PROBLEM POSING: A REVIEW OF SORTS

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In this paper we offer a review of sorts of the studies conducted around issues of problem posing in mathematics education research. We first ground the work on problem posing in the seminal work of Polya and of Brown and Walter, which influenced most studies on this subject. We then propose two perspectives taken on problem posing: the implicit and the explicit. These illustrate the varying emphasis concerning the conception of what is meant by problem posing, one being about actual requests for creating a problem and the other about defining the nature of problem solving processes. We conclude by discussing the significance of this categorization for making theoretical advances in problem posing research.

Keywords: Cognition; Problem Solving

Context

Issues about problem posing have been around for a number of years in mathematics education research. This being so, recently there has been a resurgence of studies on the topic, illustrated through Working Groups (e.g. PME 2009, 2011), Special Issues (e.g. ESM, *83*(1)), and books (e.g. Singer, Ellerton, & Cai, 2015). Through this spread of studies on problem posing, however, numerous orientations have been developed, and often one is at a loss in making differences or even finding similarities between the perspectives taken. Far from being a negative aspect of the field, as it shows its richness and enlargement, there is however a need to distinguish and categorize the kind of work being conducted in order to develop clearer views on what problem posing means and how to study it. Other researchers have also attempted classifications in the past (e.g. Voica et al., 2013; Christou et al., 2005). We re-use and deepen these classifications, combining them with the work of Polya (e.g. 1957) and of Brown and Walter (e.g. 2005), who are seen as pioneers on the theme. In addition, we outline another line of studies to which little attention has been paid to, that is, studies focusing on the activity of problem solving defined as an activity of problem posing. Thus, in this paper we extend the current categorization of studies on problem posing, leading us to varied views of what is meant by problem posing in the community of mathematics education researchers.

To do this, we first situate the field on problem posing through an overview of the work of Polya and of Brown and Walter. We then offer a first category of studies, termed the *explicit perspective*, which focuses on explicit requests to students to participate in an activity of composing problems. We then offer a second category of studies called the *implicit perspective*, which focuses on studies that define the activity of problem solving as one of problem posing.

This being said, as expected, we do not claim to offer an exhaustive list of all work ever conducted on problem posing. In this sense, we do not offer a review, but mainly a review *of sorts*. The intention with this review of sorts is to offer fruitful distinctions related to the underpinnings of what is considered an activity of problem posing. Through these distinctions, we aim to take a step forward in the direction of Silver's (2013) suggestion for more developed theoretical frameworks to support studies in problem posing.

Pioneers of Problem Posing: The Work of Polya and of Brown and Walter

Numerous researchers have mentioned being influenced, directly or indirectly, by the work of Polya or Brown and Walter, making them important sources in the problem posing literature. We thus refer to their work as a way of grounding and contextualizing this review of sorts.

George Polya's Problem Posing

Polya's (e.g. 1957) work on problem solving focuses on helping and pushing students to analyse the problems they solve and to think of other interesting problems in relation to them. In so doing, for Polya, teachers help students to consolidate their knowledge, develop their ability to solve problems and improve their solution or their understanding of it. Polya did not used the expression *problem posing* in his work, referring mostly to what he called the *Looking back* technique, which enables students to generate new ideas and investigate possible connections between mathematical problems. Having solved a problem, students are asked to look at what they have done and then to formulate new problems out of it. Polya argues that through this activity, students gain a better understanding of their solutions and increase their solving abilities. To formulate new problems, Polya suggests various heuristics of *Looking back*, four of which are discussed here. For example, consider this problem for students to solve (Figure 1):

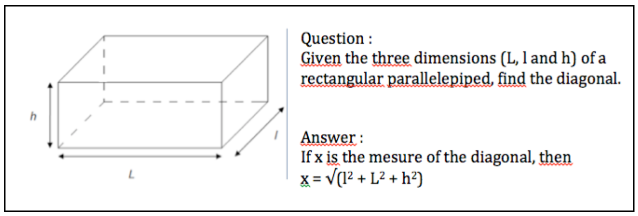

Figure 1: Polya's parallepiped problem (Polya, 1957, p. 7)

A first heuristic consists, once one knows the solution to this problem, of *generating analogous problems*, i.e. similar problems to this one. Polya gives examples of possible formulations: "a) Given the three dimensions of a parallelepiped, find the radius of the circumscribed sphere; b) The base of a pyramid is a rectangle of which the center is the foot of altitude and the sides of its base, find the lateral edges; c) Given the rectangular coordinates (x_1, y_2, z_3), (x_1, y_2, z_3) of two points in space, find the distance of these points." (1957, p. 66) These problems allow students to go back to the initial solution, but for other contexts, which requires them to rethink the solution and not only apply the formula. A second heuristic consists of applying the formula found by *modifying* the problem and its data. For example, the initial problem requires looking for the diagonal of the parallelepiped in relation to its width, length, and height. Another problem can be formulated by asking to find the height of the parallelepiped depending on the diagonal, the length, and the width. This heuristic requires interchanging the role of the various givens of the problem. Polya's third heuristic is *generalizing/specifying*. Generalizing consists in solving the same problem, but for an entire category of numbers or givens. For the above problem, one possible generalization could be: "Find the diagonal of a parallelepiped, being given the three edges issued from an end-point of the diagonal, and the three angles between these three edges." (1957, p. 67), which requires e.g. aiming for algebraic letters to represent the needed values of the problem. Also, a way of specializing the problem would be to look for specific cases, like finding the diagonal of a cube knowing one of its edges. A fourth heuristic is *studying variations*, that is, studying the effect of varying some of the data in the problem. For example, in the analogous problem of the circumscribed sphere, one can vary the radius of the sphere and study its effect on the problem and solution, leading to three possible cases: the sphere is entirely contained in the cube, the sphere is circumscribed in the cube; and finally the sphere encompasses the cube. Polya's heuristics are illustrations of his *Looking back* approach. For him, binding problem posing to problem solving allows students to see the possible mathematical connexions between various problems. By looking back at their solution, by reconsidering and examining the solution and the path they have followed, he argues that students consolidate their knowledge and develop their problem solving skills.

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Brown and Walter's Problem Posing

The main goal of Brown and Walter's (e.g. 2005) problem posing is to study mathematics by working on students' questions and reflections on a given topic. For Brown and Walter, questions that arise in the classroom must not only be instrumental (i.e. posed to ensure understanding and performance of what the teacher asks students to do), but rather should help student to develop their mathematical skills, understanding and autonomy. In *The Art of Problem Posing*, Brown and Walter present two perspectives of problem posing: *Accepting* and *What-if-not?* (WIN). These perspectives are to help teachers to develop strategies for using problem posing in class with their students. The *Accepting* perspective refers to students accepting a concept suggested by the teacher (e.g. the concept of prime numbers defined as natural numbers that have exactly two natural divisors), and then finding interesting problems/questions about it. In the case of prime numbers, it could be questions like: *How many prime numbers are there? How to find the next prime number?* They argue that this leads students to explore and work mathematically on a concept, in order to develop an understanding of it.

The WIN perspective consists, on the other hand, in seeing what happens when instead of "accepting" the concept, one contests its characteristics. Brown and Walter suggest various levels of WIN, which they illustrate with the example of the Pythagorean theorem. A first level is for students to list the attributes of the Pythagorean theorem. For example, it may be noted that all number are squared or that the variables are connected by an equal sign. A second level consists of asking a WIN question for each of the attributes listed. For example, *What-if* the variables were *not* connected by an equal sign, but by an inequality? This question opens and becomes a new route to be explored for both students and teacher. A third level, called the *What-if-Not-ing* level, requires combining the negation of two of the attributes listed. In this case, it could be by looking at what happens when the variables are not linked by an equal symbol *and* all numbers are not squared. This also opens a new route to explore. For Brown and Walter, these mathematical explorations allow students to understand the Pythagorean theorem through the importance of its mathematical attributes, as well as developing their ability to formulate questions, explore mathematics, and solve problems. In this sense, the authors argue that after solving a problem, a person does not fully understand the meaning of what he/she has done unless new interconnected problems are formulated and analyzed, which affords a better grasp of the concept worked on. Thus Brown and Walter's problem posing is related to an inquiry process that leads to the exploration of concepts for understanding them better, arguing also for openness toward mathematical questions and thoughts that occur in classrooms.

Conceptualizing Problem Posing: Explicit and Implicit Perspectives

Grounded or not in Polya or in Brown and Walter's ideas, various meanings about problem posing are found in the literature. We distinguish these meanings by suggesting two perspectives. The explicit perspective refers to an explicit request for students to compose problems, whereas the implicit perspective refers to something that occurs implicitly in the activity of problem solving, i.e. every act of problem solving is seen as an activity of problem posing in itself.

The Explicit Perspective: A Pragmatic View of Problem Posing

In the explicit perspective, we distinguish three categories of studies being conducted, highlighting their diverse but complementary nature. We discuss these and give examples for each of them. In our description, we use the word *learners* to refer whether to students or (prospective) teachers who are doing the various kinds of problem posing.

Category 1: To compose a problem without any context or constraint. This first category refers to asking learners to compose a problem without imposing any context or constraints. In short, they need to compose from scratch. This category of problem posing can be linked to what Stoyanova and Ellerton (1996) call a free problem posing situation where students have to formulate

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new problems in an open situation. For their part, Christou et al. (2005) refer to this kind of problem posing as tasks that require students to pose a problem in general, in free situations. In this category, we find, for example, the work of Ellerton (1986) and of Crespo (2003). In her work, Ellerton asks students to compose a problem that would be difficult, a challenge, for a friend to solve. The students then have a blank card to compose mathematical problems of any kind related to the concepts that they wish. In Crespo's study, elementary students are paired with prospective teachers and correspond one-on-one by sending each other letters. Through this, Crespo aims at placing prospective teachers in an authentic experience of generating problems by asking them to write mathematical problems in their letters for their elementary student. The prospective teachers have no constraints on the type of problem to compose or the mathematical concepts to use.

Category 2: To generate problems from specific constraints. Another category refers to asking learners to generate problems on the basis of specific constraints. Many, if not most, studies conducted on problem posing can be placed in this category. In fact, this category can even be subdivided into three subcategories covering the constraints given to learners for generating problems: (a) generate from a general context; (b) generate from specific constraints; (c) generate from a previously solved problem.

The *generate from a general context* subcategory contains studies that ask learners to generate a problem in a general context. Brown and Walter's (e.g. 2005) *Accepting* perspective of problem posing is an example of this. In the example given above about *Accepting*, the context is the prime numbers, which are introduced to students who then have to generate problems about this mathematical concept without other indications. Work conducted by English (1998) is also of this type, where students have to compose problems in an informal mathematical context free of symbolic representation. For example, she asks students to make up a story problem about what they see in a large photograph of children playing with a set of brightly coloured items. A general mathematical context is then given to learners who have to generate problems from it. The subcategory *generate from specific constraints* refers to studies that ask learners to generate problems within or in relation to specific constraints. Silver (1994) refers to this subcategory as *problem generation*, where the goal is the creation of new problems from a situation prior to any problem solving. This subcategory can also be linked to what Christou and *al.* (2005) call a task that requires students to pose a problem with a given answer, a problem that contains certain information, a question for a problem situation, or a problem that fits a given calculation. Brown and Walter's (e.g. 2005) WIN perspective, as discussed above, takes place in this subcategory as it asks students to generate problems based on a initial mathematical situation using the WIN technique. The WIN technique is seen here as a constraint because it gives insight into the kind of problem students have to generate. Lavy's works (Lavy and Bershadsky, 2003; Lavy and Shriki, 2007), using the WIN technique in class with prospective teachers in a geometry context, is another example of this subcategory. We can also refer to studies of Silver and Cai (1996) and Silver, Mamona-Downs, Leung and Kenney (1996), in which before solving a mathematical task, students are asked to compose three problems that can be solved from the information/data given in an initial problem. When they have composed a new problem based on this one, students are asked to solve eight related problems. The researchers then studied the nature of the composed problems and the relationship between their ability to compose and to solve problems. This kind of problem posing contrasts with the other subcategories because the specific constraints (the technique or the problem) guide the kind of problem that learners would be more likely to compose. The last subcategory, *generate from a previously solved problem*, contains studies asking learners, *after* having solved a specific problem, to create other problems based on this solved problem. The problems then created are modifications of the goals or the conditions of the previously solved problem. Silver et al. (1996) above mentioned study is also in this subcategory, where in another part of their study students have to generate a problem from previously solved ones. Polya's (e.g. 1957) heuristics of the *Looking back* technique (*analogies*, *modifying*, *generalizing-specifying* and *studying variations*) are also examples of this.

Category 3: To transform an initial problem. The third category of problem posing is intricately linked to problem solving strategies, as it contains studies that ask learners to transform an initial problem in order to solve it. This kind of problem posing occurs during the problem solving process, when students are invited, as an efficient solving strategy, to reformulate for themselves the given problem. For example, strategies given to students are to decompose the problem into sub-problems, to simplify or modify the original problem or to solve a related problem. Students use these strategies to achieve one goal: to be able to solve the original problem. The problem posing is then seen as a *means* of solving the given problem. Silver (1994) refers to this category of problem posing as occurring during the process of solving when students must ask themselves “How can I formulate this problem so that it can be solved?” (p. 20). Kilpatrick (1987) mentions that problem posing consists of reformulating an existing problem in order to make it one’s own; seeing problem posing (what he calls problem formulating) as an important companion to problem solving. Mason, Burton and Stacey’s (1982) *Thinking Mathematically* book explains this approach in detail to help learners solve a problem. In a similar vein, Polya’s (1966) video on *Guessing* amply illustrates this category of problem posing. In the video, Polya tells his students that if a problem is too difficult to solve, they should pose easier sub-problems, which could prepare them to solve the bigger problem; examples explored are to consider the problem in 2D instead of 3D, reducing the number of constraints/givens in the problem, and so forth. The aim is that, as they solve these sub-problems, learners gain a better sense of the original problem and prepare themselves for solving it.

The Implicit Perspective: An Epistemological View of Problem Posing

Studies under what we term the *implicit perspective* are less frequently, if ever, accounted for in reviews on problem posing. In this perspective, we integrate studies that conceptualize the problem solving process as events of problem posing. Thus whereas in the first perspective the notion of problem posing was related to *explicit* requests for creating problems through varied contexts, in this second perspective the notion of problem posing happens *implicitly*, without any request, as it defines the activity of problem solving itself. In various ways, work conducted under this lens makes the argument that when students solve problems, they are in fact posing their own problems, as we show below.

Category 1: Problem posing that influences the problem solving path. This category comprises studies that focus on the link between problem posing and problem solving, emphasizing the influence of the posed problems on the solving process. The work of Sevim and Cifarelli (2013) illustrates this. They argue that when solving a problem, a solver creates his/her own goals and purposes. These change as the solver progresses in the solution and also indicate the path of solving that the solver chooses. Armstrong’s (2013) work is another example of this, as she records and studies students’ questions that arise while they are solving a problem, and which influences the course of the solving. In her work, she looks at the questions that a group of students working together ask themselves when solving a task. For example, students would ask questions like “What is meant by an interval?” “Is it a square root?” or “What if there are x people?” (p. 67), in order to think about the task and arrive at a solution. Each group of students asks different or similar questions, and some groups also ask questions more than once during the same problem solving process. Armstrong made a “tapestry” schema out of this that shows the problem posing path followed by each group when solving their problem. She illustrates that the process used by learners directly influences their problem solving process. Stoyanova and Ellerton (1996) definition of problem posing, “the process by which, on the basis of mathematical experience, students construct personal interpretations of concrete situations and formulate them as meaningful mathematical problems” (p. 518), can be linked to this implicit problem posing. In order to solve a problem,

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learners formulate for themselves meaningful problems on the basis of their mathematical experience; this is not done as a request or as a strategy for solving, but mainly reflects what they do, their solving processes.

Category 2: Gaps between teachers' and students' task. Researchers like Perrin-Glorian, Robert and Rogalski (see e.g. Perrin-Glorian & Robert, 2005; Robert & Rogalski, 2002; Rogalski, 2003) have also worked along those lines to develop meaning about students' problem solving processes. Like Polya, they have not used the expression *problem posing*, but have focused on students' interpretations of the problems given to them, where they formulate for themselves, they pose, what the problem to be solved is. Rogalski (2003) identified various natures that tasks can assume when presented by teachers in the classroom, all of this happening implicitly during the activity of problem solving. First, the teacher prescribes a task to students, which consists essentially in the formulation of the problem. This *prescribed task* may be directly observed, as it consists of the instructions presented by the teacher to the students. The teacher has expectations about the task that the students have to work on: this is the *expected task*. On the other hand, students do not necessarily work on the teachers' prescribed task, but on one that has been redefined from that prescribed task. The *redefined task*, therefore, represents the student's personal representation of the task; somehow his/her *implicit* posing of the task in his/her own terms. Finally, the *effective task* is the actual one to which the student responds, which is not necessarily identical to the one he/she thinks he/she is responding to; this leads to a dynamical interrelation between the *redefined* and *effective* task. These redefined and effective tasks illustrate the problem that the student actually asks/poses him/herself and intends to solve (again, all of this happening implicitly in the solving process). The studies conducted within this framework focus on the gaps between teachers' expectations and students' mathematical activity in solving problems, their problem posing process.

Category 3: Problem posing versus problem solving. Work in this third category is related to Varela's (1996) epistemological definition of problem posing, which he contrasts with problem solving. For Varela, problem solving implies that problems are already in the world, independent of us, waiting to be solved. Varela explains, on the contrary, that we specify the problems that we encounter through the meanings we make of the world in which we live, leading us to recognize things in specific ways. We do not choose problems that are out there in the world independent of our actions. Rather, we bring problems forth, we pose them: "The most important ability of all living cognition is precisely, to a large extent, to pose the relevant questions that emerge at each moment of our life. They are not predefined but enacted, we bring them forth against a background." (p. 91). The problems that we encounter, the questions that we ask, are thus as much a part of us as they are a part of our environment: they emerge from our interaction with/in it. The problems that we solve are relevant for us as we allow them to be problems. Working in this perspective, René de Cotret (1999) notes that one cannot assert that instructional properties are present in the tasks presented and that these *causally determine* solvers' reactions. As Simmt (2000) explains, it is not tasks that are given to students, but mainly prompts that are taken up by students who themselves create tasks with. Prompts become tasks when students engage with them, when, as Varela would say, they pose them as problems. This posing, as we show in Proulx (2013) about mental mathematics contexts, determines the task solved, hence the strategy developed for it. Students *make* the "wording" or the "prompt" a multiplication task, a ratio task, a function task, an algebra task, and so forth, and solve accordingly, which leads to varied strategies and answers because they often start from "different" posed problems.

In this implicit perspective, students play an important role in what the problem to solve is: not because they have created it, but because a student is always solving his/her own problem, from a given prompt. In this sense it is implicit, that is, it is not an explicitly requested task by someone external to the student (as in the first explicit perspective), but something implicit in the solving process when students engage with the(ir) problem to be solved. In sum, they *implicitly* create a problem in the action of solving the problem; whereas in the first explicit perspective they were explicitly asked to create a problem from various context and data given. Both perspectives and their categories are summarized in Table 1.

Table 1. Summary of Implicit and Explicit Perspectives

Explicit perspective on problem posing (as an explicit request to learners)	Implicit perspective on problem posing (as defining the problem solving process)
To compose a problem without any context or constraint	Problem posing that influences the problem solving path
To generate problems from specific constraints	Gaps between teachers' and students' tasks
To transform an initial problem in order to solve it	Problem posing versus problem solving

Final Remarks on these Categorizations of Problem Posing

What do we learn from this? This categorization of work conducted on problem posing is more than a review: it is an extension of the field. By integrating the work under the implicit perspective, which we have seldom encountered in reviews and activities about problem posing (Working Groups, books, Special Issues), we extend what are normally considered as studies in problem posing by opening the way to epistemological considerations about students' mathematical activities. Whereas problem posing as a field is widely known in terms of an activity to plunge students into, as a teaching device or as a strategy for solving problems (see e.g. Voica et al, 2013; Christou et al., 2005), what we have grouped under the implicit perspective is much less known and tackles epistemological issues related to students mathematical activity in itself. Epistemological questions/issues are not new in problem posing work, as some of them can be seen and felt through the work of Stoyanova and Ellerton (1996) and of Kilpatrick (1987), however they oscillate between a view of problem posing as a request on students and as representing students' mathematical activity. The distinction offered here between explicit requests for problem posing and the implicit problem posing activity happening in students' mathematical activity appears fruitful for developing a sharper understanding of what is meant by problem posing and clarifying where one's focus is. As mentioned, this can be felt as a step forward in the direction of Silver's (2013) suggestion for developing finer theoretical frameworks to support studies in problem posing.

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