INVESTIGATING SECONDARY MATHEMATICS TEACHERS' ATTITUDES TOWARD ALTERNATIVE COMMUNICATION PRACTICES WHILE DOING PROOFS IN GEOMETRY

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We used multimedia surveys to investigate secondary mathematics teachers' reactions to storyboards that represented episodes of instruction. Participants were asked open-ended questions about the storyboards. We analyzed the responses to the open-ended questions for evidence of the attitudes *(Martin & White, 2005) that participants conveyed about the episodes. We found that, when presented with storyboards that depart from what is hypothesized to be routine instruction, participants' open responses included significantly more negative than positive linguistic markers of attitude. At the same time, when participants were shown storyboards that represented what routinely happens in classrooms, markers of positive and negative linguistic markers of attitude occurred with equal frequency.*

Keywords: Instructional Activities and Practices; Geometry

Introduction

For as long as mathematics has been taught in US public schools, there have been initiatives that have attempted to improve the quality of mathematics teaching in classrooms. A fundamental challenge for such initiatives is the paradox of *change without difference*: reform efforts that, in principle, could bring about fundamental shifts in classrooms emerge, in practice, as "shadows of their original intent" (Woodbury & Gess-Newsome, 2002, p. 763). One reason for this paradox is that the patterns of classroom interaction that practicing teachers have honed through years of experience are robust. Initiatives that aim to effect change in the way that mathematics is taught thus need to contend with the realities of the already established practice of mathematics teachers (Cobb, Zhao, & Dean, 2009). To understand how reform efforts might contend with such realities we need to raise a natural question: When teachers encounter reasonable departures from routine instruction, how do they relate to such actions?

To answer this question, we conducted a study that used multimedia surveys to investigate secondary mathematics teachers' reactions to storyboards that represented episodes of instruction. Participants were asked open-ended questions about the storyboards. We analyzed the responses to the open-ended questions for evidence of the *attitudes* (Martin & White, 2005) that participants conveyed about the episodes. We found that, when presented with storyboard representations of reasonable departures from what we hypothesized to be routine instruction, participants provided open responses that contained more negative than positive linguistic markers of attitude. At the same time, when participants were shown storyboards that represented what routinely happens in classrooms, positive and negative markers of attitude occurred with equal frequency.

The analysis reported below is part of a larger study whose objective was to investigate instructional routines that pertain to discipline-specific communication practices. The communication skills used by disciplinary experts have traditionally been thought to be gradually and tacitly developed by novices as the novices are apprenticed into a field (Lemke, 2013; Thurston, 1994). But recent work in analyzing mathematical communication suggests that discipline-specific ways of communicating are practices that can be described and taught (Fang, 2012; O'Halloran, 2011; Yore, Pimm & Tuan, 2007). *Doing proofs* is one classroom activity during which students could develop discipline-specific communication practices. As the geometry classroom has historically been the principal instructional setting in which students are introduced to mathematical proof (Knuth, 2002),

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the instructional situation of doing proofs in geometry (Herbst & Brach, 2006) was the focus of this study.

Theoretical Framework

Classroom activity can be modeled as a social system in which an agent playing the role of teacher and other agents playing the roles of students act in accordance with tacit but mutually held norms (Herbst & Chazan, 2012). Such norms help characterize instructional situations: stable segments of classroom activity in which students' work is exchanged for claims that they have acquired items of knowledge (Herbst, 2006; Herbst & Brach, 2006). Though the instructional activity of doing proofs in geometry has been criticized by mathematics educators for being a misrepresentation of the work of proving in mathematics (Schoenfeld, 1988; Martin & Harel, 1989; Lockhart, 2009), it endures as an instructional setting where students are introduced to the notion that there is such a thing as mathematical proof. The goal of this work was to describe norms of the instructional situation of doing proofs that pertain to how student proofs are presented and checked in geometry classrooms.

We use *norm* to refer to those aspects of social situations that not only regularly happen but also that participants (in social situations) *expect* to happen (Garfinkel, 1963). In social situations, when people confront departures from what they expect, they can react with anxiety, bewilderment, or anger (Mehan & Wood, 1975). Such negative reactions are ways in which people mark that a norm has been breached. The work reported here used the notion of a *breaching experiment* (Garfinkel, 1963) to investigate secondary teachers' reactions to episodes of instruction in which hypothesized norms[1] of instructional situations were breached.

Method

Our method of inquiry combined a planned-comparison study with a virtual breaching experiment (Herbst, Aaron, Dimmel, & Erickson, 2013) in an instrument that we call a *virtual breaching experiment with control* (Dimmel & Herbst, 2014). The instrument was a multimedia survey that used storyboards to represent episodes of geometry instruction that were inspired by video records of actual geometry classrooms. Our use of storyboards to probe for recognition of norms is analogous to how scripted classroom videos have been used to probe teacher professional knowledge (Kaiser, 2014) and is an application of the cyclical use of records of practice (Jacobs, Kawanaka, & Stigler, 1999). Each participant[2] in our study viewed two sets of parallel storyboards: one set of parallel storyboards represented departures from hypothesized norms (i.e., *breach* storyboards), and the other set of parallel storyboards represented instances of instruction that were hypothesized to be routine[3] (i.e., *control* storyboards). The storyboards that were designed to represent routine instruction (i.e., the control storyboards) were based on video records of actual geometry classrooms, hence our claim that these storyboards represent the instruction that might typically occur in geometry classrooms. The storyboards in a set were parallel in the sense that they targeted the same hypothesized norm.

After viewing each storyboard, participants were given four opportunities to provide open-response data. The first question that participants were asked is: "What did you see happening in this scenario?" The purpose of prompting participants with this broad, open-ended question was to capture participants' overall reactions to the instances of doing proofs (hereafter: situation instances) that were represented by the different storyboards. This general open response question has been used in previous virtual breaching experiments (Herbst, Aaron, Dimmel, & Erickson, 2013) as a means to capture participants' reactions to storyboards. Participants had three other opportunities to provide open responses, following their review of each storyboard. These open response fields followed episode—how appropriate was the teacher's review of the proof in this scenario?—and segment-specific—how appropriate were the teacher's actions in this segment of the storyboard?—appropriateness rating questions.[4] Following each rating question, participants were prompted to "Please explain your rating."

Responses to the four open response questions were coded using a scheme derived from the *attitude* system of the appraisal framework (Martin & White, 2005). Coding for attitude is a means to capture participants' ways of feeling (Martin & White, 2005) toward the situation instances represented by the storyboards. The attitude system differentiates statements of affect, judgment, and appreciation. Statements of *affect* convey personal feelings through linguistic markers of emotion, such as "sad", "happy" or "angry" (Read & Carroll, 2012). Statements of judgment convey evaluations of people and their deeds, such as "he is a good teacher." Statements of appreciation convey aesthetic evaluations of non-person things in the world (goods and services), such as "that is a clear proof" (Read & Carroll, 2012). The scheme we developed coded each response for all instances of attitude. Attitudes were classified by type—is it judgment, affect, or appreciation?—target—e.g., the teacher, the proof—and polarity—positive or negative.

An example of a response that conveys a positive judgment of the teacher is: "teacher is guiding students effectively." In this response, the teacher is the target of the attitude and "effectively" is a positive evaluation of the action—guiding—that the teacher is described as doing in the scenario. Since the response is about the quality of how a person (i.e., the teacher) performs an action, it is coded as a positive judgment. An example response that contains a negative judgment of the teacher is: "This teacher is being a bit ridiculous." These examples were coded as judgments because, in each case, the targets of the appraisals are people and their deeds. An example response that contains an appreciation is: "the math proof was not accurate." In this example, a mathematical proof is the target of the appraisal. It was coded as a negative appraisal because the response states that the proof is " not accurate."

Reliability of the Attitude Coding Scheme

The *attitude* scheme was tested for reliability by comparing coded responses of two independent coders[5]. The coders applied each scheme to 100 randomly selected texts in the corpus—25 of each of the 4 response types, roughly 10% of the total number of responses. Before each text was coded, it was blinded with respect to whether the response was provided for a storyboard in which the norm was breached or a storyboard in which the norm was not breached. The purpose of blinding the data was to minimize bias. The *kappa statistics* for the *attitude* coding for which there were sufficient instances of the codes to warrant the statistics are .79 for *negative judgments* of the teacher; .49 for *positive judgments* of the teacher; .77 for *negative mathematical* appreciations; .49 for *positive mathematical* appreciations. These *kappa* scores indicate moderate (.49), high (.76, .77, .79), and very high (.89) agreement between the coders.

Data

Data was gathered from 73 secondary mathematics teachers located within a 60-mile radius of Midwestern University. Participants completed the instrument during in-person and online data collection periods that occurred during the 2013-2014 academic year. The multimedia survey (described above) that contained the four storyboards was one of several instruments participants completed during a day-long data collection event.

Results

Each response in the corpus was coded for judgments, appreciations, or statements of affect, and each instance of an attitudinal appraisal was coded in each response. This means that it was possible for a response to contain multiple instances of the same kind of statement of attitude (e.g., there could have been several judgments of a teacher), as well as instances of different kinds of statements of attitude (e.g., a judgment of the students, an appreciation of the proof), and statements of attitude that had different polarities (e.g., a response could contain both positive and negative judgments of the teacher).

We hypothesized that, in the case of storyboards that represent a breach of a norm, participants would react negatively. These negative reactions would be evident by higher numbers of negative statements of attitude. By contrast, based on the premise that the control storyboards represent routine teaching, we hypothesized that open responses associated with control storyboards would contain roughly equal numbers of positive and negative statements of attitude. Such a distribution of positive and negative statements of attitude could be explained on the basis of individual differences (among participants) alone.

Table 1 shows the total number of positive and negative statements of attitude throughout the open responses in the corpus. The results are reported according to storyboard condition. The entire corpus contained 1168 open responses to the 4 different open response questions, with equal numbers of responses to the breach and control conditions (584 responses per condition). These were divided equally (146 per question) among the 4 open response questions that were described above. The results reported in Table 1 are for the entire corpus across all 4 question types. Furthermore, the results reported in Table 1 are simple counts of the number of statements of positive or negative attitude that were coded in the open responses to the storyboards in the breach and control conditions.

Table 1: Counts of statements of attitude, tallied by polarity and storyboard condition.

Storyboard Condition	Statements of Positive Attitude	Statements of Negative Attitude	Total
Breach Storyboards	211	473	684
Control Storyboards	309	310	619

The results reported in Table 1 are consistent with the hypotheses stated above. The open responses to the storyboards in which a hypothesized norm was breached had more statements of negative attitude than statements of positive attitude. In the case of the control storyboards, there were nearly equal numbers of positive and negative statements of appraisal. A chi-square test indicates that there is a significant association between storyboard condition and the number of positive or negative statements of attitude ($\chi^2 = 48.49$, $p < .001$).

The unit of analysis for the results reported in Table 1 is a statement of attitude. This means that each statement of attitude in a response was included in the totals. To further investigate the relationship between attitude polarity and storyboard condition, we recoded the data to eliminate multiples, by polarity, within each response. Thus, if a response contained 3 positive statements of attitude and 2 negative statements of attitude, it was recoded as (1) for *positive attitude* and (1) for *negative attitude*. Table 2 reports tallies of statements of attitude after applying this reduction. The unit of analysis for the results reported in Table 2 is an open response (n = 584 for each storyboard condition).

The results reported in Table 2 are consistent with those reported above. Across the corpus for the control storyboards, there were 248 responses that contained at least one statement of positive

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Table 2: Counts of open responses that contain positive or negative statements of attitude, by storyboard condition.

Storyboard Condition	Statements of Positive Attitude	Statements of Negative Attitude	Total
Breach Storyboards	176	352	528
Control Storyboards	248	245	493

attitude and 245 responses that contained at least one statement of negative attitude. By contrast, across the corpus for the breach storyboards, there were 176 responses that contained at least one statement of positive attitude compared to 352 responses that contained at least one statement of negative attitude. A chi-square test of association indicates that there is a significant relationship between storyboard condition and attitude polarity ($\chi^2 = 29.54$, $p < .001$).

The results reported in Table 2 are a refinement of the results reported in Table 1 because multiples have been eliminated. The results reported in Table 3 (below) refine these results further by distinguishing 4 categories of response: those that contain only positive statements of attitude, those that contain only negative statements of attitude, those that contain both positive and negative statements of attitude, and those that contain no statements of attitude.

Table 3: Counts of open responses that contain only positive attitude, only negative attitude, both, or neither.

Storyboard Condition	Statements of Positive Attitude	Statements of Negative Attitude	Both Positive and Negative Attitude	None	Total
Breach Storyboards	78	254	98	154	584
Control Storyboards	166	163	82	173	584

The results reported in Table 3 are consistent with those reported in Table 1 and Table 2. The breach storyboards contained more responses that contained only negative statements of attitude than responses that contained only positive statements of attitude. By contrast, the control storyboards contained nearly equal numbers of responses that contained only positive statements of attitude and responses that contained only negative statements of attitude. A chi-square test of association indicates a significant relationship between storyboard condition and the categories of attitude in Table 3 ($\chi^2 = 54.12$, $p < .001$)

The results reported above provide evidence to support our hypotheses: Throughout the corpus, the responses to the breach versions of the storyboards yielded more negative statements of attitude than positive statements of attitude. That responses to the breach versions of the storyboards produced more negative statements of attitude is consistent with the results of the breaching experiments conducted by Garfinkel (1963) and the virtual breaching experiments conducted by others (Herbst, Aaron, Dimmel, & Erickson, 2013). In contrast with the breach storyboards, responses to the control storyboards contained roughly equal numbers of positive and negative statements of attitude. Because the breach and control storyboards were the same except during those frames where the teacher is shown breaching (or complying with) a hypothesized norm, it follows that the teacher's breach of the norm is what prompted participants to react negatively to the storyboard.

Conclusion

We began with the question: When teachers encounter reasonable departures from the routine, how do they react? The results reported above provide evidence that secondary mathematics teachers react more negatively to episodes of instruction that represent breaches of hypothesized classroom norms than to episodes of instruction that represent instruction that is hypothesized to be routine. We provide context for these findings here by describing the nature of the breaches.

The storyboards representing instances of *doing proofs* were scripted to investigate communication practices expected by secondary teachers when proofs are presented (by students) and checked in geometry classrooms. An example of a routine practice for presenting proofs is that of a student going to the board and creating a mark-for-mark reproduction of an already completed proof—an act we call *proof transcription*. Such transcriptions of proofs by students do not match the practices used by disciplinary experts, where a proof is described verbally (with accompanying gestures) as it is generated at a blackboard (Artemeva & Fox, 2011; Greiffenhagen, 2014; Núñez, 2009).

In our study, one set of breach storyboards depicted teachers interfering with student transcriptions of proofs, for example, by requiring students to provide labels on a diagram before using those labels in a proof. The teacher's interference could be defended as reasonable on the grounds that the teacher is steering the student presenter toward staging a discovery—as opposed to a reproduction—of the proof that shows how the student engaged with the material artifact of the proof (Livingston, 1999). Such a move on the part of the teacher could be seen as an effort to bring the student's proof presentation practices more in line with disciplinary practices for presenting proofs (Greiffenhagen, 2014). In fact, some participants in the study remarked on the positive instructional value of the teacher's interference in such storyboards. Yet on the whole, the attitudes that participants expressed toward the teachers that interfered with the student transcriptions tended to be negative. What are we to make of these findings?

One implication is that it is possible that teachers could recognize, in the abstract, the value of an instructional alternative yet prefer, in actuality, the routines they have developed. The tendency toward routine is not evidence of a deficiency in teachers but is rather a fact of social life (Garfinkel, 1963). We see teachers' preference for routines as a resource that could be used to design instructional alternatives that are likely to have greater uptake by practicing teachers. For the work of managing student presentations of proofs in geometry classrooms, the expectation that students create mark-for-mark reproductions of proofs could be the basis for alternatives that would help students develop discipline-specific communication practices. An example of such an alternative could involve asking students to present proofs in pairs, where one student is responsible for generating the transcription and the other student explains the proof as the transcription is being completed. Such an alternative practice would recognize the value in the existing routine—i.e., that the proof that is displayed on the board is an accurate record of the work the student completed—while at the same time provide a scaffold for students to develop the proof presentation skills that are used by mathematical experts.

Endnotes

[1]The target norms were: (1) hypotheses about the details students are expected to include in a proof, and (2) hypotheses about how students are expected to present proofs during class.

[2]Participants were randomly assigned to one of five different treatment groups.

[3]The design of the study was described in a prior report. See (Dimmel & Herbst, 2014) for details.

[4]Analysis of the closed-ended responses to the rating questions were reported in a prior study (Dimmel & Herbst, 2014).

[5]We acknowledge the support of Nicolas Boileau for assisting with the reliability study.

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