

USING LINGUISTICS TO EXAMINE A TUTORING SESSION ABOUT LINEAR FUNCTIONS

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We applied techniques from systemic functional linguistics to examine how a student and a tutor construed meaning related to linear functions during a 1-1 tutoring session. The student and tutor varied in how they discussed rates of change. This difference highlights that there are multiple correct ways to use this term in algebra, although small differences in speakers' use may create the potential for confusion. Additionally, the different ways that the student and tutor spoke about rates of change illuminate how any scenario may be represented by multiple different linear functions. This study has implications for providing teachers in a variety of settings information about how students construct meaning through their discussions.

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In most school districts in the U.S., Algebra 1 is a required course for students to graduate from high school and go on to post-secondary opportunities. In this way, Algebra 1 can be viewed as a “gatekeeper course”; in other words this course is a border between the work students are required to complete and the opportunities that come beyond it. Many students struggle to learn the foundational concepts of a typical Algebra 1 course, and disparities in student performance are greater along racial and socioeconomic lines (Chazan et al., 2007; Lubienski, 2002). There are efforts in place to provide one-on-one or small group tutoring services outside of students' typical mathematics classes, so that students can receive individualized attention and instruction (Hord, DeJarnette, & Marita, 2015; Hord, Marita, Walsh, Tomaro, & Gordon, in press). We conducted a project in which a group of university pre-service teachers provided tutoring for struggling eighth-grade students in Algebra 1 at an urban, high-needs public school. We sought to understand the features of students' and tutors' interactions that supported students' achievement in the course and conceptual understanding of algebra.

In this study, we focus specifically on one tutoring session between a tutor, whom we call Emily¹, and an eighth-grade student named Tanisha. We pose the question, *what are potential sources of ambiguity in a conversation between a student and a tutor about linear functions?* We use Systemic Functional Linguistics (SFL) (Halliday & Matthiessen, 2014) as an analytical tool to identify potential ambiguity in the pair's discourse. By looking for ambiguities in conversation, we bring up some of the barriers in communication that may surface when experts and novices talk about mathematics. Through a better understanding of how teachers and students construct meaning through discourse, we see opportunity to overcome these barriers and support all students to be successful in and beyond Algebra 1.

Theoretical Framework

We draw on a social semiotic framework to inform this study. This perspective emphasizes the importance of our choices in representation for constructing meaning through social activity (Kress & van Leeuwen, 2006; O'Halloran, 2014). Interactions in mathematics classrooms are multi-semiotic, in that communication between teachers, students, and textbooks requires a variety of representation systems including visual representations, symbolic notation, and gesturing (Alshwaikh, 2011; Arzarello & Edwards, 2005; Chapman, 1993; Dimmel & Herbst, 2015; O'Halloran, 2003, 2005; Radford, 2009). The use of spoken language can be considered one of the

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primary means through which academic subjects such as mathematics are taught and learned (Lemke, 1988). In particular, spoken interactions can provide a primary means of support for struggling students in mathematics (Hord et al., in press; Ketterlin-Geller, Yovanoff, & Tindal, 2007; Scheuermann, Deshler, & Schumaker, 2009). For these reasons, we attend to the interactions between a student and her tutor through the lens of how the pair co-constructed meaning related to linear functions through their conversations about a task.

Thematic analysis is a method within the theory of SFL, which focuses on the ways that ideas are connected to one another (Lemke, 1990; see also Chapman, 1993; Herbel-Eisenmann & Otten, 2011; O'Halloran, 2005). The primary assumption guiding thematic analysis is that meaning is not made through individual words alone. Instead, meaning is given to words and phrases through the ways in which they are connected to other words and phrases. In an example provided by Herbel-Eisenmann and Otten (2011), the base of a triangle refers to two different constructs depending on how the term is used. In one context, the *base* of a triangle refers to a tangible part of the triangle, specifically the edge of the triangle that serves as the base. In another context, the *base* of a triangle refers to a measurement of that edge. Neither use of the term is more correct, but the term “base” is given meaning based on how it is used in relation to other words. Although communication is achieved through choices in representation—in this case, choices about the ways in which words and phrases are connected to one another—those choices are not always made explicit at the moment of an interaction (Lemke, 1990). Thematic analysis provides an analytical framework to examine the implicit connections between ideas.

Data and Methods

Setting of the Study

We conducted this study in a large, urban public school serving grades 7–12. All students at this school take Algebra 1 during their eighth-grade year, and students who do not pass the course are required to re-take the course during the summer. In collaboration with the eighth-grade mathematics teacher and the special education teacher, the second author established a tutoring program with the school. Beginning in December 2014, and running through the end of the school year, pre-service teachers went to the school on a weekly basis to work individually or in small groups with struggling students on their current classwork and homework. Students were selected for the tutoring program by the mathematics and special education teachers, who identified students that they expected to benefit from individualized attention.

In this study, we focus on one particular student, Tanisha, and her work with a tutor named Emily. Emily had developed rapport with Tanisha prior to the implementation of the present study. Specifically, Emily was an undergraduate pre-service special education teacher at a local university, and she had worked with Tanisha during a prior semester as part of a field placement focusing on literacy interventions. Tanisha was struggling in Algebra 1 and requested to participate in tutoring. Tanisha and Emily had a positive relationship, and Tanisha often requested to work with Emily. Tanisha was especially talkative around Emily, and she seemed highly motivated to use her tutoring time to make progress on her classwork.

Data Collection

We audio recorded all of the tutoring sessions, in addition to making copies of student work and taking field notes during the sessions. In total, we have records of 7 different tutoring sessions with Tanisha, ranging from March-May of 2015. For this analysis, we focus on one particular session, which took place between Emily and Tanisha in early May, when students were preparing for final exams. The task, included below, presented a scenario in which an individual named Katie was saving money from her part-time summer job.

Task: Each week in the summer Katie earned \$95 as a lifeguard. Katie deposited 10% of her earnings into her account. At the start of the summer, Katie had \$60 in her account. Write an equation to represent the amount of money Katie had in her bank account after a certain number of weeks.

At this time of the year, students had studied linear functions, and they were accustomed to setting up linear equations of the form $y=mx+b$. Students had previously learned about using linear functions to represent real-world scenarios. The above task served as a practice problem for students to prepare for their final exam. The conversation about the task lasted approximately 2 minutes and 30 seconds. We produced transcripts of the pair’s conversation based upon the audio records, field notes, and work samples.

Methods

After transcribing the interaction between Tanisha and Emily, we identified the semantic relations between terms and phrases related to the rate of change. *Semantic relations* (Halliday & Matthiessen, 2014; Lemke, 1990) refer to relationships between words or phrases in a text. For example, when the *base* of a *triangle* refers to a specific part of the given triangle, the semantic relationship between the terms *base* and *triangle* would be that of part/whole (Herbel-Eisenmann & Otten, 2011). The part/whole relationship is used when a particular term refers to an object or item that is part of some larger object (Lemke, 1990).² Alternatively, when the *base* of a *triangle* refers to the length of that specific part of the triangle, the term *base* would be related as a measure of the *triangle* (Herbel-Eisenmann & Otten, 2011). By identifying the semantic relations among terms, it is possible to describe how words and phrases are connected to one another through speech and, thus, how meaning is construed in these interactions.

We focus our analysis for this presentation on Tanisha and Emily’s use of ideas specifically related to the rate of change of a linear function. To do so, we first identified the key terms and phrases that Tanisha and Emily used in their comments about rates of change. For example, the pair used phrases such as “the amount that Katie earns per week,” and “she gets that much money,” in addition to phrases such as “rate of change.” After noting these key phrases, we described the semantic relations between those words and phrases. Prior work using SFL has identified many of the relations that may exist between mathematical terms (Table 1) (Chapman, 1993; Herbel-Eisenmann & Otten, 2011; Lemke, 1990; O’Halloran, 2005). These relations build upon the identification of semantic relations among participants within and between clauses (e.g., Halliday & Matthiessen, 2014). We include in Table 1 only the semantic relations that are most relevant to this study.

Table 1: A Partial List of Semantic Relations Connecting Mathematical Terms

Relation	Description	Example
Part/Whole	An object or figure is part of a larger figure.	Katie deposits 10% of the \$95 she earns.
Identified/Identifier	An object or figure is identified in a specific way.	\$95 is the amount that Katie earns per week.
Synonyms	Two terms or phrases are equivalent.	The amount Katie earns is the amount of money Katie gets per week.
Subcategory/Category	A category fits inside a broader category.	Katie’s total earnings are one example of varying quantities.

Analysis and Findings

We present our analysis and findings together, focusing on the conversation Tanisha and Emily had about the task presented above. We include specific excerpts of the interaction where Emily and Tanisha discussed the rate of change of the function. We share parts of the transcript with our analysis, to exemplify our findings related to potential sources of ambiguity in the conversation. Numbers in parentheses indicate turn numbers. We identify speakers by their first initials. We bold parts of the transcript to highlight specific points in the analysis. To begin, Emily posed a question about Katie's earnings.

Emily: So how much does **Katie earn per week?** (17)

Tanisha: **95 dollars.** (18)

Emily: Cool. So she **earns** a certain amount of money per week. So each week she **gets that much money**, right? (19)

Tanisha: So **that's** the **rate of change.** (20)

Emily: Yes, awesome. (21)

Tanisha: So **that's m.** (22)

Emily: Yes, wonderful. (23)

With this section of the transcript we can begin to identify the semantic relations among ideas in Emily and Tanisha's talk. We see that \$95 identifies the amount that Katie earns per week, and Katie's earnings are synonymous with how much money Katie gets (lines 17-19). After establishing that, Tanisha made a statement that *that* (i.e., \$95, or the amount of money Katie earns) was synonymous with the rate of change, which is denoted by m in a typical linear equation (lines 20-22). At this stage in the conversation, *Rate of change* seemed to be synonymous with *Katie's earnings per week*, at least as described by Tanisha in line 20.

After determining the starting value of the linear function to be 60, Emily returned to the issue that Katie would only be depositing 10% of her weekly earnings into her bank account.

Emily: So what are we gonna do with the **10%**? The tricky part... (35)

Tanisha: Probably **put it in for x.** Won't we put, $y=mx+b$, for m we're gonna put 95, b we're gonna put 60, and then that **x we're gonna put 10.** (36)

Emily: You're right there. You're so close, and it's crazy. Cuz you're so – Okay, so **how much money does she put in her account per week? That's what m is gonna equal, each week how much money she puts into her account.** (37)

Following the question about what to do with the 10%, Tanisha suggested that they might substitute 10% for x in the equation $y=mx+b$ (lines 35-36). From an outside observer's perspective, Tanisha's response might indicate a complete misunderstanding of linear functions. However, consider the scenario from Tanisha's perspective. She had already determined that Katie's earnings were synonymous with the rate of change of the function, which would be represented by m . Following that, the pair had quickly determined the starting value to be \$60, which would be represented by b . With those assumptions, Tanisha was limited in how she might use the value of 10% in an equation of the form $y=mx+b$. At this point, from Tanisha's perspective, x was the only variable that had not yet been assigned a value.

The source of ambiguity in the conversation between Katie and Emily starts to become clear in line 37, when Emily noted, " m is gonna equal, each week how much money she puts into her account." This statement seems to be in direct contradiction to the earlier exchange in lines 17-22, when Emily and Tanisha established that m would be equal to the amount of money Katie earns per week. To make sense of this contradiction, we consider the possible differences between the semantic relations used by Emily and Tanisha in making sense of the rate of change. For Tanisha, the semantic relation between *Katie's earnings* and *Rate of change* seemed to be a synonym relationship

(Figure 1). In other words, the rate of change was equivalent to Katie's earnings, which had been identified as the specific value of \$95 per week.

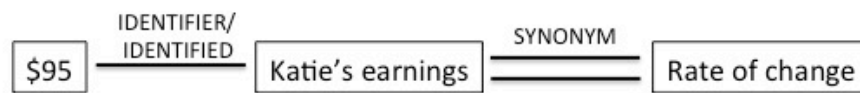


Figure 1. Tanisha's semantic relations between earnings and rate of change.

Once Emily clarified that the value of m would be determined by how much money Katie deposited per week, Tanisha needed to redefine the rate of change of the function. They agreed that Katie deposited 10% of her earnings, so Tanisha suggested that perhaps 10% would be the rate of change of the function.

Tanisha: I mean, I meant, put **10%**, right? (42)

Emily: Mm hmm. (43)

Tanisha: So it'll be $y=10x+60$? (44)

Emily: Close. Close. (45)

Here, Tanisha replaced *Katie's earnings* with *Katie's deposits* as synonymous with the rate of change of the function (Figure 2). Tanisha substituted 10 for m in the equation $y=mx+b$, as 10% identified specifically what Katie deposited on a weekly basis.



Figure 2. Tanisha's semantic relations when replacing earnings with deposits.

The semantic relation in Figure 2 seemed to replace Tanisha's earlier construction. Because Katie's earnings and deposits were not the same thing, it would be impossible for them to both be synonymous with the rate of change of the function. In order to account for the 10% deposit, Tanisha needed to disregard the previous information about Katie's \$95 weekly earnings.

Emily seemed to be using a slightly different semantic construction to describe the relationships between earnings, deposits, rate of change, and the value of m . After determining with Tanisha that 10% of \$95 is \$9.50, Emily made the following comment:

Emily: So **that is our new m** . Because since she doesn't, since she doesn't deposit all 95 dollars, then that's not our rate of change. Our **rate of change is how much she deposits per week**. (55)

Importantly, in turn 55, Emily noted that the value of 9.5 represented a new m . To suggest that Katie's deposits represented a new value of the rate of change indicates that, for Emily, rate of change did not refer to a unique item. Instead, *Katie's earnings* and *Katie's deposits* could both be considered sub-categories of a broader category of *Rate of change* (Figure 3). Ninety-five dollars identified the specific value of Katie's weekly earnings, which provided one example of a rate of change. Additionally, 10% represented the part of that \$95 that Katie deposited in her bank account, another example of rate of change.

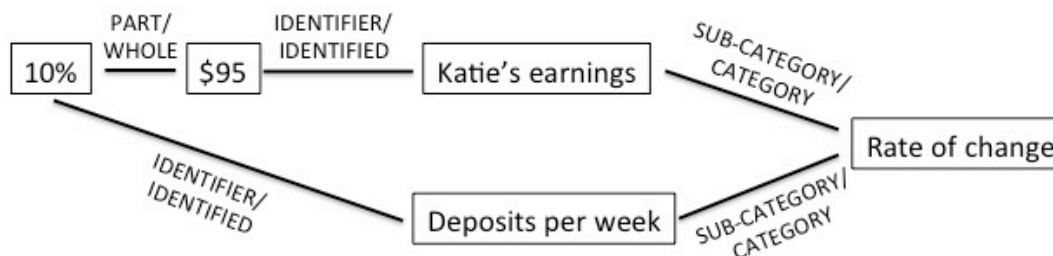


Figure 3. Semantic relations according to Emily.

In summary, there was one key difference in the semantic relations used by Tanisha and Emily, which seemed to be a source of ambiguity in their conversation. After establishing Katie's earnings as the rate of change of the function in turns 19-20, Tanisha used *Katie's earnings* synonymously with *Rate of change*. This became apparent, and also problematic, when Tanisha needed to account for the 10% deposited each week. Having already accounted for all of the parameters in the equation $y=mx+b$, Tanisha's only remaining option was to replace *Katie's earnings* with *Katie's deposits* as the rate of change. To make sense of Emily's comments, on the other hand, one can recognize Emily employed a sub-category/category relationship between *Katie's earnings*, *Katie's deposits*, and *Rate of change*, rather than a synonymous relationship. This difference helps explain Tanisha's apparent confusion about how to account for the value of 10% in the linear function.

Discussion and Conclusion

A primary point that we emphasize in our observations of Emily and Tanisha's conversation is that there is no single correct way to use specific terms in algebra. In a discussion about semantic differences in a science classroom, Lemke (1990) made an even stronger argument, noting, "words do not necessarily 'have' meanings in themselves. A word in isolation has only a 'meaning potential,' a range of possible uses to mean various things" (pp. 34–35). In this study, we saw that phrases such as "Katie's earnings" or "amount deposited" had potential for different meanings related to the rate of change of a linear function, depending on how the interactants used those phrases. Because Tanisha and Emily did not recognize the difference between their semantic constructions, when Emily proposed a new value for the rate of change, Tanisha attempted to accommodate this information into her existing semantic construction.

A natural question to pose is whether Tanisha or Emily was more or less correct in her semantic constructions. Emily, in fact, agreed with Tanisha when Tanisha suggested, "that [Katie's weekly earnings] is the rate of change." Only later did Emily propose that the \$9.50 that Katie deposited per week would represent the rate of change. However, we keep in mind that the use of the term *is* can be ambiguous in how it relates terms or ideas (Schleppegrell, 2007). For Tanisha, to state that \$95 *is* the rate of change seemed equivalent to stating that a dog *is* a canine; in each case the two phrases are synonyms for the same thing. For Emily, to agree that \$95 *is* the rate of change seemed equivalent to suggesting that a dog *is* a mammal. In this case, the two terms are not synonymous. Instead, a dog is one subcategory of the category of mammals, which also includes many other things. Examples of this subcategory/category relationship can be identified in other areas of mathematics as well. For example, a square *is* a rectangle in the sense that squares constitute one subcategory of rectangles. From this study, we see that a construct such as "rate of change" can be viewed as a category of things, with multiple different subcategories. Because small differences like this in semantic relations are often left implicit in interaction (Lemke, 1990), research making these distinctions explicit is necessary to support teachers and students to recognize the nuances in their use of spoken language.

Beyond individuals' use of spoken language, this study highlights the way in which any given scenario can be represented by multiple different functions. The task as it was given required

students to write a function representing how much money Katie had in her bank account each week. Given the same information, one could represent how much money Katie had earned as a function of the number of weeks, using Tanisha's initial equation $y=95x+60$. Alternatively, one could represent the amount of money Katie had deposited as a function of how much she had earned using the equation $y=.10x+60$, similarly to Tanisha's third attempt at constructing the linear equation. Students in a variety of settings struggle to translate problem contexts into linear equations (Capraro & Joffrion, 2006). Moreover, students' errors in setting up linear equations stem partly from their attempts to make sense of the meaning of a problem (MacGregor & Stacey, 1993). We see it as a strength that students like Tanisha make connections between algebraic symbols and the scenarios they represent. To be able to correctly express a problem context symbolically, students must be able to interpret the relationship between the various quantities and components of that context. Students need opportunities to develop understanding of a context using informal language in order to be successful representing that context with symbols (Kieran & Chalouh, 1993). The specific function requested in the task could have been somewhat arbitrary, and an alternative question could have been posed for which one of Tanisha's linear equations would have been appropriate. The more that students can develop the skills to understand the semantic relationships between components of a problem context, the more equipped students will be to select an appropriate symbolic representation of that context.

Finally, because low-achieving students often opt out of whole-class discussions in mathematics classrooms (Baxter, Woodward, & Olsen, 2001), individualized or small group settings can be ideal for struggling students and students with learning disabilities to engage in conversations about mathematics (Woodward, 2006). In order to make the most of these opportunities, it is essential to understand how students and instructors work together to construct mathematical meaning through their talk. The analytic tools of SFL, and specifically analysis of the semantic relations among ideas in a text, provide a resource for ensuring that students and instructors are able to understand one another's meanings.

Endnotes

¹ We use pseudonyms for all names and institutions.

² Lemke (1990) and Herbel-Eisenmann and Otten (2011) use the more formal linguistic terms meronym/holonym to describe the part/whole relationship. We use the more colloquial terms here for brevity and clarity.

References

- Alshwaikh, J. (2011). *Geometrical diagrams as representation and communication: A functional analytic framework* (Doctoral dissertation) University of London, England.
- Arzarello, F., & Edwards, L. D. (2005). Gesture and the construction of mathematical meaning. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 123–154). Melbourne: PME.
- Baxter, J., Woodward, J., & Olson, D. (2001). Effects of reform-based mathematics instruction in five third-grade classrooms. *Elementary School Journal*, 101(5), 529–548.
- Capraro, M. M., & Joffrion, H. (2006). Algebraic equations: Can middle-school students meaningfully translate from words to mathematical symbols? *Reading Psychology*, 27(2-3), 147–164.
- Chapman, A. (1993). Language and learning in school mathematics: A social semiotic perspective. *Issues in Educational Research*, 3(1), 35-46.
- Chazan, D., Leavy, A., Birky, G., Clark, K., Lueke, H. M., McCoy, W., & Nyamekye, F. (2007). What NAEP can (and cannot) tell us about performance in algebra. In P. Kloosterman & F Lester (Eds.), *Interpretations of the 2003 mathematics assessment of the National Assessment of Educational Progress* (pp. 169–190). Reston, VA: National Council of Teachers of Mathematics.
- Dimmel, J. K., & Herbst, P. G. (2015). The semiotic structure of geometry diagrams: How textbook diagrams convey meaning. *Journal for Research in Mathematics Education*, 46(2), 147–195.

- Halliday, M. A. K., & Matthiessen, C. M. I. M. (2014). *An introduction to functional grammar* (4th ed.). London, UK: Hodder Arnold.
- Herbel-Eisenmann, B. A., & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education*, 42(5), 451–485.
- Hord, C., DeJarnette, A. F., & Marita, S. (2015). Justification in the context of linear functions: Gesturing as support for students with learning disabilities. In T. G. Bartell, K. N. Bieda, K. Bradfield, & H. Dominguez (Eds.), *Proceedings of the 37th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 587). East Lansing, MI: Michigan State University.
- Hord, C., Marita, S., Walsh, J. B., Tomaro, T. M., & Gordon, K. (in press). Conversational scaffolding for students with LD. *Mathematics Teacher*.
- Ketterlin-Geller, L. R., Yovanoff, P., & Tindal, G. (2007). Developing a new paradigm for conducting research on accommodations in mathematics testing. *Exceptional Children*, 73(3), 331-347.
- Kieran, C., & Chalouh, L. (1993). Pre-algebra: The transition from arithmetic to algebra. In D. T. Owens (Ed.), *Research ideas for the classroom: Middle grades mathematics* (pp. 179–198). New York: Macmillan.
- Kress, G., & Van Leeuwen, T. (2006). *Reading images: The grammar of visual images*. London: Routledge.
- Lemke, J. (1988). Genres, semantics, and classroom education. *Linguistics and Education*, 1(1), 81-99.
- Lemke, J. (1990). Talking science: Language, learning, and values. Norwood, NJ: Ablex.
- Lubienski, S. T. (2002). A closer look at black-white mathematics gaps: Intersections of race and SES in NAEP achievement and instructional practices data. *The Journal of Negro Education*, 71(4), 269-287.
- MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education*, 24(3), 217–232.
- O'Halloran, K. L. (2003). Educational implication of mathematics as a multisemiotic discourse. In M. Anderson, A. Sáenz-Ludlow, S. Zellweger, & V. V. Cifarelli (Eds.), *Educational perspectives on mathematics as semiosis: From thinking to interpreting to knowing* (pp. 185–214). New York: Legas.
- O'Halloran, K. L. (2005). *Mathematical discourse: Language, symbolism, and visual images*. London, England: Continuum.
- O'Halloran, K. L. (2014). The language of learning mathematics: A multimodal perspective. *The Journal of Mathematical Behavior*. Advance online publication. doi:10.1016/j.jmathb.2014.09.002
- Radford, L. (2009). Why do gestures matter? Sensuous cognition and the palpability of mathematical meanings. *Educational Studies in Mathematics*, 70(3), 111–126.
- Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, 32(2), 103-120.
- Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading and Writing Quarterly*, 23(2), 139-159.
- Woodward, J. (2006). Making reform-based mathematics work for academically low-achieving middle school students. In M. Montague & A. K. Jitendra (Eds.), *Teaching mathematics to middle school students with learning difficulties* (pp. 29-50). New York: Guildford Press.