This study investigated the impact of a dynamic geometry (DG)-centered teacher professional development program on high school geometry teachers’ content knowledge and their students’ geometry learning. 64 geometry teachers were randomly assigned to an experimental (DG) group and a control group. Both groups received appropriate and relevant professional development. Classroom observation data and the teachers' responses to the implementation questionnaires revealed that most teachers in the DG group were faithful to the DG instructional approach. Teachers in the DG group scored higher on a conjecturing-proving test than did teachers in the control group. The students of teachers in the DG group scored significantly higher than the students of teachers in the control group on a geometry achievement test.

Keywords: Teacher Education-Inservice (Professional Development); Technology

Introduction

Dynamic geometry (DG) refers to an active, exploratory study of geometry carried out with the aid of interactive computer software available since the early 1990’s that allows for learner knowledge construction and exploration. The most widely used current DG software packages include the Geometers’ Sketchpad (Jackiw, 2001), Cabri Geometry (Laborde & Bellmain, 2005) and Geogebra (Hohenwarter, 2001) as well as variations that are applications within handheld graphing calculators or applets on web sites. DG environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena (CCSSI, 2010). With distinguishing features of dragging and measuring, DG software can be used to help students engage in both constructive and deductive geometry (Schoenfeld, 1983) as they build, test and verify conjectures using easily constructible models.

In a funded four-year research project, we conducted repeated randomized control trials to investigate the efficacy of an approach to teaching high school geometry that utilizes DG software as a supplement to regular instructional practices. Our basic hypothesis was that the use of DG software and DG teaching methods that engage students in constructing mathematical ideas through experimentation, observation, data recording, conjecturing, conjecture testing, and proving would result in improved geometry learning experiences for most students. The use of DG software and teaching methods was referred to as the DG approach in the project. The DG software used by the project was the Geometers’ Sketchpad (GSP).

In this paper, we report the results from the second year of the project on teacher content knowledge and student achievement. We investigated the impact of the professional development of teachers and their students’ geometry achievement in the DG group. The study built upon related research studies on mathematics teachers’ professional development (e.g., Carpenter et al., 1989), including those concentrating on technology-centered (and especially DG-centered) professional development (e.g., Meng & Sam, 2011).
Theoretical Framework and Research Questions

An integrative framework (Olive & Makar, 2009) drawing from Constructivism, Instrumentation Theory and Semiotic Mediation was used to guide the study. Within this framework, as teachers and students interact in DG environments, their interactions with the DG technology tool influence the next act by each person, and continue in an interplay between the tool and user. As a user (teacher or student) "drags" an object and observes outcomes from that act, the user adjusts her or his thinking, which in turn influences the next interaction with the tool. Because DG technology allows users to adjust their geometry sketches and the relationships within them, users are transforming the tool, their use of the tool, and their thinking.

This study addresses the following research questions:

- Did teachers in the experimental (DG) group develop stronger conjecturing and proving abilities than did teachers in the control group?
- How well did the teachers implement the DG approach with fidelity in their classrooms?
- Did the students of teachers in the experimental group over a full school year achieve significantly higher scores on a geometry test than did the students of teachers in the control group?

Method

The participants in the study were sampled from the geometry teachers at high schools and some middle schools in Central Texas school districts. The study followed a randomized cluster design, randomly assigning 64 teachers to either an experimental group or a control group receiving relevant professional development, implementing the instructional approaches respectively assigned to them, helping the project staff in administering the pre- and post-tests of the participating students, and participating in other data collection activities of the project.

Professional Development and the DG Treatment

In order to effectively implement the DG approach in their classrooms, teachers must first master the approach. Without professional development, “teachers often fail to implement new approaches faithfully” (Clements et al., 2011, p. 133). So teachers’ professional development (PD) was a critical component of the project. For our PD to be effective, it had to be sustained, rigorous, and relevant to participating teachers, with substantial support from their school districts. Based on these guiding ideas, a weeklong summer institute was offered to the participating teachers in the DG group, followed by 6 half-day Saturday PD sessions during the school year. The PD was planned and implemented collaboratively by project staff that included mathematics and education university faculty members and school-based master teachers selected based on their success as mathematics teachers and their experience with DG software. The project team and master teachers served as partner facilitators for all PD sessions.

The teachers in the experimental (DG) group were actively involved in each PD session and focused on developing their conceptual understanding of mathematics using the DG software as a tool. They worked on challenging problems and developed important geometric concepts, processes, and relationships while building DG skills and teaching methods. They experienced how DG environments encourage mathematical investigations by allowing users to manipulate their geometric constructions to answer "why" and "what if" questions, by allowing them to backtrack easily to try different approaches, and by giving them visual feedback that encourages self-assessment.

Typically, each activity in a PD session consisted of the following instructional events: 1) Presenting a task (exploring concepts/relationships or solving a problem) to the teachers; 2) Requesting teachers to use DG tools to construct the related geometric object or problem situation.
(with help if necessary) or providing them with a prepared DG environment; 3) Asking teachers what conjecture(s) they can make based on their initial observation; 4) Requesting teachers to use dragging, measuring, and multiple, linked representations to experiment with the constructed or provided DG environment, and observe what characteristics change and what remain the same; 5) Asking teachers to further make and test conjecture(s); 6) Reminding teachers to redo events #4 and #5 in a new aspect or at a higher level, as appropriate; 7) Asking teachers to summarize and reflect on what they have conjectured; and 8) Helping teachers develop explanations to prove or disprove their conjecture(s).

In each PD session, teachers either worked individually at a computer or in small groups. In either case, PD facilitators encouraged teachers to share ideas and help each other. The facilitators circulated, observed (to monitor the progress) asked questions, and provided necessary assistance. They also initiated whole group discussions as needed.

In terms of content, the summer PD sessions concentrated on important and commonly taught topics of high school geometry: triangle congruence and similarity, properties of special quadrilaterals, properties of circles, and geometric transformations. School year follow-up PD aligned with the course scope and sequence determined by the participating school districts.

The PD facilitators modeled what teachers were expected to do with their students in geometry investigations. To help teachers change their instructional practices, their engagement of students, and how they facilitated student learning, mathematical explorations were always followed by discussions on questions such as “How will you teach this content using DG software?” and “How will you lead your students in conjecturing and proving using DG software?” The PD facilitators valued teachers learning from each other and sharing ideas and also sought to provide opportunities to apply new teaching skills. Therefore, teachers were encouraged to present their insights on and experiences with the DG approach and to describe problems they might have experienced or anticipated with other teachers offering suggestions to address the concern. Teachers also prepared lesson plans that they shared with the entire group.

The Control Group

The teachers in the control group taught geometry as they had done before. They also participated in a PD workshop that addressed the same mathematical content as the DG group but without the use of technology. The PD sessions for the control group utilized teaching methods with which teachers were already familiar. The PD facilitators lectured and involved teachers in activity-based instruction. Participants engaged in problem solving without using technology tools. They spent the same amount of time in PD training as the teachers in the DG group. The control group PD was included in the research design to control the variables tied to professional development and ensure both groups experienced sustained, rigorous, and relevant development in high school geometry teaching. Since all teachers participated in PD sessions and all were presented with the same mathematics content, any differences measured between the two groups would be attributed to the presence (or lack of) the interactive DG learning environment (since it was the only instructional difference between the two groups).

Measures and Data Analysis

Measures  

A measure of teachers’ conjecturing-proving knowledge. A conjecturing-proving test was developed by the project team to measure teacher knowledge. As a result of a thorough literature review, geometry construct development, item construction, Advisory Board members’ review, and several pilot tests with resulting revisions, a test consisting of 26 multiple-choice items and 2 free-
response proofs was produced. The test was administered to the teacher participants as both a pre-
and post-test at the PD summer institute.

**Teachers’ implementation fidelity and classroom observations.** The DG approach involves
intensive use of dynamic software in classroom teaching to facilitate students’ geometric learning.

The critical features of the DG approach include using the dynamic visualization to foster students’
conjecturing spirit, their habit of focusing on relationships and explaining what is observed, and their
logical reasoning desire and abilities. To capture these critical features of the DG approach, two
measures of implementation fidelity (the DG Implementation Questionnaire [DGIQ] and the
Dynamic Geometry Observation Protocol [DGOP]) were developed. The DGIQ was adapted from a
teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and
her colleagues) in an NSF-funded project, based on the critical features of the DG approach. The
final version of the DGIQ consisting of six multiple-choice items and ten open-response questions
was administered to the teachers in the experimental group six times across the school year. A
different version of the questionnaire was administered to the control group teachers (also six times)
to examine how they teach geometry without using dynamic technology.

The DGOP was developed to address the critical features of the DG approach. It was adapted
from the *Reformed Teaching Observation Protocol* (Sawada et al., 2002). The final version of the
DGOP consisted of 25 items with a 4-point Likert response scale from *Never Occurred* to *Very
Descriptive* addressing four different aspects: (1) *Description of intended dynamic geometry lesson*,
(2) *Description of implemented dynamic geometry lesson*, (3) *Assessment of quality of teaching*, and
(4) *Assessment of engagement and discourse*. For the control group, an observation protocol (CGOP)
was developed by removing from the DGOP items related to the implementation of DG software
functions such as dragging and dynamic measuring. The DGOP or CGOP was administered in 16
Geometry classrooms (8 selected from each group). Each classroom was visited by two observers.
Each selected teacher was observed four or five times across the school year.

**Student level measures.** Two instruments were used for measuring students’ geometry
knowledge and skills: (for the pre-test) Entering Geometry Test (ENT) used by Usiskin (1981) and
his colleagues at University of Chicago; and (for the post-test) Exiting Geometry Test (XGT). The
XGT was developed by selecting items from California Standards Tests – Geometry. The final
version for XGT had 25 multiple-choice items. (See Jiang et al., 2011 for the details of the two tests.)

All research instruments mentioned above, except the student geometry pre-test, were developed
by the project team. For all project-developed measures, the Cronbach's Alpha statistical values were
within the acceptable ranges for reliability (e.g., reliability was calculated with Cronbach’s alphas of
0.957 and 0.952 for the DGOP and CGOP, respectively.) Item Response Theory (IRT) scoring
routines were applied to the DGIQ and students' post-test data providing evidence for the
instruments' construct validity.

**Data Analysis**

Two-level hierarchical linear modeling (HLM), other statistical methods, and the constant
comparison method (Glaser & Strauss, 1967) were employed to analyse the quantitative and
qualitative data.

**Results**

**Findings about Teacher Content Knowledge from the Conjecturing-Proving Test**

The participating teachers completed the conjecturing-proving test at the beginning and end of
the summer PD institute. A statistic for teacher content knowledge as measured by the instrument
was calculated by adding the number of correct multiple-choice responses with points from free-
response items. Average scores were 20.49 on the pre-test and 21.86 on the post-test with an average gain of 1.37. A paired-sample t-test showed that this gain was statistically significant (p = .003). These results show that the PD had a positive effect on teachers’ conjecturing and proving capabilities. The teachers in the experimental group showed a greater average gain (1.56) than the teachers in the control group (1.18); however, this difference was not statistically significant (p = .670).

Findings about DG Approach Teaching from the Classroom Observations

Table 1 provides the results of the DGOP administration measuring the levels of fidelity of the dynamic approach implementation in the DG group. If we focus our attention to the mean scores (with a maximum score of 4) for the DG group, we observe that the three aspects with the highest scores were Good Lesson Design, Use of DG Features, and Teachers’ Knowledge. The data provides evidence that the teachers in the DG group demonstrated an intention to implement the DG approach and to some extent they demonstrated knowledge about how to integrate the dynamic approach to teaching geometry. Overall, teachers in the DG group were implementing the DG approach at a moderate level (2.28). In part, this moderate level of implementation was explained by the challenges reported during the school year such as the inaccessibility of computer labs in the first several weeks and the pressure to spend time preparing for the state required tests. However, the majority of the classrooms observed can be described as being faithful to the DG teaching approach.

**Table 1: Comparison between DG and Control Groups**

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Sub-aspect</th>
<th>Mean DG</th>
<th>Mean Control</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intended Dynamic Lesson</td>
<td>Good lesson design</td>
<td>2.81</td>
<td>1.85</td>
<td>.032*</td>
</tr>
<tr>
<td></td>
<td>Use of dynamic features</td>
<td>2.75</td>
<td>0.70</td>
<td>.000*</td>
</tr>
<tr>
<td>Implementation</td>
<td>Actions beyond use of software</td>
<td>2.06</td>
<td>1.33</td>
<td>.095</td>
</tr>
<tr>
<td>Quality of Teaching</td>
<td>Cognitive demand</td>
<td>2.30</td>
<td>1.78</td>
<td>.113</td>
</tr>
<tr>
<td></td>
<td>Teachers’ knowledge</td>
<td>2.89</td>
<td>2.84</td>
<td>.924</td>
</tr>
<tr>
<td></td>
<td>Conjecture/Proof</td>
<td>1.93</td>
<td>1.40</td>
<td>.206</td>
</tr>
<tr>
<td>Engagement and Discourse</td>
<td></td>
<td>2.37</td>
<td>2.29</td>
<td>.735</td>
</tr>
<tr>
<td>Overall DGOP</td>
<td></td>
<td>2.28</td>
<td>1.68</td>
<td>.088</td>
</tr>
</tbody>
</table>

Comparing the two groups, Table 1 also shows the mean values of the CGOP and the p-values assessing the significance of the treatment effect computed using a mixed effect ANOVA. Results confirm the efficacy of the DG treatment by showing significant differences in the two aspects related to the intention to implement a dynamic lesson. As a whole, lessons in the DG group had a significantly better design aligned with the DG teaching approach, moving students from initial conjecture, to investigation, to more thoughtful conjecture, to verification and ultimately to proof. Further, lessons in the control group did not use dynamic features in teaching geometry. With respect to the other aspects of the DGOP (or CGOP), the two groups did not differ significantly, however all the DG ratings were higher than those of the control group. Note that most of those aspects assessed elements of the lesson that were not related to the use of dynamic features.

Findings about DG Approach Teaching from the Implementation Questionnaire

The purpose of the DGIQ was to assess the DG group teachers’ effectiveness and comfort in using GSP in teaching geometry. Also, the questionnaire results provided the frequency of teacher and student use of GSP. Figure 1 shows how the teachers rated themselves on their effectiveness and comfort in using GSP. Out of 31 teachers who completed the questionnaire, 29% felt that they were
at the high level of effectiveness, 61% at the middle level, and 10% at the low level. However, the majority of the teachers (97%) felt very comfortable or somewhat comfortable in using GSP in teaching. Overall, the teachers felt more comfortable than effective in using GSP with only one teacher not feeling comfortable in using GSP in teaching of geometry.

Figure 1: Effectiveness in Using GSP and Level of Comfort in Using GSP

Figure 2 shows average teacher and student use of GSP throughout the school year for those in the DG group. The “average teacher use of GSP” represents the average number of times per week the teacher used the demonstration computer in his/her classroom to do GSP presentations and demonstrations. The “average student use of GSP” represents the average number of times per week students worked in a computer lab doing hands-on explorations with GSP. Out of the 31 teachers who completed the questionnaire, 77% of them used GSP at least one time per week and 38% at least two times per week. However, the student use was lower, with 61% of them using GSP at least one time per week, and only 10% two times per week.

Figure 2: Average Teacher Use of GSP and Average Student Use of GSP per Week

Therefore, in terms of “taking students to the computer lab to do hands-on activities with GSP,” the teachers’ implementation of the DG approach was at the medium level of intensity. This finding is consistent with data from the classroom observations. However, almost all teachers were positive or enthusiastic in using GSP in geometry teaching. Again, considering the challenges that the teachers experienced during the school year, data supported the conclusion that most of the teachers implemented the DG approach faithfully.

Even though some teachers in the DG group might have not felt as effective in using GSP because of the students’ limited use of GSP (one time or less than one time per week), some of them might still be considered very effective if we focus on the ways they used the DG approach. One teacher provided such an example. He felt “somewhat effective” and his students used GSP on average one time per week, but his classroom observations showed very effective use of GSP. During one of the observations, his students were exploring the midsegments of a triangle and their goal was...
to come up with as many conjectures as possible. Students completed the constructions on their own, made initial conjectures based on their observations, used measurements to confirm their conjectures, and wrote their final conjectures. The teacher circulated among students and provided guiding questions when needed. One student made many measurements but no conjectures. The teacher asked this student, “Do you notice any relationships? What conjectures can you make?” These questions helped the student focus on the objective of the lesson and form conjectures based on the measurements and observations. During the lesson, students also engaged in conversations with one another to discuss their observations and conjectures. Students were actively involved in their learning and the teacher took on the role of a guide by prompting his students through questioning. This lesson not only showed effective use of GSP, but also addressed higher-level thinking.

Findings about Student Achievement from the Geometry Test

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the DG approach on overall student geometry achievement measured by the student post-test (XGT). The model was analysed using student pre-test (ENT) scores as a covariate. The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectation and quality of the students in each of these levels were very different, the factor Class Level was included in the model. Additionally, the covariate Years Exp (number of years of classroom experience) was included in the model. The results of the model indicated that the DG effect was strongly significant (p = .002). Comparing the means, the DG group was higher than the control group in each level of Geometry and the effect size (.45) was substantially larger at the Regular Geometry level. (See Jiang et al., 2011 for the details of the HLM analysis results.)

Using the integrative framework (Olive & Makar, 2009) as a lens, further quantitative and qualitative data analysis on the impact of the DG professional development is ongoing.

Discussion

The HLM model taking pretest, class level, and teaching experience into account provided evidence that the students of DG group teachers scored significantly higher than the students of control group teachers on the Exiting Geometry Test. Given that teachers were randomly assigned to the two groups and both groups received comparable sustained, rigorous, and relevant professional development on the same geometry topics, the results of this study provide evidence to support the finding that the DG professional development positively impacted the students’ geometry achievement. Both DG and control group teachers demonstrated significant gains on the Conjecturing-Proving Test through the one-week summer PD institute. This result suggests that both the PD sessions designed for the DG group and those designed for the control group had an effect on teachers’ conjecturing and proving ability. Although the DG and control teachers did not differ significantly on their mean gain scores, the DG teachers’ mean gain score was 32% higher than that of the control teachers. Classroom observation data revealed that lesson plans that the DG group teachers prepared were designed significantly better than the control group teachers’ lessons by facilitating students’ conjecturing and proving abilities. The teachers’ DGOP ratings (overall and in each sub-scale) were consistently higher for the DG group although most of the differences were not statistically significant. In summary, the results of this study suggest that the DG professional development offered to the participating teachers had a significant positive effect on the teachers’ mathematics conjecturing-proving content knowledge and their ability to implement a dynamic geometry approach to teaching. The teachers, in turn, helped their students achieve better geometry learning.
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