emphasized. The discourse in Instructor 2’s classroom maintained the level of cognitive demand of the task.

Conclusions

When working on mathematical tasks, preservice teachers tend to focus too much on getting the “correct answer” and fail to step back and look at the big ideas for which the tasks are intended. This is especially true because of their familiarity and experiences with certain procedures or algorithms. It is the role of the instructor to draw out the idea that getting only the correct answer is not sufficient and that it is more important to examine the mathematical structure and the ideas behind the tasks. Teacher educators can accomplish this goal by focusing on questioning strategies built around challenging student assertions and creating generalization rather than simply steering and rephrasing student responses (Knott et al., 2008). By emphasizing this ideal, preservice teachers in Instructor 2’s class were involved in more active engagement with mathematical ideas (Walshaw & Anthony, 2008) and participation in a communicative reasoning process (Lerman, 2001). Instructor 2’s questioning strategies led to preservice teacher discourse that maintained the high level of cognitive demand of the surface area task. Preservice teachers’ own educational backgrounds often lack sufficient mathematical understanding, therefore it is critical that the cognitive demand of tasks remain high so that they can provide effective instruction in the future. Questioning strategies that elicit strong discourse among preservice teachers is one way to ensure rigorous learning of mathematics content and specialized content knowledge in preservice elementary teacher classrooms.

References


