INVESTIGATION OF MATH TEACHERS' CIRCLE THROUGH A ZONE THEORY LENS

Gulden Karakok
University of Northern Colorado
Gulden.karakok@unco.edu

Diana White
University of Colorado Denver
Diana.white@ucdenver.edu

Math Teachers' Circles (MTCs) are an innovative, problem-solving focused approach to professional development. This model provides teachers opportunities to develop their problem-solving skills as well as help them to communicate with others on classroom implementation of problem-solving activities. As with any professional development with teachers, it is important to explore the impact of this model in terms of teachers' learning and development. In this report we provide our implementation of a zone theory lens provided by Goos as a way to investigate the MTC model. Initial analysis implementing this particular theoretical lens helps us gain insights in ways to improve this new model of professional development activities for future participants.

Keywords: Teacher Education-Inservice; Problem Solving; Mathematical Knowledge for Teaching; Assessment and Evaluation

Introduction

Problem solving is one of the critical topics in mathematics education that has been the focus of research and curriculum reform internationally (e.g., Common Core State Standards Initiatives 2010; NCTM 2000; OECD 2004 & 2010). As its importance in students' learning of mathematics is palpable, in-service teachers also need support in developing and improving their problem solving abilities as well as its classroom implementation (Anderson 2005; Hiebert et al. 1996). One innovative professional development approach to address these areas is the Math Teachers' Circle (MTC) model, which emphasizes developing and improving teachers' problem solving skills in the context of significant mathematical content. It is designed after Math Circles for secondary students, which originated in eastern Europe, migrating to the US in the mid-1980s. Secondary students in these circles engage in solving challenging mathematics problems with guidance from the mathematicians who facilitate these sessions. In this spirit, participating teachers in an MTC engage in solving math problems geared towards the level of teachers, rather than the level of their students, designed and facilitated by mathematicians. During each session teachers solve problems, discuss problem-solving strategies and different solutions, as well as possible implementations of problem-solving activities in their classrooms. Generally, each MTC provides a weeklong summer workshop, followed by monthly evening sessions during the academic year. This model has been gaining momentum in the US, with 71 active MTCs in 36 states. White, Donaldson, Hodge and Ruff (2013) and White (2015) provide additional information about the MTC model.

Being a relatively new model (since 2006), the evaluation of MTCs requires an investigation of effectiveness from various lenses-both theoretical and practice based ones. The purpose of this report is to describe our initial attempt at implementing a theoretical model described by Goos (2009) to explore possible contributions of the MTC models to teachers' development in the area of problem solving.

Theoretical Framing

The evaluation of professional development activities is a complicated task. Many researchers provide practice-based perspectives to design, implement and evaluate professional developments and outline effective practices. For example, Mewborn's (2003) review of research identifies three key elements of successful practices. In general professional developments are stated to be effective if they (1) provide opportunities in which teachers engage with mathematical concepts and also focus

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on their students' learning of such concepts; (2) are situated in school-context in which teachers could implement and authenticate ideas; and (3) provide opportunities in which teachers discuss issues related to their and their students' learning within a supportive group of participants and network.

These key elements are part of a more theoretical model proposed by Goos (2009), which provides additional aspects contributing to the outlined key elements of effective professional developments. This particular perspective is an extension of Valsiner's (1997) model for understanding learner's development that stems from Vygotsky's Zone of Proximal Development construct (1978).

In their model, Goos et al. (2007) describe three zones, each of which focuses on different aspects of teacher learning and development: the Zone of Proximal Development (ZPD), the Zone of Free Movement (ZFM), and the Zone of Promoted Action (ZPA). The ZPD in this model refers to teachers' *knowledge* and *beliefs* on content and pedagogy. In other words, teachers' development of content knowledge, pedagogical content knowledge, their beliefs about mathematics and about teaching and learning of mathematics are considered within this zone. The ZFM, on the other hand, focuses on "constraints and affordance within the professional context" (p.26). In particular, teachers' *perceptions* related to their profession such as insights on their students' ability, views on curriculum, standards, and assessments they implement, and connections to other teachers and districts, and how such perceptions develop or change through participation in professional development activities are considered in this zone. The third zone, the ZPA, focuses on the *professional development strategies* that are being introduced either formally through structured workshops or informally through communication with other colleagues.

Goos et al. (2007) observed that these zones complement each other in the effort to describe teacher's learning and development. In particular, Goos et al. state, "For teacher learning to occur, professional development strategies [ZPA]must engage with teachers' knowledge and beliefs [ZPD]and promote teaching approaches that the individual believes to be feasible with their professional context [ZFM]" (p. 26).

In this exploratory study, we investigated ways in which the MTC model contributes to participants' learning and development by identifying aspects related to each zone. Though this particular theoretical lens was only implemented in the data analysis phase of the evaluation, this exploration provided insights on improving the planning stages of the professional development model, as discussed in our conclusion.

Methods

Participants and Data

The participants of this study were 129 in-service teachers who attended one of the six MTCs summer workshop hosted during 2011 summer. The number of participants and their years of teaching varied by site, as summarized in Table 1. There were more MTC sites during this particular

Table 1: MTC sites and participants

Site	Number of participants	Years Teaching	
		Mean (SD)	Range
A	23	7.26 (5.2)	1-18
B	20	9.4 (8.8)	0-31
C	16	12.3 (10.3)	0-35
D	19	11.7 (8.7)	0-28
E	20	8.2 (5.3)	1-19

F	27	14.6 (6.9)	2-30
All	129	10.8 (8.0)	0-35

summer; however only six of them both qualified and agreed to participate in this exploratory study. To qualify, a site had to offer at least a workshop of four full days in length and with primarily middle-school level teacher participants, though many also allowed secondary teachers to participate. These sites were geographically diverse. All sites focused on problem-solving activities by engaging participants to solve challenging problems. Some sites also included activities in which participants explicitly discussed pedagogical approaches and implementation of problem solving in their classrooms.

Analysis

In this study we focus on data collected by two instruments: the Learning Mathematics for Teaching (LMT) instrument developed by Hill, Schilling and Ball (2004) and an exit survey developed by one of the authors. All sites administered two subscales of the LMT instrument at the beginning and the end of the workshop. These subscales were used to measure mathematical knowledge for teaching (MKT) of Number Concept and Operations, and Proportional Reasoning. These particular items were used to get an insight on the participants' development of content and pedagogical content as described in the ZPD.

In addition to the LMT instrument, we investigated the ideas relating to the ZPD by qualitatively analyzing the end of workshop exit survey. These questions focused on participants' overall experiences, asking them to comment on their thoughts about the workshop, their learning, other participants' impact on their learning, their anticipation of changing their teaching practices as a result of this workshop, useful aspects of the workshop as well as their comparison of this workshop to other professional developments. Participants' responds were transcribed and analyzed using open and axial coding procedures as described by Strauss and Corbin (1990). Each researcher developed codes that describe participants' experience as a learner and a teacher. These codes were refined by the research team and themes were developed. At the final stage of the analysis, themes relating to the ZPD, ZFM and ZPA from each site were analyzed across workshops to get an insight of the overall experience of all participants.

Results

The LMT instrument was used to explore the participants' development of mathematical knowledge for teaching, addressing aspects outlined in the ZPD. As a condition of using the instrument, we must report standardized scores only by converting raw LMT scores to standardized (*z*) scores using the scoring tables provided at the instrument-training workshop. An example of the

Table 2: The LMT: Standardized Scores for Number Concepts

Site	Number and Operations		
	Pre	Post	Difference
A	-.41 (1.0)	.06 (1.0)	.48 (.53)*
B	.23 (1.0)	.58 (.74)	.35 (.86)*
C	.12 (.85)	.30 (.91)	.18 (.75)
D	-.50 (.66)	-.21 (.73)*	.28 (.63)*
E	.65 (1.1)	.91 (1.1)	.26 (.72)*
F	.37 (.98)	.56 (.96)	.18(.47)
All	.05(1.0)	.33(97)	.29(.67)*

*Notes.(1)*Scores are standardized and are presented as *M (SD)*. *Pre* = pretest score; *Post* = posttest score; *Difference = Post – Pre*.

(2) *= Planned comparisons showed a significant difference between pre- and posttest scores ($p < .05$).

standardized pre-, post and differences is provided in Table 2 for the Number Concept and Operations.

The analysis of the LMT indicates that the planned comparison t-tests show an increase in the Number Concept and Operations scores that was significant with all sites combined, *M(SD)=.29(.67)*with $p < .00001$.However, we have not observed a similar increase in Proportional Reasoning content that was also administered in the LMT instrument.

In addition, we conducted the repeated measure ANOVA, which revealed a significant main effect of Test Form, $F(1, 112) = 76.31$, $p < .001$. Overall, Proportional Reasoning scores were significantly higher than the Number Concept and Operations scores (M(SD) = .29(.67) and -.10(.67), respectively), and this pattern was consistent across all six sites. The interaction of Test Form x Workshop Site was not significant, $F(5, 112) = 1.46$, $p = .21$.

Table 3: Repeated Measures ANOVA: LMT Standardized Scores

Source	SS	df	MS	F	p
Between subjects					
Site	68.27	5	13.65	5.48**	.00
Error	279.19	112	2.50		
Within subjects					
Test	21.03	1	21.03	76.32**	.00
Test * Site	2.02	5	0.40	1.44	.21
Test (Error)	30.86	112	0.28		
Time	0.96	1	0.96	4.28	.04
Time * Site	0.476	5	0.10	0.42	.83
Time (Error)	25.13	112	0.22		
Test * Time	4.45	1	4.45	18.82**	.00
Test * Time * Site	0.40	5	0.08	0.34	.89
Test * Time (Error)	26.49	112	0.237		

Note. Site = Workshop Site 1, 2, 3, 4, 5 or 6. Test = Number Concepts and Operations or Proportional Reasoning. Time = Pretest or Posttest. ***p* < .01.

There was also a significant main effect of Time Administered, $F(1, 112) = 4.28$, $p = .04$. Post-test scores were higher on average than pre-test scores (M(SD) = .45(.92) and .36(1.05), respectfully). The interaction of Time Administered x Workshop Site was not significant, $F(5,112) = .424$, $p = .83$, indicating that all six sites shared a pattern of differences between pre- and post-test administration and supporting the combination of data across sites for the planned comparisons tests.

There was significant interaction effect of Time Administered x Test Form, $F(1,112)=4.452$, $p<.001$. Overall, there were gains from the pre- and post-test scores for Number Concept and Operations scores, whereas there were losses for the Proportional Reasoning scores (M(SD) = .29(.67) and -.10(.67), respectively),and this pattern was consistent across 5 of the 6 sites. However, the interaction of Test Form x Time Administered x Workshop Site was also not significant, $F(2, 112) = .35$, $p = .889$. This indicates that the pattern of pre- and post-test scores for each form was not significantly different across sites and further supports combining the data across sites for analysis in the planned comparison tests.

The exit survey results were analyzed qualitatively to capture general themes within all zones: ZPD, ZFM, and ZPA. The main themes reflecting ideas that relate to the ZPD were different types of learning such as learning of math content, problem-solving strategies, teaching strategies and overall comments on learning. The themes that captured the ideas referring to the ZFM were teachers' perspectives on student learning, plans for classroom teaching and problem-solving activities and teaching, and the Common Core State Standards perspectives. The main themes observed reflecting the ideas for the ZPA were the challenges that participants experienced during the workshop, collaboration and engagement experience, and general comments on the structure of the workshop.

Responses to the question, "please tell us your thoughts about the workshop," were varied, demonstrating that the culture within the individual sites was unique. However, the dominant themes were reported by the participants mostly related to the ZFM and ZPA, such as collaboration with other teachers; engagement in the workshop activities and perspectives on teaching strategies in the classroom. Even though themes related to the ZPD were also observed, these were not observed in the cross-case analysis. For example, Site A participants reported that learning problem-solving strategies was impactful.

Most participants (64.12%) commented that collaboration with others was the most important support they received during the workshop. This particular theme directly relates to the structure of the MTC model, referring to the ZPA. This theme was observed in response to the question, "please comment how the support you received from others impacted your learning." A participant from Site E commented that the support from others was most impactful in a way that it was, "positive, reflective, [they] bounced ideas off each other, check[ed] mechanisms, asked/answered why—had seen/shared several different approaches to the same question."

Another theme indicating a possible development in the ZPD but also ideas referring to the ZFM was observed in the responses to the question, "Do you anticipate changing anything about how you teach mathematics as a result of the workshop. If so, in what ways?" The majority of participants (63.84%) stated that they learned teaching strategies in the workshop that they planned on transferring to their classroom. An exemplary quote from a participant referring to the ZFM was about this teacher's perspective on student learning is, "I realize that it's ok to give 'hard' problems for students to solve. The process to solve takes patience and, as a teacher, I need to encourage students to be creative in their thinking to build good reasoning."

Participant engagement was the most common theme that highlights the unique structure of the MTC model in area of the ZPA. This theme emerged in response to the questions, "Please comment on any differences or similarities that struck you about this workshop compared with other professsional development workshops you have attended in the past (if applicable)" and "Please comment on what you considered to be the most useful aspects of this week." Participants from workshop Sites A, C, D, E, and F all commented that the MTC workshops were more engaging than other professsional development workshops they had attended. A participant from site F commented that,

> most workshops are how to get children excited about learning. This workshop goes beyond with a chain reaction. The instructor was excited, making the teacher excited, which in turn will make the students excited. Some workshops I have attended in the past, I felt like the teacher link was the smaller part. Each link needs to be equal to gain strength.

The most common theme coming from participants of workshops at sites B, C, and D was in regards to the usefulness of collaborating with other teachers: this was also the most common theme from the combined responses.

Overall, this initial analysis to investigate the MTC professional development model through a zone theory perspective highlights the possible impact that the MTC could provide to participating teachers in the ZFM and ZPA. In addition to the possibility of increasing participants' development and learning of mathematics and mathematical knowledge for teaching the exploration of other zones provides a framework to structure the future MTC workshops.

Conclusion

In this report we shared a theoretical approach to explore the MTC professional development model. The MTC model is an innovative professional development in which participants develop their abilities and beliefs on problem solving which are the main ideas of the ZPD. However, as previous research on effective professional development address the development in the ZPD is not enough to make change in teachers' professions and special focus and investigation are needed on the areas that are highlighted in the ZFM and ZPA. We use this zone theory approach was to gain insights on possible impacts of the MTC model. This particular approach provided a mean to understand aspects such as importance of engagement with other teachers and impacting teachers' personal motivation during their participation. This initial attempt needs further exploration by collecting data from facilitators of each MTC on the structure of the MTCs, observing each MTC during workshops and conducting follow-up interviews and surveys with the participants.

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