IMPLEMENTATIONS OF CCSSM-ALIGNED LESSONS

Jeffrey Choppin
University of Rochester
jchoppin@warner.rochester.edu

Jon Davis
Western Michigan University
jon.davis@wmich.edu

Amy Roth McDuffie
Washington State—Tri-Cities
mcduffie@tricity.wsu.edu

Corey Drake
Michigan State University
cdrake@msu.edu

We analyzed 52 middle school mathematics lessons from multiple states and curriculum contexts to understand how teachers were enacting the CCSSM. The teachers stated that all of the lessons were CCSSM-aligned. We categorized curriculum materials according to two approaches, with one approach associated with curriculum programs funded by NSF and the other representing curriculum programs commercially produced, typically from a large publisher. We analyzed the nature of mathematical activity and level of interactions in the lessons. We found significant differences across curriculum approaches in the mathematical activity categories related to cognitive demand and in the level of interaction. The implications are that curriculum programs strongly mediated the enactment of the CCSSM.

Keywords: Curriculum Analysis, Instructional Activities and Practices, Standards, Middle School Education

The *Common Core State Standards for Mathematics* (CCSSM) (National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010) were initially adopted by 45 states plus the District of Columbia, and, despite the rollback in some states, the CCSSM or CCSSM-based standards are still in place in most states. Thus, there is a relatively common articulation of content and the progression of content across the grades in the CCSSM-adopting states. This provides researchers an opportunity to consider the impact of curriculum programs on how the CCSSM get taken up by schools and teachers. Furthermore, the CCSSM framers were agnostic with respect to curriculum and instructional approaches (McCallum, 2012), which provides potentially enhanced roles to curriculum materials as teachers work to interpret and implement the standards. Consequently, the widespread adoption of the CCSSM presents an opportunity and need for researchers to understand how districts and teachers interpret standards and to understand the role of curriculum materials in the process of interpreting and enacting those standards.

Teachers' initial interpretations of the CCSSM, when compared to prior state standards, were that the CCSSM required a greater emphasis on problem-solving, discovery, communication, and conceptually-driven instruction (Davis, Choppin, Roth McDuffie, & Drake, 2013; Choppin, Davis, Drake, & Roth McDuffie, 2013). Although teachers expressed a relatively strong view of the student-centered nature of the CCSSM, prior research based on teachers' enactments of similar recommendations in the National Council of Teachers of Mathematics (NCTM) Standards documents (NCTM 1989, 1991) shows that even the most reform-minded teachers did not implement the recommendations in ways beyond superficial features (Spillane & Zeuli, 1999). However, much of the research on the implementation of the recommendations in the NCTM Standards primarily occurred before curriculum programs based on those documents were developed and widely disseminated. In this study, we explore the association between district-adopted curriculum programs (e.g., *the designated curriculum* [Remillard & Heck, 2014]) and instruction that resulted when teachers used those programs to plan and enact lessons (e.g., *the enacted curriculum*). These lessons

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– and the designated curriculum – were ostensibly aligned with the CCSSM (e.g., *the official curriculum*).

Framework

We frame our analysis by distinguishing between two broad approaches to curriculum and instruction. In describing each approach, we connect conceptions of curriculum design to conceptions of classroom instruction. In the first approach, we connect the notion of *curriculum as delivery mechanism* to that of *direct instruction*. In the *curriculum as delivery mechanism* approach, knowledge is detached from an authority or expert (i.e., textbook, teacher) and transmitted to novice learners (students), allowing those far from classrooms to exert control over content (Datnow & Park, 2009). Curriculum design is based on explaining and modeling concepts and procedures and presumes that learners have minimal understanding of the subject matter or intuitive understandings. Mastery focuses at the level of lesson or topic, with fluency expected on one topic before proceeding to the next. The treatment of language in curriculum materials from this approach mirrors the treatment of mathematics content. There is typically an emphasis on early formalization and precision, with little validation of less formal or everyday terminology, and terms are defined and explained before students have opportunities to explore the content. These conceptions of curriculum align with models of direct instruction, as defined by Munter, Stein, and Smith (in press). Munter and colleagues explain that direct instruction is dominated by teacher explanation and demonstration of procedures or definitions, which students then practice to develop accurate and fluent reproduction of those procedures or definitions.

In the second approach, we connect *curriculum as epistemic device* to *dialogic instruction*. In considering curriculum as epistemic device, the primary goal of curriculum is to provoke interactions that generate understanding. The role of tasks in curriculum materials is to elicit and progressively refine student thinking, individually *and* collectively, as contrasted with serving as a delivery mechanism for content. This conceptualization of curriculum design builds from a notion of text as *thinking device* that promotes dialogic interaction (Wertsch & Toma, 1995). A primary characteristic that shapes task affordances is the potential for heterogeneous approaches that vary in terms of their entry points and sophistication, or what has been called *low-threshold, high ceiling* tasks. This metaphor aligns with *dialogic instruction*, as described by Munter and colleagues (in press), which emphasizes students' collaborative work on challenging tasks and the positioning of students as co-participants in classroom discourse and as emerging mathematical authorities. These distinctions allow us to parsimoniously characterize distinct approaches evident in teachers' interpretations and enactments of the CCSSM, and to tie characteristics of curriculum to characteristics of instruction.

Curriculum Types

Building from the two approaches outlined above, we characterized curriculum programs into two types. Programs developed in ways aligned with the epistemic device approach, comprised exclusively of National Science Foundation (NSF)-funded curriculum programs, we labeled as ED Programs. These programs have some or all of the following characteristics:

- Problem contexts serve as the basis of exploration for multiple lessons.
- Students explore a problem and/or mathematical concept *before* concepts, procedures, and/or mathematical terms are formalized.
- Mathematical practices, particularly problem solving, reasoning, and argumentation, are considered essential in teaching and learning.
- Representational fluency is promoted through work with individual representations and the connections among representations.

- Grouped work is collaborative, used for high level tasks, and often involves a group product and/or presentation (Lappan & Phillips, 2009).

Programs aligned with the delivery mechanism approach, we labeled as DM Programs. These programs, comprised almost exclusively of publisher-produced programs, are geared primarily towards procedural fluency, with characteristics of direct instruction, including:

- Problems may be set in context, but the contexts vary with each problem are not focused on students reasoning analogically about the mathematics.
- Problem solving steps and procedure are described or provided through examples.
- Formal definitions are presented before students use terms or constructs associated with specific terms, and precise use of language and efficient procedures are considered essential to developing conceptual understanding.
- Group work and/or seat work are used primarily to practice problems demonstrated by the teacher (*cf*., Battista, 1999).

Methods

We analyzed 52 lessons from the video recordings of the lessons. The teachers stated that the materials and lessons were aligned with the CCSSM content and practice standards. We developed an observation tool designed to distinguish between direct and dialogic forms of classroom instruction. The tool was originally adapted from the instrument used in a large scale study (Tarr et al., 2008) to characterize the extent to which lessons aligned with what they termed *standards-based instruction*. To that end, the instrument emphasized conceptual understanding, multiple solutions and representations, and recognizing and building from student thinking. We ultimately transformed the instrument and associated analytic techniques by utilizing a modified time-sampling approach. We transcribed most of the whole class portions of the lessons and group work as the audio quality permitted. We parsed the transcripts into roughly two- to four-minute chunks delineated by participation structures and topical foci. We bounded the analytic segments first by participation structures (e.g., seat work, whole class discussion, group work), then by a combination of duration and topical focus, similar to what Mehan (1979) termed a *topically related set*. Given that lesson ratings were derived as a ratio of the number of lesson segments in which a code occurred divided by the total number of lesson segments, we wanted to maintain roughly similar time intervals for each lesson segment. Consequently, if a discussion around one problem extended beyond three or more minutes, we divided that discussion into multiple segments of roughly two minutes each. Similarly, if there were a series of rapid resolutions to a set of problems that were based on the same kind of mathematical activity, we combined these sets into one segment for analytic purposes. Further description of how we coded each segment is described below.

Data Sources

The data came from a larger NSF-funded study that explored teachers' perceptions of the CCSSM, the ways teachers were prepared to teach the CCSSM, how teachers drew upon curriculum materials to plan lessons they viewed as CCSSM-aligned, and how they enacted those lesson plans. We had comprehensive data sets for 52 teachers, including lesson observations from the 2013-2014 School Year. Of the 52, 21 of the lessons came from teachers using ED materials, and 31 came from teachers using DM materials. Sixteen of the ED lessons involved the second (CMP2) or third (CMP3) edition of Connected Mathematics Program, four of the lessons involved College Preparatory Mathematics (CPM), and one involved Core-Plus Mathematics. For the DM materials, 13 used Glencoe, five used *digits*, four used Prentice Hall, three used Math in Focus, and others used similarly organized curriculum programs. For 13 of the teachers using ED programs, their districts

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had recently adopted the programs specifically to address the CCSSM. Districts for seven of the other teachers had adopted programs before the CCSSM were adopted. Twenty-five of the 31 teachers using DM programs worked in districts that had recently adopted the programs specifically to address the CCSSM. Of those 25, eight used other materials regularly, including in the lessons we observed. Six of the DM teachers' districts had not adopted new materials; these teachers used a range of materials aligned with the DM approach.

Data Analysis

We coded lessons using three broad sets of analytic categories. We coded lessons using three broad sets of analytic categories. The categories were: *Nature of Mathematical Activity*, *Lesson Mode*, and *Elicitation and Presentation of Student Explanations*. The first set, *Nature of Mathematical Activity*, included five sub-categories, three based on levels of cognitive demand from the work of Stein and colleagues (Stein, Grover, & Henningsen, 1996), and two that were inductively developed as a result of the data analysis, as described below. The categories derived from the cognitive demand literature included: *recall, memorization, or basic application of definition or rule*; *procedural or computational routine*; and *procedure plus*. These three categories correspond to the first three cognitive demand categories of *memorization*, *procedures without connections*, and *procedures with connections*, respectively, but were revised to provide additional descriptive detail to facilitate coding and to reflect trends that emerged in our data. We developed two new categories to represent patterns identified in the mathematical activities observed in the lessons. The first, *interpreting or generating representations*, refers to tasks that involved creating or interpreting information a table, graph, equation, or other representation. This category involved mathematical activity that required students to translate information across types of representations or to extract and describe a pattern evident in a representation. As such, the activity extended beyond simple recall or application but did not necessarily involve a procedure. Consequently, we deemed it as higher cognitive demand than the *recall* category and aligned more with *procedure plus*, though it didn't involve a procedure. The second new category, *developing definitions or formulas*, refers to when tasks involved creating a definition or formula. This is different from when the teacher simply provided a definition, which was categorized as recall or memorization. We found this category most often occurred in geometry lessons, though Munter et al. (in press) emphasize the need for teachers and students to co-construct definitions across all strands when appropriate.

The Lessons Mode categories were used to make two distinctions. The first distinction was whether a segment predominantly occurred before students had an opportunity to work (alone or in groups) on a problem or occurred after students had an opportunity to work on a task. We considered cases when the teacher presented and explained examples in whole class discussions as having occurred before students had an opportunity to work on a problem, even if the teacher engaged in recitation-style interactions with students. The second distinction characterized the interactions within the segment. If the teacher strictly or primarily engaged in Initiate-Respond-Evaluate (IRE) interactions or spent the entire segment explaining the mathematics or the solutions to problems, then we characterized that as low-interactive. Segments were characterized as low-interactive if they were primarily comprised of a series of rapid evaluations of the accuracy of students' responses. A segment was considered high-interactive if the teacher elicited responses without engaging in immediate evaluation and if the teacher pressed students to explain their answers and to respond to the answers of other students. These two distinctions produced four modes, a pre- and post-work low interactive, and a pre- and post-work high interactive. We also included a fifth lesson mode, termed *directions or administrivia*, to code segments when the teacher was providing directions, taking attendance, attending to classroom rules, or the like. These lesson modes provided us a way to characterize the lessons as involving direct or dialogic instruction.

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The third set of codes, Elicitation and Presentation of Student Explanations, included three categories that are relatively self-explanatory: teacher elicited student strategies or interpretations; teacher pressed students for steps and justification for steps; and students explained solution strategies.

We coded each segment with at most one code from the Mathematical Activity and Mode categories, and as many as appropriate from the Student Explanations categories. There were segments for which the nature of mathematical activity was not clear, so no code was applied. For each lesson, we divided the number of segments in which a code occurred by the total number of segments and then multiplied by eight in order to avoid having the ratings clump around one rating number, usually the lowest numbers in the scale given the relatively low occurrence of some codes. We added a one to each rating to avoid having values of zero, and rounded each rating to get an integer scale from one to nine. We then averaged the codes for each category across each curriculum type and used Excel to apply a two-tailed t-test of different samples of equal variance, with a significance level of 0.05.

Results

We found significant differences in the lesson segment codes across the ED and DM materials. In terms of the Nature of Mathematical Activity, the ED lessons had significantly fewer segments coded as *recall, etc*. ($p < 0.01$) or *procedural or computational routine* ($p < 0.01$) and significantly more segments coded as *interpreting or generating representations* ($p < 0.05$) and *procedure plus* ($p < 0.05$). Although there were relatively more ED than DM segments coded as *developing definitions or formulas*, the **difference was not significant** ($p < .225$). See Table 1 for a summary of the Mathematical Activity categories.

Table 1: Frequency and p-value for Mathematical Activity Categories

	Recall, memorization, or basic application of definition or rule	Interpreting or generating representations	Developing definition or formula	Procedural or computational routine	Procedure Plus
Frequency - ED	2.0	2.7	1.7	1.8	2.2
Frequency - DM	3.4	1.6	1.3	3.1	1.4
p-value	0.002	0.015	0.225	0.006	0.04

Similarly, in the Lesson Mode categories, there were significantly fewer ED segments coded as low-interactive / pre-work ($p < 0.01$) and significantly more ED segments coded as interactive (both pre- and post-work) ($p < 0.01$ in both cases). The only codes not significant were low-interactive post-work, though there were relatively fewer ED segments with this code ($p = 0.17$), and *directions or administrivia*, which occurred relatively equally across types $p = .33$). See Table 2 for a summary of the Lesson Mode results.

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Table 2: Frequency and p-value for Lesson Modes

	Presentation and explanation of task and mathematics in the task, primarily characterized by teacher explanation and IRE exchanges that evaluate students' understanding of facts and procedural knowledge (pre-work, non-interactive)	Teacher engages students in non-evaluative exchanges to establish the meaning of the problem context or to establish the mathematical focus of the task (pre-work, interactive)	Explanation of task and mathematics, typically after students had an opportunity to do work, primarily characterized by teacher explanation and IRE exchanges that evaluate students' accurate completion of the task (post-work, non-interactive)	Probing for, sharing of, or discussion of student strategies, characterized by non-evaluative exchanges in which the teacher elicits and probes student understanding of the task or strategies (post-work, interactive)
Frequency – ED	2.9	1.9	2.6	2.9
Frequency – DM	4.9	1.1	3.3	1.16
p-value	0.0004	0.002	0.17	0.0001

In the Elicitation and Presentation of Student Explanations categories, there were significantly more ED lesson segments coded as *teacher elicited student strategies or interpretations* ($p < 0.01$) and *teacher pressed students for steps and justification for steps* ($p < 0.01$). Though there were more ED segments coded as *students explained solution strategies*, the difference was not significant ($p = 0.15$).

Discussion

There was a strong association between curriculum types and the type of instruction, as characterized by the nature of mathematical activity, the level of interactivity, and emphasis on student explanations. That is, curricula deemed as aligning with curriculum as delivery mechanism were strongly associated with instructional forms that were predominantly encapsulated by Munter and colleagues' (in press) description of direct instruction, and curricula deemed as aligning with curriculum as epistemic device were strongly associated with instructional forms that were predominantly encapsulated by Munter and colleagues' description of dialogic instruction This finding is consistent with earlier case studies of eight teachers using two different curriculum programs, as part of the larger project. This occurred even though multiple curriculum programs, schools, and states were present in each of the Type 1 and Type 2 samples. Even though some of the differences may be due to underlying perceptions of the CCSSM (which we are exploring), the results lend credence to the notion that curriculum materials are strong mediating factors in enactments of the CCSSM, and more generally, that the designated curriculum is a strong mediating factor between the official curriculum and the enacted curriculum.

Although strong majorities of participants in the survey samples and interview samples from the larger study reported that the CCSSM required more communication, problem solving, exploration, and overall rigor than past standards (Davis, Choppin, Roth McDuffie, & Drake, 2013; Choppin, Davis, Drake, & Roth McDuffie, 2013), the lessons involving Type 2 materials typically lacked these features. Few teachers using the Type 2 programs expressed dissatisfaction with the fit between their materials and the CCSSM, suggesting that the designated curriculum served as the de facto representation of the official curriculum, and the teachers felt as long as they were using the materials, they were addressing the CCSSM. The results from the Type 2 teachers are consistent with results from lessons observed soon after the release of the NCTM Standards documents in which most teachers struggled to incorporate the recommendations in their lessons beyond surface features (Spillane & Zeuli, 1999), lessons conducted without the benefit of curriculum programs designed to comprehensively integrate the recommendations.

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A plausible explanation for the differences in the observed instruction is that there are characteristics of the Type 1 materials that contribute to instruction rated as having higher cognitive demand mathematical activity, greater emphasis on interactivity, and a greater focus on student explanations. One possible explanation is that the materials convey underlying pedagogical messages, and the teachers create more opportunities for exploration and communication to follow what they perceive as the wishes of the curriculum designers. A second possible characteristic is the inclusion of task sequences in which students are first presented tasks "to which students do not have an immediate solution, but must wrestle with for a while without the teacher's interference" and then presented tasks that "help them become more competent with what they already know" (Munter et al., in press, p. xx). These design features are built into the Type 1 programs (Lappan & Phillips, 2009), and, even though there is typically wide variation in which these features are taken up by teachers (Tarr et al., 2008), there was nevertheless a stark overall contrast in our data between the Type 1 and Type 2 lessons, suggesting the presence of such features contributed to teachers' instructional decisions.

There were four categories that did not result in significant differences. The first, *developing definitions or formulas*, was most strongly associated with geometry lessons, of which there were few, and the lack of presence of this activity in other strands made the overall mean low across both types. The lack of significant difference in the post-work non-interactive Lesson Mode reflects the relatively even distribution in the Type 1 post-work lesson segments between interactive and non-interactive segments. That is, teachers using Type 1 materials engaged in IRE-style interactions and teacher-led explanations almost as much as they facilitated more interactions or emphasized student explanations. This compared to the Type 2 lessons, in which post-work lesson segments were rated as non-interactive fourteen times as much as they were rated interactive. Nevertheless, the Type 1 lessons included a similar quantity of non-interactive segments, so the difference between types was not significant. The third category whose difference was not significant was *directions or administrivia*, reflecting the relatively equal occurrence of this code across both types, suggesting that providing directions, taking attendance, managing behavior, and so forth is a staple of all lessons. The fourth category, *students explained solution strategies*, like the *developing definitions or formulas* code, did not occur frequently, with low means across both types. So, while teachers often elicited and probed students' explanations, there was either inadequate follow-up so that the student actually provided a full explanation, or the students were not able to provide a full explanation. Nevertheless, the low means (1.62 for ED, and 1.19 for DM, with 1 indicating no occurrence) across both types suggests that students still rarely have opportunities to provide comprehensive explanations for their solutions, approaches, or strategies.

The consistent presence of interactive forms of instruction in the ED lessons deviates in substantive ways from prior results of large-scale observations of middle school classrooms (Jacobs et al., 2006; Stigler & Hiebert, 1999), studies that largely took place before the widespread dissemination of NSF-funded materials. The results, however, are more in line with the findings of Tarr et al. (2008), whose study included NSF-funded programs. Our results, along with those of Tarr et al. (2008), suggest that NSF-funded programs can mediate longstanding lesson structures to make instruction more focused on student thinking and students' explanations.

Implications

An implication from our findings is that the choice of curriculum programs – whether by district, school, or teacher – is associated with instructional approach. An underlying question, then, is what these entities were responding to in their choice of curriculum programs, especially if the ostensible goal – to align instruction with the CCSSM – was the same across the contexts we studied. What messages or information were the schools and districts responding to, and what messages did they want to send with their choice of programs? What does their choice of curriculum program say about the depth with which or the evidence with which decisions about curriculum programs were based?

A second implication is that interpretations of the official curriculum – the CCSSM – are heavily mediated by decisions and curricular choices at the local level. That is, the designated curriculum is potentially the strongest mediating factor in the ways that the CCSSM are being enacted. This suggests challenges for policy makers who hope to change classroom instruction without providing a stronger articulation of what classroom practices should look like or providing materials that have been developed with these practices in mind.

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