

Supporting Clear and Concise Mathematics Language:

Say This, Not That

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Supporting Clear and Concise Mathematics Language: Instead of That...Say This...

Juan, a child with a mathematics disability, is learning about addition and subtraction of fractions. Juan's special education teacher, Mrs. Miller, has tried to simplify language about fractions to make fractions easier for Juan. During instruction, she refers to the "top number" and "bottom number." At the end of chapter test, Juan reads the problem: What's the least common denominator of $\frac{1}{2}$ and $\frac{2}{5}$? Juan answers "1."

Upon returning his test, Mrs. Miller asks Juan how he arrived at his answer. He told Mrs. Miller that he didn't know what denominator meant, so he used the word "least" to choose the number that was least. Mrs. Miller expressed that "denominator" was the formal term for the "bottom number." As soon as Mrs. Miller explained that, Juan exclaimed, "I know how to find the least common bottom number!" Mrs. Miller did not intend to make mathematics confusing for Juan; she tried to make mathematics easier. But, in simplifying her language without connecting this informal language to formal mathematics language, she did Juan a disservice.

Children with disabilities, like Juan, perform lower than their peers without disabilities in mathematics, and this gap widens from ages 7 to 13 (Wei, Lenz, & Blackorby, 2013). Even more concerning is that fifth-grade children with mathematics disabilities continue to perform in the bottom quartile of their grade in high school (Shalev, Manor, & Gross-Tsur, 2005). This trend leads educators to ask the question: With multi-tiers of instruction, why do low-performing children in the elementary grades continue to struggle with mathematics?

One influence contributing to this trend may be the imprecise use of mathematics language. Educators may not interpret mathematics as a second (or third) language for children, when, in fact, all children are mathematical-language learners (Barrow, 2014). The numerals,

symbols, and terms that explain mathematics concepts and procedures are plentiful and complex. The language of mathematics, especially vocabulary terms, is necessary for understanding mathematics in oral and written forms (Ernst-Slavit & Mason, 2011; Riccomini, Smith, Hughes, & Fries, 2015). Mathematics vocabulary is often difficult for children because many terms have meanings in general English and meanings specific to mathematics (Rubenstein & Thompson, 2002; Schleppegrell, 2007). For example, *factor* could mean a contributing element (e.g., one factor contributing to mathematics difficulties is vocabulary that requires language code switching) or two or more numbers multiplied together to produce a product (10 and 12 are factors of 120). Even in the mathematical definition of factor, *produce* and *product* may have multiple meanings!

Language plays an important role in learning mathematics. In Juan's case, his special education teacher was trying to make fractions easier for Juan to understand, but because of the simplified language used in instruction, Juan did not understand grade-level questions presented with mathematical vocabulary. Just as there are rules in mathematics that expire in later grade levels (e.g., multiplication always results in a *bigger* number; Karp, Bush, & Dougherty, 2014), there are mathematics terms (e.g., *bottom number*) that only help children temporarily. One way for educators to improve Juan's mathematics performance is by developing an understanding of and sensitivity to mathematics language that facilitates conceptual and procedural understanding. Children should learn mathematics skills in accurate contexts that provide a solid foundation on which to build more complex skills in later grades. Therefore, teaching language that is mathematically correct and holds true across grade levels can help children generalize mathematics across concepts (Townsend, Filippini, Collins, & Biancarosa, 2012).

Educators need to plan for using clear and concise language, which includes identifying inaccurate terms, preferred language, and why changes in language matter. For example, by the end of first grade, there are over 105 novel mathematics vocabulary terms that children are expected to understand and apply (Powell & Nelson, 2016). The number in fifth grade is above 325 (Powell, Driver, & Roberts, 2016). With children expected to know hundreds of mathematics vocabulary terms and the meaning of those terms, exposure to clear, concise, and uniformly used language is a must (Monroe & Orme, 2002). Specificity with language is an important consideration for special educators because children who experience mathematics difficulty often experience reading difficulty (Hart, Petrill, Thompson, & Plomin, 2009). As the majority of mathematics high-stakes items involve reading and interpretation of mathematics vocabulary terms, special educators must provide explicit instruction related to mathematics vocabulary.

The purpose of this article is to discuss mathematics language that supports accurate and conceptual understanding of mathematics. In the following sections, we discuss five domain areas within the elementary Common Core standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). We provide a brief description of why it is important to use clear and concise mathematics language within the domain area and then provide educators with a ‘instead of that...say this...’ guide.

Counting and Cardinality

A solid foundation for success in mathematics begins with understanding that numbers have given values. Correct counting requires stable order (i.e., a sequence of counting words), one-to-one correspondence, and cardinality. Cardinality refers to the understanding that the last number word of a counting sequence (e.g., “one, two, three, four, *five*” for a set of five toy cars)

represents the quantity in the set. Given that these skills provide the foundation for future success in calculation (e.g., addition; Noël, 2009), it is not surprising that counting and cardinality comprise almost one-third of kindergarten Common Core standards. It is important, however, that children are exposed to counting and cardinality language in a way that allows for integration of higher level, complex mathematical understanding. In terms of language, educators must be mindful of how they talk about numbers within the counting sequence (see Figure 1).

<Insert Figure 1 about here>

Our language suggestions pertain to the counting sequence. Purposeful language may promote students to extend mathematics understanding as they learn new concepts that compliment the continuum of learning. For example, while teaching the concept of negative integers in kindergarten is not developmentally appropriate, educators can present counting and cardinality using a number line that extends in both directions, requiring children to locate 1 (or another number) when counting. Educators should not indicate that any number is *first* when counting or looking at a number line because this indicates there are no numbers less than the *first* number. Instead, educators should talk about counting as *starting* at a specified number. In a similar manner, educators should not indicate an end to the counting sequence (e.g., 10). Many children have difficulty with the teen numbers and beyond (Geary, 2000) and, as too many mathematics trade books about counting finish with 10 (Powell & Nurnberger-Haag, 2015), it is important to use language that supports numeral and quantity understanding beyond 10. Two problematic examples pertaining to 10 are songs with lyrics “7, 8, 9, and the last one’s 10” or “8, 9, and 10.” Using accurate language that promotes counting as a continuum on a number line

provides children with an accurate, base understanding of numbers so that they can flexibly incorporate new, complex understanding when it is developmentally appropriate.

Number and Operations in Base Ten

As children develop counting and cardinality skills, they also acquire an understanding of number and operations in base 10. This begins with composition and decomposition of two-digit numbers. For example, the number 15 can be represented as one bundle of 10 and five ones or 15 ones. Children also learn to compare two-digit numbers using symbols and words (i.e., $>$, $=$, and $<$). Not only do children need to develop a flexible understanding of numbers, but they need a solid foundation in mathematics language to discuss numbers. For example, children need to know the following terms in order to compare numbers: *greater than*, *less than*, *place value*, *digit*, *more*, and *fewer*. Children's understanding of place value builds in complexity with each grade, and so do the language demands. By fifth grade, children must understand and describe the relationship between each place value (e.g., a digit in the hundreds place represents 10 times as much as the digit in the tens place), compare decimals to the thousandths, and multiply or divide whole numbers and decimals using place value strategies. Unfortunately, many children struggle to develop proficiency in this domain (DeWolf, Grounds, Bassok, & Holyoak, 2014), and deficiency in number and base-10 operations greatly impacts children's performance in other domains of mathematics.

When teaching this domain, educators should incorporate language that supports place value understanding and develops flexibility in mathematical thinking. Although it may seem easier to teach children chants, tricks, or rules for solving place value and computation problems, these approaches often use inaccurate language that does not support conceptual understanding.

It is critically important that educators model and maintain accurate language that facilitates conceptual understanding of place value.

<Insert Figure 2 about here>

Educators should keep two things in mind when teaching this domain: instruction should be tied to place value understanding and developing flexibility in mathematical thinking (see Figure 2). As mentioned within the section about Counting and Cardinality, children often have difficulty with number names beyond 10 (Browning & Beauford, 2011). Therefore, educators should practice number names (e.g., 324 as *three hundred twenty-four*) and discuss digits and their value (e.g., “the 2 is in the tens place so 2 means 2 sets of ten”). Educators should explain that each *numeral* within a *number* represents the value of the *digit*. It should also be noted that there are different ways of referring to the base-10 system (e.g., *ones, tens, and hundreds; singles, sticks, and sheets; ones, rods, and flats*). When deciding which mathematical terms to use, it is recommended that educators refer to the Common Core language for guidance; however, the terms used should be consistent across the grades and mathematical in nature (i.e., not *singles, sticks, and sheets*). When educators ask children to represent numbers by place value, children *compose* or *decompose* rather than *make* or *break apart*.

Educators should also be mindful of language about comparison and calculation. For comparison, educators should use accurate language such as *greater than* or *less than*. These terms replace the consistent description that, “The alligator eats the bigger number.” In this example, no conceptual understanding is established with *eats*. Also, numbers are not *bigger* but *greater* or *more*. The term *bigger* can cause difficulty years later with addition and subtraction of positive and negative integers. The same goes for the term *smaller*. With calculations based on the four operations (i.e., addition, subtraction, multiplication, and division), educators must be

mindful of the language used to explain the operator symbols (+, −, ×, ÷), inequality symbols (<, >), and equivalence symbols (=, ≠). The plus (+) sign means to *add*, but it does not signal *plussing*. The plus sign may be explained as *putting together*, but this definition is short-lived as children are encouraged to start with a set and add on to the set (Fuchs et al., 2009). Another symbol that causes difficulty in later grades is the equal sign. Educators could use language, such as *the same as* or a *balance*, to help children understand the equal sign as a balance between two sides of an equation (e.g., Powell, Driver, & Julian, 2015). Educators should avoid saying the equal sign means *write your answer* or *compute*. As children use symbols to perform multi-digit computation, educators must be aware of using language that supports the concept of regrouping (e.g., *regroup*) rather than the notational procedure (e.g., *borrow*).

Numbers and Operations with Rational Numbers

Number and operations with rational numbers address mathematics involving fractions, decimals, and percentages. Rational numbers tend to be one of the most problematic areas for children, and difficulty with rational numbers impacts later mathematics learning (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Any difficulty with rational numbers is concerning given research linking rational number achievement to later success in mathematics (e.g., Bailey, Hoard, Nugent, & Geary, 2012). It is not alarming, however, when we consider how properties of rational numbers may differ from properties of whole numbers. Language terms taught early about whole numbers may no longer apply when working with fractions or decimals (see Figure 3).

<Insert Figure 3 about here>

Many of the language suggestions for rational numbers pertain to how we communicate fractions to children using words or images. For example, a fraction is a unique *number* whose

magnitude or *value* can be identified on a number line. A fraction is comprised of *numerals* (e.g., 3 and 8) that create a single *number* (e.g., $\frac{3}{8}$). The denominator of a fraction represents the *equal parts* of the relative whole (i.e., whole, group, set, measurement), not just *parts*. Children commonly divide a rectangle into eight unequal parts and shade three of the parts to represent $\frac{3}{8}$ without an understanding that the whole must be divided into *equal parts*. If an educator merely added *equal* to *parts* every time the denominator was mentioned in instruction, children might develop a better understanding of wholes and parts. Educators must also provide exposure to fraction concepts that do not fit the *equal parts in a whole* meaning. For example, a fraction can be *parts of a set* or a *position on a number line*.

Educators should use the technical terms *numerator* and *denominator* under most circumstances and relegate *top* and *bottom number* language to descriptions of the position of the numerator and denominator. Top number and bottom number are strictly terms that describe the position of the numbers in the numerator and denominator, and fractions are not always presented with a fraction bar (e.g., $\frac{1}{8}$), which negates these terms). In a similar way, describing a fraction as a number *over* a number does not help children understand that a fraction is a single quantity. Students' misunderstanding that fractions consist of two separate numbers separated by a *line* may lead to errors such as adding across the *top* number and adding across the *bottom* number when adding fractions. As children learn to *reduce* a fraction to lowest terms, some children believe this means the value of the fraction changes. If educators describe determining an *equivalent fraction* in *simplest form*, this would eliminate the need to use the term *reduce*.

Decimals communicate similar information as fractions but are based on powers of 10. As such, the language used to read a decimal can support the relationship. For example, reading 5.4 as “five *and* four tenths” naturally connects fractions to decimals by sharing how fractions

can be written as decimals. Casual language, such as saying *point* (e.g., “five point four”) as the placeholder for the decimal point, spills over to discussion on how decimals are manipulated (e.g., “move the decimal point over”) instead of building conceptual understanding of the base-10 system. Another language consideration is introduced with *out of* and ratios. Many educators use *out of* to describe the parts of a whole (e.g., “three out of four” for $3/4$), but with ratios, *out of* does not convey the same meaning (e.g., 3:2 is not “three out of two” but “three to two”).

Geometry

Children typically start school with a basic understanding of shapes (Clements & Sarama, 2000). Within the Common Core, Geometry appears as a domain area at kindergarten and all subsequent grades through eighth grade. Much of geometry in the elementary grades focuses on two-dimensional (2D) and three-dimensional (3D) shapes. In the late elementary grades, children are expected to understand lines and angles and how these relate to properties of shapes and coordinate planes. Children with mathematics difficulty struggle with geometry concepts through high school (Dobbins, Gagnon, & Ulrich, 2014); therefore, it is necessary to provide a consistent and strong geometry background to children across grade levels. Often, general vocabulary is used to describe geometric concepts, yet children are expected to interpret formal geometric vocabulary. Educators should show the connection between informal and formal terms (see Figure 4).

<Insert Figure 4 about here>

Many of the issues around language with Geometry pertain to preciseness of vocabulary. At the earliest grades, educators may use informal names for shapes, like *ball* for *circle*, when a *ball* is actually a *sphere*. Children must understand the term *circle* and use it across grade levels, so introducing this term early and applying it consistently is necessary. The same is true for

square and *rectangle*; a *square* is a *rectangle*, but a *rectangle* is not always a *square*.

Mathematical language accuracy is also important for understanding that the space between intersecting lines is an *angle* and not a *corner*. Children do not measure corners, but they do measure angles.

In the late elementary grades, language used to describe 2D shapes can change for 3D shapes, so educators must explicitly help children identify these changes and connect the concepts. A cube may be described as having six *sides*, but these sides are *faces*. The *sides* are actually *edges*, and *edges* meet at *vertices*, not *points*. When calculating the volume, two of the *faces* of the cube are *bases*. Another language concern is around the term *same*. An educator may use *same* to describe figures that are similar, congruent, and symmetrical. Using *same* may be helpful in the short term, but as children are asked to find similar and congruent shapes, *same* does not help with this task.

As children learn transformations in the early elementary grades, educators often describe these as *flips*, *slides*, and *turns*. While these terms describe the action of a transformation, children in the later elementary and middle school grades must be familiar with the formal terms of *reflection*, *translation*, and *rotation*. Specificity with the term is necessary for children to have gained adequate exposure to the term for practice within textbooks and on high-stakes assessments. In a similar manner, shapes do not *shrink* or *stretch*. Instead, these are *dilations* of a shape.

Measurement and Data

The domain area of Measurement and Data is mentioned specifically across kindergarten through fifth grade. At kindergarten, children are expected to describe length and weight and compare objects with measureable attributes. Measurement of objects continues across the

elementary grades with a focus on standard and nonstandard units of measurement. In first grade, children start learning about telling time, and children are introduced to money in second grade. Starting in third grade, measurement becomes intertwined with Geometry as children measure perimeter and area of shapes, continue with measurement of angles in fourth grade, and measurement of volume in fifth grade. Representing data is a theme across the elementary grade levels. Figure 5 highlights important components of language when educators teach measurement and data concepts.

<Insert Figure 5 about here>

Similar to the Geometry domain area, many of the language issues with measurement relate to being precise with language and not using certain terms interchangeably. When educators introduce telling time, the hands should be referred to as the *minute hand* and *hour hand* so that children can understand which hand indicates minutes and which indicates hours. As children compare quantities, it is important to use language correctly. For example, “Gabe’s amount is *less* than Marta’s” is grammatically correct over “Gabe’s amount is *fewer* than Marta’s.” Numbers should not be described as *bigger* but instead as *greater* because *greater* is associated with quantity.

For detailed measurement, *length* refers to the measurement of a side or edge. Educators may use *weight* and *mass* interchangeably, yet *weight* refers to how much an object weighs down on a surface, whereas *mass* refers to the matter within an object. On a high-stakes assessment, a child may be presented with a pictorial representation of a liquid measuring cup filled with a liquid. The question may ask about the *capacity* of the cup and the *volume* of the liquid. In order to understand the task, the child must understand that *capacity* and *volume* have different meanings but similar calculations. For interpretation of data, educators should be specific with

the types of data representations (e.g., *chart, graph, picture, pictograph*) so children can create appropriate representations of data.

Conclusion and Implications to Field

Taking steps to prevent mathematics difficulty for children is important. One way to support children and promote progressive understanding of mathematics is to use precise and accurate language embedded within teaching strategies that progress and generalize across standards and grade levels. In this paper, we share several examples of ways to adjust common errors in mathematics language; however, the lists provided do not encompass all possible language errors and faux pas. Many of our language suggestions help to support conceptual understanding of mathematical concepts, and this is often difficult for children with mathematics disabilities. The clear and concise mathematics vocabulary shared can be incorporated into existing evidence-based practices and instruction with ease. For example, when using manipulatives to demonstrate the concepts of multiplication and regrouping, educators can focus on describing base-10 blocks as *hundreds, tens, and ones* and reinforcing *regrouping* or *exchanging*. Because clear and concise mathematical language sets children up for success, educators in subsequent grade levels may not have to reteach so many misconceptions related to language and rules (Karp et al., 2014).

Additionally, it is important for special educators to consult the standards and curricula across grade levels to understand language and expectations for future successes. For example, a third-grade educator may benefit from looking at, not only third grade standards, but also fourth, fifth, and sixth grade standards. Teaching children so that they are successful in mathematics requires that educators plan for not only short-term success, but long-term success. We encourage educators to use the information shared in this article to attend to the importance of

mathematics language in instruction and evaluate personal use of correct mathematics language to support longitudinal learning.

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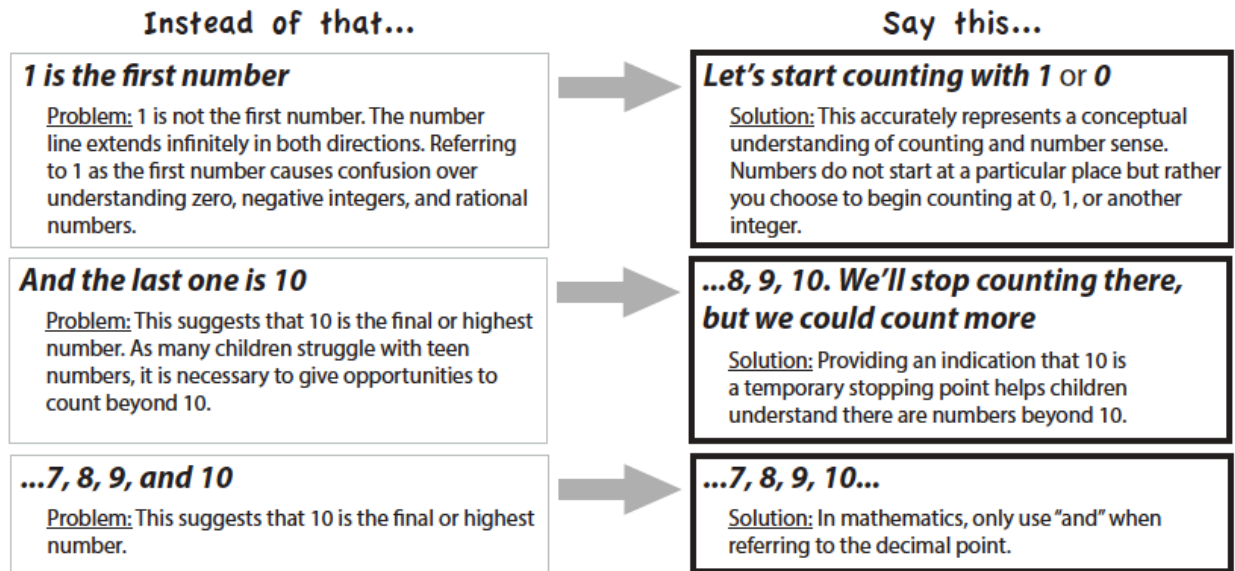


Figure 1. Counting and Cardinality.

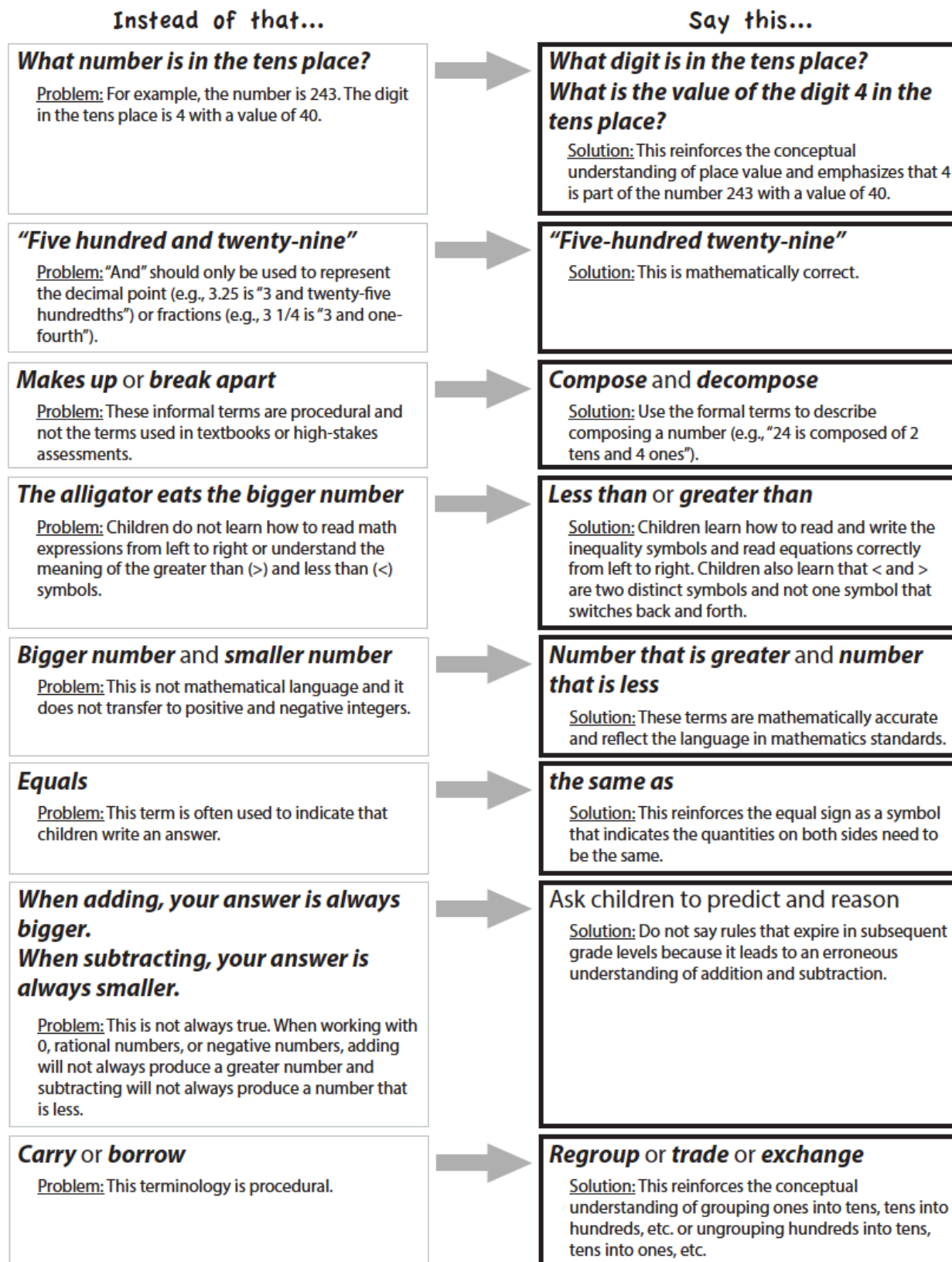


Figure 2. Numbers and Operations in Base Ten.

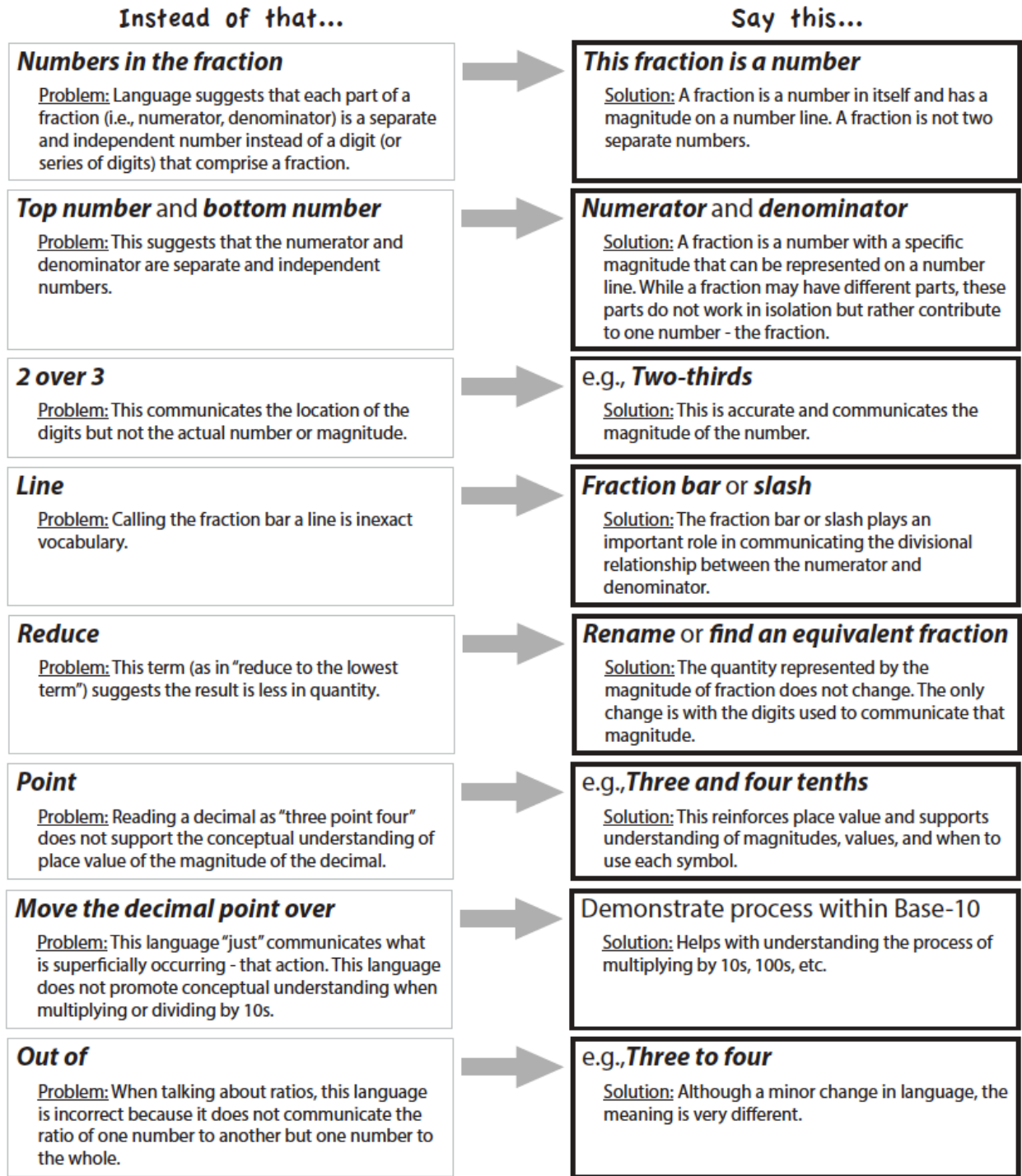


Figure 3. Numbers and Operations with Rational Numbers.

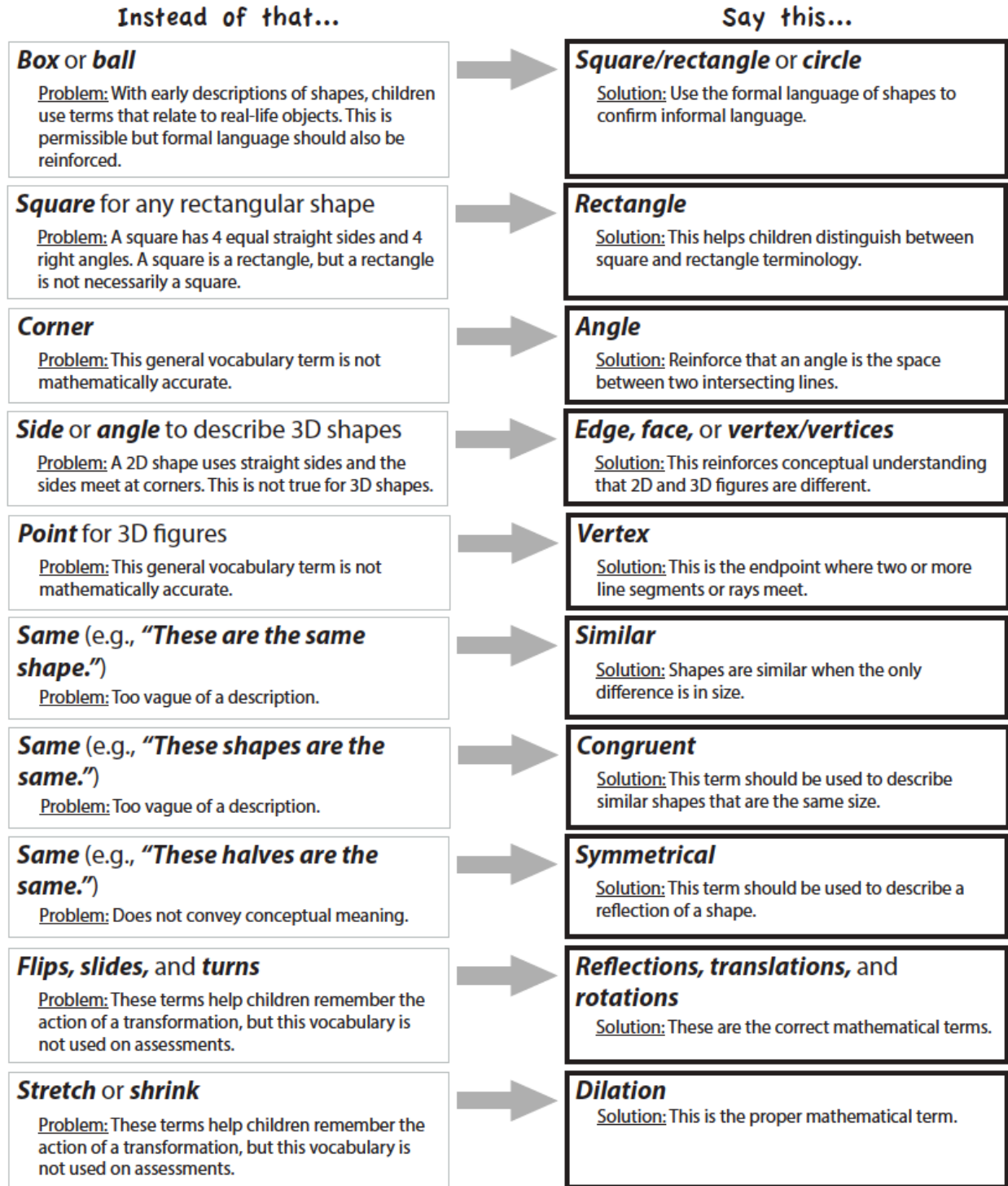


Figure 4. Geometry.

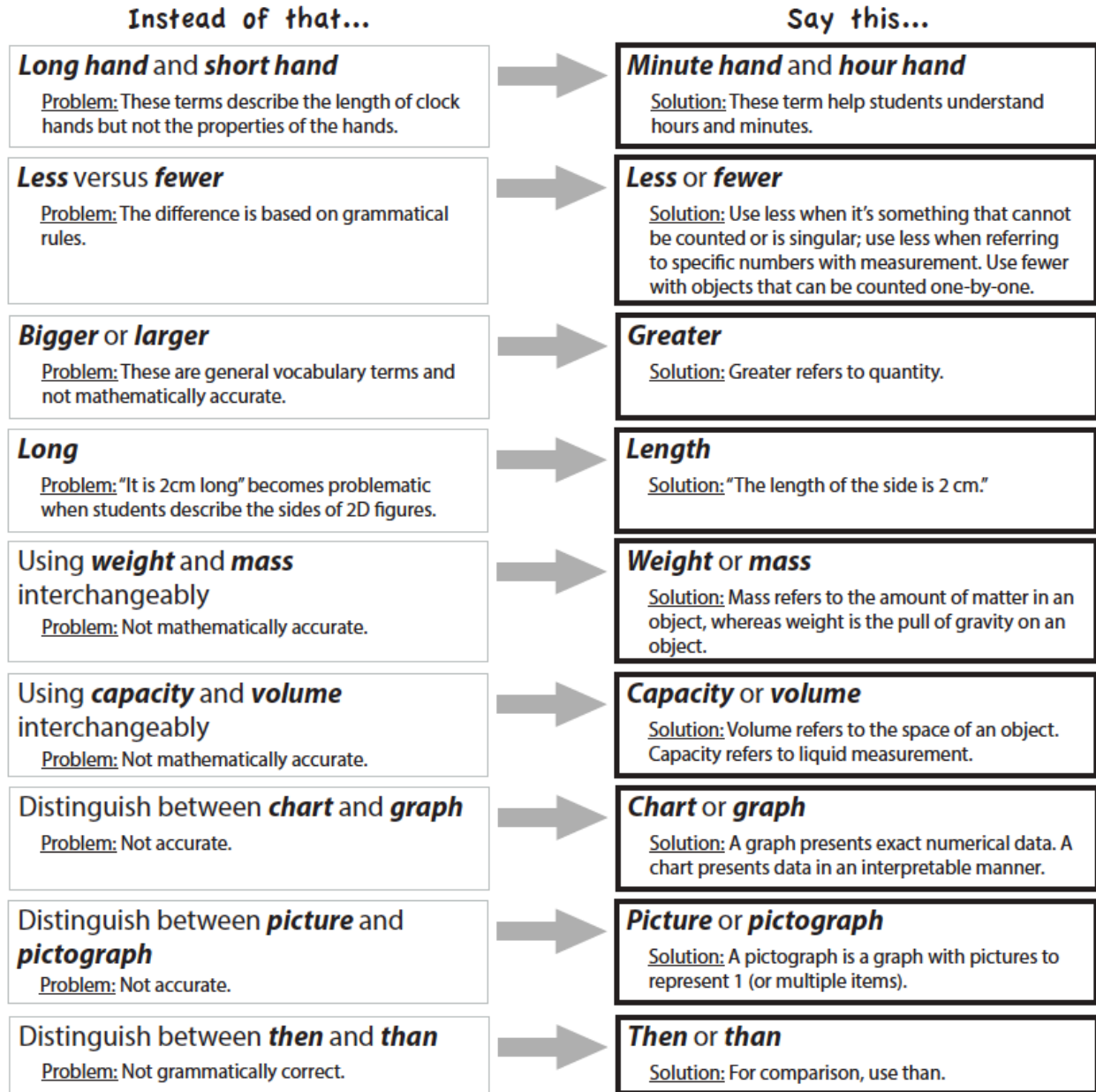


Figure 5. Measurement.