SENSE-MAKING PRACTICES OF EXPERT AND NOVICE READERS

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Mathematics textbooks are a significant pedagogical tool, particularly in light of the growing interest in "flipped" classrooms. However, there has been little research on how mathematics students read and comprehend textbooks. This article uses the ideas of the implied reader, sense-making frames, and sense-making gaps to analyze students' reading of a section of a calculus textbook. In order distinguish potential weaknesses in students' content knowledge from their reading abilities, the article also compares students' reading strategies to the reading strategies of "expert" readers: professors in technical fields other than mathematics.

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As "flipped" and blended classroom pedagogies become more widely used, students are increasingly expected to read and learn from various text materials (e.g., Staumsheim, 2013). However, research has suggested that mathematics students struggle to read their textbooks effectively (e.g., Shepherd, Selden, & Selden, 2012). Consequently, it is important for us to understand how students make sense of reading mathematics text materials.

As Osterholm (2008) noted, there is little research that describes how students read and comprehend mathematical texts. Shepherd, Selden, and Selden (2012) found that undergraduate calculus and precalculus students struggled to read their textbooks effectively. Shepherd and van de Sande (2014) compared the reading practices of first-year mathematics students, mathematics graduate students, and mathematicians, and characterized their reading strategies based on background knowledge, use of resources, and self-monitoring of comprehension.

We aim to expand the current research on how students read and understand mathematics textbooks by exploring the ways that students make sense of mathematical texts and to distinguish the role of content knowledge from the role of reading ability by comparing the students reading practices with those of "experts."

Theoretical Framework

Sense-making

In order to describe the aspects of the text that students focus on and the ways in which they interpret the text, we use the idea of a *conceptual frame*, which is "a mental structure that filters and structures an individual's perception of the world by causing aspects of a particular situation to be perceived and interpreted in a particular way" (Weinberg, Wiesner, & Fukawa-Connelly, 2014, p. 169). From this perspective, readers experience and seek to organize a collection of phenomena as they read texts; they use their knowledge and experience to create frames, and these frames then determine which phenomena are noticed and how they are interpreted.

Also central to the sense-making process are the ideas of gaps and bridges. Gaps are "questions that must be answered in order for the student to engage in or construct meaning for the mathematical situation or activity" (Weinberg, Wiesner, & Fukawa-Connelly, 2014, p. 170). A bridge is the answer that the student constructs. The ideas of frames, gaps, and bridges are complementary: the frame influences the nature of the gaps that arise, and, after constructing a bridge the student may notice different aspects of the text or interpret aspects of the text in a new way. Weinberg, Wiesner, and Fukawa-Connelly (2014) identified four types of frames:

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- *Content*: Noticing mathematical aspects of the situation (e.g., symbols, definitions, facts, and concepts) and seeking to understand the meaning of the mathematical content or how to use it in an example that is being presented.
- *Communication*: Noticing the instructor's spoken, written, and gestural actions for organizing and presenting mathematical ideas and seeking to understand the ways the instructor is categorizing or connecting ideas, the ideas communicated by board layout, and the instructor's organizational cues.
- *Situating-mathematical purpose*: Noticing mathematical aspects of the situation and seeking to understand the usefulness or mathematical significance of the concept.
- *Situating-pedagogical purpose*: Noticing communicational aspects of the situation and seeking to understand how the instructor's pedagogical actions and decisions—such as choosing and ordering lecture content—are related to the meaning or significance of the mathematical ideas.

Implied and Empirical Readers

One of our goals is to identify the characteristics of the reader that might explain the differences in what people actually learn from reading a textbook. To do this, we use the idea of the *implied reader* of the text, which is "the embodiment of the behaviors, codes, and competencies that are required for an empirical reader to respond to the text in a way that is both meaningful and accurate" (Weinberg & Wiesner, 2011, p. 52). A *code* is "a way of ascribing meaning" to the symbols, words, and formatting (Weinberg & Wiesner, 2011, p. 53). *Competencies* are the "mathematical knowledge, skills, and understandings [that are required] to work within the established context" (Weinberg & Wiesner, 2011, p. 55). For example, in order to understand a description of the limit definition of the derivative, a reader would need to have some knowledge of functions, limits, rates of change, and their various representations.

Research Questions

Based on the goals and theoretical framework, our research questions are:

- What gaps do students encounter as they read a math textbook, and how do they bridge these gaps?
- What sense-making frames do students use while reading?
- How can mismatches between the implied reader and the actual reader explain the gaps that students encounter and the bridges they construct?
- How do the practices of "expert" readers compare to the students?

Methodology and Methods

Student Participants

All 22 students in the second author's second-semester calculus class were invited to participate in the study. Five students volunteered and participated in the interviews. All of the students had previously completed a standard first-semester calculus course in college or received AP credit for first-semester calculus.

The textbook used in the class—and the interviews—was *Calculus: Single Variable* (Hughes-Hallett et al., 2012). Throughout the semester, the students had been regularly asked to read sections of the textbook outside of class and complete various activities, such as annotating the textbook and writing summaries of textbook sections in groups. Thus, by the time the students participated in the interviews, they had considerable experience reading their textbook.

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Faculty Participants

In order to distinguish the role of content knowledge from the role of reading ability, we compared students with "expert" readers. We thought of an "expert" reader as a person who was used to reading dense, technical articles—and had some background in the concepts of calculus—but was neither a mathematician nor an expert in the specific concepts in the texts.

To recruit "expert" readers, we contacted colleagues in the physics, chemistry, biology, economics, and computer science departments at our institution. All contacted professors agreed to participate: two from the physics department, and one each from chemistry, biology, economics, and computer science. With the exception of the physicists, the professors interviewed had not actively engaged with introductory calculus ideas for (at least) the past ten years. The physics professors had encountered introductory calculus concepts in their teaching but had not recently encountered the specific topics we used in the interviews.

Texts

Our goal in selecting excerpts of the textbook was to find sections for which students should have the necessary background knowledge and weren't purely procedural (i.e., only presenting a formula or a step-by-step procedure). We selected the excerpts from the "applications of integration" and "systems of differential equations" chapters that presented formulas as well as conceptual explanations for how these formulas were derived. In this report, we describe an interview based on sections from Hughes-Hallett et al. (2012) Chapter 8.2, *Applications* [of the integral] *to Geometry*": the introduction and the section "*Arc Length*".

Interview Methods

Each student participated in two interviews, approximately five and ten weeks into the (15-week) semester. Each professor participated in a single interview. The interviews were video-recorded to capture the interviewee's gestures and writing, and the audio was transcribed.

In order to elicit the interviewee's perspective and experience of reading the textbook, our methods involved interviewing participants to identify (1) how they perceived the situation; (2) the gaps they encountered; and (3) they way they drew upon their resources to bridge the gaps. We used a *message q/ing* protocol (Dervin, 1983), where participants were asked to read the text and stop at places where they had questions or were confused, and engage in discussion. When they were finished reading the text, the participants were asked to describe the main ideas of the section. Then, we interviewed the participants using an *abbreviated timeline* method (Dervin, 1983): we asked them to explain the meaning of each graph and/or formula, how the terms in the graphs and formulas had been derived, how the graphs and formulas were connected to each other, and why the text's explanation of the connections and derivations made sense.

Analytical Methods

Sense-making frames, gaps, and bridges. We viewed a gap as a question that a reader has while reading the text. However, a reader may not be consciously aware of the gap when it occurs, and is unlikely to consciously think of it in terms of an explicit question. Thus, gaps may be either recognized by the reader while reading the text or during the subsequent interview.

To identify gaps, we individually read each transcript line-by-line and identified collections of utterances that appeared to be evidence that the reader had had an explicit question about the text, had constructed a bridge, or where there were verbal hesitations or pauses. In order to identify the sense-making frame that the reader used, we identified the aspects of the text that were the focus of the explanation or question.

Implied reader. We used the concept of the implied reader as a tool to provide theoretically-grounded explanations for why a particular reader may succeed or fail to construct mathematically correct conceptions from the text. We compared all of the interviews to identify places where at least one interviewee experienced a gap or described the concepts incorrectly. Then, we described the various behaviors, codes, and competencies that might be required to construct a mathematically accurate interpretation of these aspects of the text and used these descriptions as a working hypothesis for why some interviewees struggled while others did not.

Results

Types of Gaps

In many cases, the gaps that students and professors experienced appeared to be related to their lacking one or more of the aspects of the implied reader. Most of the were related to missing mathematical competencies: the meaning of derivatives of functions and approximating a derivative at a point, the meaning of Riemann sums and their connection to definite integrals, and the reasoning underlying the derivations of formulas. The interviewees typically used content sense-making frames when they identified these gaps. For example, when Frank read the text's explanation of arc length, he appeared to think of the Riemann sum as computing area under the curve shown in Figure 1 and described several options for the method that you could use to do this—specifically, using left-hand, right-hand, or trapezoid rules. Thus, he encountered a gap in which he appeared to ask "which type of Riemann sum is being used?" This gap may be attributable to Frank's lack of understanding of Riemann sums as a general computational tool that isn't tied to a specific geometric method:

A definite integral can be used to compute the *arc length*, or length, of a curve. To compute the length of the curve $y = f(x)$ from $x = a$ to $x = b$, where $a < b$, we divide the curve into small pieces, each one approximately straight.

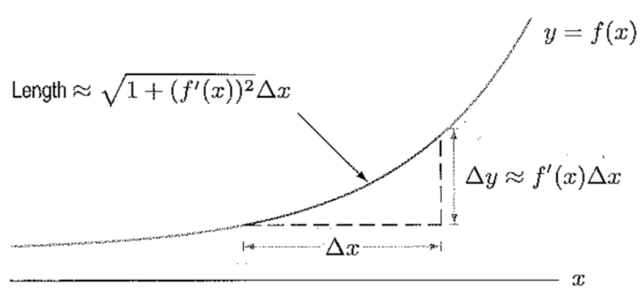

Figure 1. Excerpt from text showing construction of arc length approximation.

Frank: If you have, um, like a left or a right hand sum, where it's just a bunch of blocks, you know, making up the curve, um, it's kinda hard to imagine a curved line going through those blocks. But when you have a Riemann sum—more specifically, the Trapezoid Rule—it's a little easier, I guess, to envision, that like a bunch of, um, angled lines making up a curved line, as opposed to just a bunch of boxes making up a curved line.

Interviewer: Hmm. And you've mentioned the Trapezoid Rule. So how are they using the Trapezoid rule, where do you see that in their explanation?

Frank: Well it's… I'm not really seeing it, but I know that the Trapezoid Rule is really just a way of instead of drawing boxes, to calculate the area under a curve, you draw it, kind of triangles to represent the area under the curve. Draw a box, and then, like, a triangle on top of it connecting two points on the line, and if you were to look side by side, like a left and right hand sum, versus a trapezoid rule, um, to find the length of a curve like this, it would, uh,

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look a little more natural, I guess. But, I don't know, that's just what I thought of when I saw that they were using Pythagorean Theorem to calculate the arc length.

Unlike the professors—who were usually able to identify the source of their gaps—the students' missing competencies resulted in their being unable to identify the origins of their gaps. For example, Frank appeared to believe that Riemann sums could only be used to find areas; this resulted in his gap being a question of *which* type of area approximation was being used, rather than what the Riemann sum represented; this led to a mathematically incorrect bridge.

Most professors and students experienced a gap when they encountered the formula $\Delta y \approx f'(x)\Delta x$, as shown in Figure 1. This gap appears to have originated due to their not possessing one or more required competencies related to understanding the derivative and how it is related to slopes of secant lines. For example, in the excerpt below, Professor D., a computer scientist, initially attempted to bridge this gap by drawing on his knowledge of other mathematical concepts (specifically, linear functions), but eventually felt that this bridge was insufficient:

Professor D.: I'm still not sure why the change in *y* is roughly…. And it has to do with the linearity assumption, like piecewise linear sort of functions. And I just forget why.... You know, it's like $y=mx+b$ kind of thing. So the derivative is like the *m* and the *b* is zero, because it's just a change from your point. So it's something like that, but that's just the only part that I'm like, not... I'm sure they explained that earlier in the book too.

Professor K., a physicist, experienced the same gap, but was unable to construct a bridge:

Professor K.: So the one thing that I am puzzling to remember is why delta-*y* is *f*-prime *x* times delta *x*. Otherwise this all makes good sense.
Interviewer: And so is it just the assertion that this is true, or how they...?
Professor K.: No, how they use it is fine. This is all very clear. But this assertion... I don't remember.
Interviewer: Is it just the notation, or is there something about the concept behind that?
Professor K.: Yeah, the notation is fine, it's just remembering why *y* is going to be equal to the derivative of *x* times delta *x*.
Interviewer: But then once you assume that's true, then everything else…?
Professor K.: Everything else is fine, yeah.

Although most students appeared to experience the same competency-related gaps as the professors, they often did not recognize these gaps until the interviewer explicitly asked them about the section of the text. For example, Peter's initial hesitation in his response suggests that he did not experience a gap while he read; he recognized this gap during the interview, but was unable to construct a bridge and experienced a lack of understanding:

Interviewer: Did it make sense when they said that the change in y was just f-prime-x times the change in x?
Peter: Um… not entirely. I don't really know... Like yeah, they kinda just threw that in there without really explaining it.

In addition to experiencing gaps related to mathematical competencies, most of the interviewees also experienced gaps related to mathematical codes. For example, several students expressed confusion about the role the symbols *a* and *b* played in Figure 1. Among the professors, these appeared to primarily include (missing) codes related to mathematical terminology and notation. For example, Professor I., a biologist, experienced gaps when reading "integrand" and "elementary antiderivative."

Professor I.: Well I would need to go back up and look at some of these vocabs. But I guess an integrand is the solution of this integral problem. Um… but this elementary antiderivative. I guess to find an integral, you have to find the antiderivative? But I don't know what an elementary antiderivative is. Um… so I was a little bit worried about that.

Bridging and "Jumping" Gaps

As shown in several of the examples above, there were numerous instances where students and professors experienced gaps that they were unable to bridge; they continued reading by "jumping" the gap. However, the professors were much more likely to make the un-bridged gap an object of reflection, and to both acknowledge the lack of a bridge and make conjectures about what a bridge might look like or how the un-bridged gap might impact their understanding. For example, Professor I. made educated guesses about how "elementary antiderivatives" might be used, and Professor K. appeared to recognize how not knowing why $\Delta y = f'(x)\Delta x$ might impact her understanding. In contrast, Peter didn't recognize his own lack of understanding about this equation, and did not appear to reflect on how this unbridged gap might impact his interpretation of the text.

Sense-Making Frames

All of the students and professors tended to use a content sense-making frame for much of the reading. However, a few of the students and all professors used *multiple* sense-making frames while reading. For example, Professor K. used and coordinated content, communication, and situating-pedagogy frames to understand the derivation of arc length:

Interviewer: Okay. How would you describe the book's method for finding arc length?

Professor K.: How would I describe the method... Um... I'm not quite sure how to describe it, except that they're following the same steps that they suggested in their box. So the first step that they have here is that they show how you can find the length by breaking this up into small pieces. So in delta-*x* it's a two-dimensional function, and so delta-*x* and delta-*y*. So they show how you could approximate it. And then they take that into a summation over very small pieces. So you go from these delta-*y*'s to derivatives. So that gives me very small pieces. And then they take that sum, and they turn it into an integral. So they're following their own steps and laying out their procedure. And then they give me a nice boxed equation that I can just go to and work from.

In examining the details of the calculation, Professor K. employed a content frame. Her reference to following steps and laying out a procedure suggest a situating-pedagogical frame, and her discussion of the boxed equation suggested she was using a communication frame.

All of the professors regularly used multiple sense-making frames—in particular, situating-pedagogical frames—and this appeared to be associated with their ability to recognize more gaps, construct (mathematically correct) bridges, and to use various resources to create these bridges. Using these frames appeared to enable the professors to bridge content-related gaps, even when they lacked some of the competencies or codes of the implied reader. In contrast, the students relied almost exclusively on content sense-making frames.

Drawing on Other Knowledge

In addition to using multiple sense-making frames, professors tended to draw on informal knowledge and their own disciplinary knowledge to make sense of the mathematical ideas. For example, Professor D., who was a computer scientist, encountered gaps related to understand the limiting process of transforming a Riemann sum into a definite integral. He described his bridge as based on understanding the integral as a sum of discrete entities:

Professor D.: I guess I see an integral just being a sum of a bunch of discrete things, because I'm a computer scientist. I'm, you know, discrete math instead of continuous math. So I actually really see the thing above it. The summation of all of these little hypotenuses being summed together. So uh, this notion of like having a discrete sum, and then as you integrate—as you integrate from, you know, you integrate the number of sections, and those go to infinity and the size of the section goes to zero, like it's kind of the same thing. And it's just summation and integral are basically the same symbol in my head.

Professor M., a chemist, lacked some of the implied reader's competencies for limits and Riemann sums. Rather than draw on his disciplinary knowledge, he used informal knowledge to bridge gaps he encountered when trying to understand how to approximate volume of a sphere:

Professor M.: If we're approximating this volume with a, with um, with a bunch of little cubes, you know, um... And smoothing it out to make a round shape, then that would be maybe what this is trying to do. I have no idea if that's right. That's... That's kinda what I'm thinking about... So if we were doing this sphere, we're taking, and we're like breaking it into Lego blocks, those blocks are easy volumes to calculate, so we take those, add up all those blocks together, and then we have to... [makes a rounding shape with his hands] to smooth out the edges.

Discussion

There were numerous similarities between the "novice" and "expert" readers. Both groups primarily used content sense-making frames and experienced numerous gaps. Most professors interviewed were not familiar with the formal notions of Riemann sums or limits and frequently did not recognize terminology; this could be, in part, due to their not having actively thought about calculus concepts since they were students in college. Although the students had recently discussed ideas of Riemann sums and limits, they also often experienced gaps that appeared to be related to the same calculus concepts as the professors.

These gaps can be viewed as the result of discrepancies between the implied and empirical readers—that is, the students and professors lacked particular competencies and codes that were required to construct mathematically accurate interpretations of the text. For the students, lacking some of the competencies resulted in them mis-identifying the source of their gaps, being unable to bridge the gaps, or constructing bridges that were mathematically incorrect.

The professors were generally better at recognizing gaps while they were reading the text. In contrast, the students tended to only notice the gaps when they were asked to focus on a specific section of the text or asked directed questions. This suggests that the professors tended to employ behaviors that enabled them to monitor their own understanding as they read, whereas the students did this less frequently.

In contrast to the students, the professors were often able to construct mathematically accurate codes and competencies in order to bridge gaps related to their lack of knowledge of calculus concepts. They appeared to identify the sources of their gaps; they drew on their other mathematical knowledge to construct bridges; they drew on both informal knowledge and knowledge from their own discipline to construct bridges; and they employed multiple sense-making frames—in particular situating-pedagogy frames. These practices appeared to enable them to generate accurate interpretations of the text, to construct missing codes and competencies in some places, and to recognize the limitations of their interpretations.

Taken together these results suggest that constructing accurate interpretations of a mathematical text requires particular background knowledge and ways of interpreting the various symbols and technical terms. However, a lack of background knowledge can be overcome in several ways. In particular, the examples here highlight the importance of attending not just to the mathematical content, but the way it is communicated and organized. This analysis suggests various ways of helping students structure their reading experience—such as asking questions that help students focus on non-content aspects of the text. It also suggests that students need help recognizing when they encounter a gap and developing strategies to bridge gaps. However, some of the practices of the "expert" readers, such as making connections with previous mathematical knowledge, might be more difficult to help students do.

These results build on the prior literature by beginning to disentangle the impact of content knowledge from reading practices. For example, Shepherd and van de Sande (2014) found that *mathematicians* employed many of the effective practices reported in this study. However, our results suggest that readers who are less familiar with the content (i.e., the professors) might read less effectively than people who are more familiar (i.e., the students who were currently taking calculus). Furthermore, using multiple sense-making frames and reading reflectively might enable readers to overcome deficiencies in background knowledge. These results highlight the importance of attending to both knowledge and reading practices when asking students to learn from reading mathematical texts.

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