USING DISCOURSE ANALYSIS TO UNDERSTAND VARIATION IN STUDENTS' REASONING FROM ACCEPTED WAYS OF REASONING

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In this study, I use a systemic functional linguistics approach to examine mathematics classroom discourse with the aim of providing a plausible explanation of how students could actively participate in productive classroom discussions without adopting ways of reasoning that were accepted in the classroom community. In this way, I work in the crossroads of a research tradition examining classroom interaction and a research tradition that examines student learning. I found that even though particular ways of reasoning about exponentials and logarithms were advanced and accepted in the classroom discourse, the way these ways of reasoning were talked about in the class did not preclude students from maintaining less sophisticated ways of reasoning Specifically, I argue that the two exponential ways of reasoning were not explicitly contrasted, which may have contributed to students seeing them as essentially the same strategy.

Keywords: Classroom Discourse, Teacher Education-Preservice

Introduction

It is not uncommon for a novice teacher to be surprised by his or her students' poor performance on an exam or assignment after class discussions had seemed to have been going well (see, for example, Price & Valli, 2005). This surprise is not unreasonable. At times, students can be integral participants in productive class discussions that seem to advance the mathematical agenda, yet later still be confused or grappling with a concept that was thoroughly discussed in class. To understand this phenomenon in greater detail, I took the lens of the emergent perspective (Cobb & Yackel, 1996) to examine classroom interactions.

The emergent perspective coordinates social and individual aspects of the classroom community to explain students' learning. Specifically, it coordinates classroom social norms, socio-mathematical norms, and mathematical practices with their individual correlates. In this study I focused on the relationship between classroom mathematical practices, which are specific ways of reasoning that become adopted in a class community, and its correlate of individual students' personal ways of reasoning. According to the perspective, this relationship is indirect and reflexive. Accepted practices arise as individual students posit ways of reasoning. These are then discussed within the class community and are either collectively accepted or rejected. In this way, individual students' conceptions and ways of reasoning give rise to accepted mathematical practices. Then, students' conceptions are influenced as they continue to participate in established math practices. In this way the relationship is reflexive. It is also indirect, meaning that there is not a one-to-one mapping between accepted math practices and students' ways of reasoning. This is acknowledged by the perspective in at least two ways. First, a math practice is not defined as the conceptions held by the majority of students, but as the social status of a way of reasoning in the classroom community. Second, Cobb and Yackel (1996) were careful to point out that participation in a practice influences, but does not determine students' ways of reasoning.

Despite the fact that this acknowledgement of variation in thinking existed from the inception of the theory, the research community still does not have many images of the nature and extent of this variation, much less a well-developed theory of why these variations occur. Of those scholars that have explicitly investigated the reflexive relationship between individual cognition and the emergence of mathematical practices (e. g. Rasmussen, Wawro, & Zandieh, 2015; Stephan, Cobb, &

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Gravemeijer, 2003; Tabach, Hershkowitz, Rasmussen, & Dreyfus, 2014), Stephan et al. (2003) gives the best image of the nature and extent of individual variation as they tracked two students' participation in a class community as various math practices emerged. While they documented significant variations in individual ways of reasoning from established practices, these differences seemed to resolve themselves through continued participation in the class. One explanation for this is that the individual ways of reasoning that varied from accepted practice did not yield correct answers, and thus, made it problematic to continue participation in class discussions while using their personal way of reasoning. This encouraged the students to reevaluate their way of reasoning and, eventually, to adopt ways of reasoning more consistent with the math practice. In contrast, my previous work (Gruver, 2016) found that mathematically significant variations in thinking can persist, even after instruction has ended. This finding shows that significant variations do not always work themselves out naturally through the course of instruction. This underscores the importance in further understanding these variations and their causes; this is the focus of the current study.

Nature of Variations From the Established Practice

In a previous study, I documented the ways that individual ways of reasoning varied from accepted classroom mathematical practices. I first determined which practices were established in a classroom of 29 prospective teachers and then compared those practices to the individuals' reasoning during a post instruction interview. I determined which practices were accepted in the class by analyzing students' arguments using the documenting collective activity method (Cole et al., 2012; Rasmussen & Stephan, 2008). Of the seven students interviewed, I found that four of them reasoned in ways that were different in mathematically significant ways from the established practice. In this section, I describe the established practice and the ways students reasoned in the interview.

The math practice emerged as students were developing an exponential number line, which would later be used to investigate exponential and logarithmic relationships. Early on, the students developed a number line in which powers of 10 were equally spaced. However, the spaces between the powers of ten were subdivided linearly. In this way, their number line had an exponential structure at the macro level, but a linear structure between the powers of 10. Eventually, this initial way of subdividing was eventually overturned in favor of a method that produced a fully exponential number line. The math practice, *Subdividing the Number Line,* consisted of two ways of reasoning that were accepted in the classroom community. These two methods for subdivision were cognitively distinct, but produced the same answer. The first method, *Subdividing Segments by Reasoning Linearly About Exponents* is characterized by students writing the number they wished to place on the number line in the form 10^b, ignoring the 10, and then determining the location of the number as if they were simply placing the exponent on linear number line. In other words, $10^{1.5}$ would go halfway between 10^1 and 10^2, because 1.5 would be halfway between 1 and 2 on a linear number line. The second method to subdivide the number line that became normative in the class was *Preserving the Multiplicative Relationship within the Segments.* This way of reasoning emerged as students noticed a constant multiplicative pattern at the macro level. Specifically, they noticed that the equally spaced powers of ten increased by a factor of ten. This differs from a linear number line where equally spaced points would increase by a constant sum rather than a constant factor. They then extended this pattern to apply to subdivisions. Thus, to determine the value of the half way point between 10^1 and 10^2, they would notice that between these two points there is an increase of a factor of 10; then, since the half way point divided the segment into two subsections, they would need to find a number that when multiplied by itself yielded 10. That number is the square root of ten. Thus, the midway point is 10 times the square root of ten. These two ways of subdividing segments on a number line emerged around the same time in class, but the first was talked about as a way to

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efficiently determine the value of subdivisions while the second was used as a way to explain why reasoning linearly with the exponents makes sense.

Three of the students coordinated these two ways of reasoning in the post interview. This means that while they may have determined the values of various points on an exponential number line using the numeric pattern in the exponents, they could also use multiplicative reasoning to justify their placements. However, the other four were not able to use the second way of reasoning, the multiplicative pattern, even when probed.

Since more than half the students interviewed did not include multiplicative reasoning in their interview responses, observers were left with the question: How could the students intellectually engage in class discussions, but not personally adopt ways of reasoning consistent with the classroom math practice? A partial response to this question will be developed in this report. In particular, I will focus on the nature of the classroom discourse as multiplicative reasoning was developed in this class to address the research question, *How might the nature of the discursive interactions in both whole class and small group settings give a plausible explanation for students' variations from the emergent math practice?* In answering this question, I examine the intersection of classroom interactions and individual student learning. This work is at the crossroads of two research traditions and contributes to a new path forward for using discourse analysis to give insights into the nature of individual knowledge construction.

Method

Data collection occurred in a math class for prospective secondary teachers (PSTs). The purpose of the course was to deepen the PSTs' mathematical knowledge of secondary topics. The current study focuses on a single unit where the PSTs explored exponential and logarithmic relationships. This unit was three weeks long. The class met twice a week for an hour and a half each time. Thus, the unit included nine hours of instruction spread over six days. Data included video and audio taped class discussions and approximately 1 hour problem solving interviews with seven students. The purpose of these interviews was to determine students' individual ways of reasoning about the content explored in class. These students were distributed among two small groups of four students each. The small group interactions of these seven focus students were also video and audio recorded.

To analyze the discourse, I used a modified version of Herbel-Eisenmann and Otten's (2011) method for thematic analysis (Lemke, 1990; Herbal-Eisenmann, 2011), a systemic functional linguistics (SFL) approach (Halliday, 1978; Halliday & Hasan, 1985). Central to this method is the assumption that words derive their meaning from their relationships to other words used in the discourse. To determine this meaning researchers examine the semantic relationships between words expressed in classroom discourse. For example, if a student said, "500 is at the midpoint," They are expressing a relationship between "500" and "midpoint." In particular, they express a located/location relationship. This helps determine the meaning of both 500 and midpoint, namely that 500 is something to be located and midpoint is a location.

I used this method to examine moments in the classroom where subdivision of an exponential number line was discussed to develop networks of semantic relationships between lexical items, words or phrases that came up repeatedly in the discourse. I developed a network for each method of subdivision based on arguments given in class as well as networks based on canonical arguments, those that are representative of how an expert might argue. Comparing the various networks revealed subtleties in the discourse and the meanings of various words.

I then analyzed discourse where students reflected on and talked about the methods of subdivision themselves. In these instances, students would explicitly refer to a particular method of subdivision as a method of subdivision. This contrasts with the other episodes of discourse where students were simply using a particular method. This means that in these instances of discourse,

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students were referring to a whole network of semantic relationships as a single lexical item. In SFL, this is called condensation (Lemke, 1990). In the episodes where students reflected on methods of subdivision, they tended to express semantic relationships in a different way than when they used the methods. In these episodes they tended to use equivalence and contrast strategies (see Lemke, 1990, p. 226) to show whether they thought two strategies were the same or different. This occurred during four episodes, though only two will be examined in this report. In the first episode, they contrasted linear and exponential methods using the discursive device of *parallel environments*. This means the speakers placed lexical items so that they have the same function in the grammar of two phrases. For example, a student might say, "in a *linear method* you do abc, but in an *exponential method* you do xyz". Here, the two methods are serving the same function in the grammar, in that they are both methods whose steps are being described. Furthermore, the methods are being contrasted and positioned as distinct, as one does something different in each instance. In the second episode, students explicitly said the two methods of subdivision were the same.

Results

Analysis of the four episodes in which students explicitly talked about the methods of subdivision themselves provided evidence for the following result: *When speaking, students distinguished between linear and exponential ways of reasoning, but did not distinguish between reasoning linearly with the exponents and multiplicative ways of reasoning.* In fact, students seemed to think of both of these methods as the same. In these instances, students may have referenced other lexical items, but these will be ignored as this analysis focused on references to the methods themselves.

Background to Episode 1

Students developed an exponential number line over the first three days of a six-day unit. On Day 1, they were asked to create a timeline that represented the history of the earth. Several approaches emerged on Day 1, but by Day 2 the teacher had encouraged them to focus on and develop a particular approach in which the timeline had a macro exponential structure, meaning powers of ten were equally spaced. However, to place events they subdivided the space between powers of ten linearly. Eventually, over Days 2 and 3, this method of subdivision was rejected and the two methods that were ultimately accepted were developed. The four episodes in which students in class reflected on the methods of subdivision occurred over these two days.

The first episode occurred near the end of Day 2. Students had already placed two dates on the timeline, the Renaissance and the Ordovician Periods, using linear reasoning to subdivide the space between two powers of ten. Presumably to problematize this way of reasoning, the teacher asked the students to place the Renaissance again, but using 1 and 1,000 and endpoints instead of 10^2 and 10^3. As part of her question, she specifically asked if 500 would end up in the same place. This led to the realization that using the different endpoints resulted in a different placement for the Renaissance. The students then considered the idea that the method they were using to subdivide was problematic. As they reflected on their method they contrasted how they subdivided the segments with the macro exponential structure, using the discursive device *parallel environments*. This contrast helps support the main claim in this paper, that over several discussions in class linear and exponential strategies were contrasted in the discourse, while the two exponential strategies for subdivision were not.

Discussing the Problem of the Renaissance Moving

The first instance of using *parallel environments* to contrast the linear relationships with exponential relationships came up as Nathan described why their linear method might be problematic:

> Yeah, so ultimately, the issue is, it seems like we're trying to apply a method that's completely linear in nature, when our graph [timeline] is not, it's exponential. That is, that's the problem, so that means that right there, the solution will not work, because why would it?

In this quote, Nathan referenced the idea that they were using a "method that's completely linear in nature" and a "graph… [that's] exponential." While Nathan argued that there was a mismatch between the nature of the method and the nature of the graph, Danna provided even more detail as to what the problem might be. She argued that a linear placement would not work, highlighting that it inaccurately predicts the placement of the known point 10^3.

> I started with first doing basically what we did up here [the linear way of reasoning]. So, we looked at the difference between 10^4 and 10^2, which was 9,900 years. And half of that [points to halfway between 10^4 and 10^2] should have been the 48 or should be…4,950, but we know that it's actually 10^3, so that right there told me linear doesn't work and it pushes the halfway mark closer to the 10^4 side. So applying the 500 to this one, I knew it was going to be closer to 10^3, just `cause, 500 it's halfway if it's linear, but when it's exponential, you know it's not, based on this [points to the number line that she used to argue "linear doesn't work"]. Then, I realized you can do it this way.

Again, Danna used the discursive device *parallel environments* to contrast linear and exponential ways of reasoning. She began using the linear method to predict the placement 10^3, a year whose placement was already determined from the macro pattern. She then extrapolated, "500 years, it's halfway if it's linear, but when it's exponential, you know it's not." Here the linear and exponential relationships have a similar grammatical function, in that she basically said, if it's linear, then 500 is halfway, but if it's exponential, then 500 is not halfway. She did this again when she said, "We were trying to look at it linearly in between each chunk, but the entire timeline is exponential." Here, she contrasted linear and exponential ways of reasoning by saying that in each chunk the structure was linear, but the structure of the whole timeline was exponential.

Background for Episode 2

Talk about methods of subdivision continued on Day 3. The day began with a student, Lacey, introducing the first way of subdividing that eventually became normative. She determined that the halfway point between 10^2 and 10^3 should be $10^{2.5}$, using the subdivision method of *Reasoning Linearly with the Exponents*. While discussing this task, Nathan justified her placement by introducing the second way of subdividing that eventually became normative, *Preserving the Multiplicative Relationship within the Segments*:

> Well, the way I did this one was I was looking at it where, in the more general sense, each tick was, …each thing apart on the bigger one is the same distance… multiplicatively apart, so we're going to do the same thing here. We have two so, we have two sections that when multiplied all together are 10. So, each side we'd had better have the square root of ten, because that's the only thing that's gonna give us 10 when we multiply it again, so I…just took the, I just figured it was, the distance away was 10^2 and then times the $\sqrt{10}$, which is 3.162. So I got 3.162 times 10^2.

Here, Nathan argued that by extending the multiplicative pattern that existed at the macro level, one can find the halfway point to be 100 times the square root of 10, or $10^{2.5}$, as Lacey had said. Presumably because the teacher noticed this was a distinct way of reasoning, she asked the students to talk about it in small group, specifically asking them to explain where they see the square root of ten coming up. However, during their small group discussions, the students said that Nathan's method and Lacey's method were the same.

Reactions to Nathan's Ideas.

In both small groups that included focus students, they failed to see the difference between Nathan's and Lacey's ways of reasoning about the subdivisions. In the first group, instead of engaging with the ideas of factors and multiplication, one student, Tanya, started the discussion of square roots by talking about the exponents. She said, "Well, the exponent one half is the square root right? ... So if it's 10^2, multiplied by $10^{1/2}$, right? So it's 100 times the $\sqrt{10}$." Here we see Tanya following the teacher's prompt to attend to the square root, however Tanya is arriving at the square root in a much different way than Nathan did. Instead of continuing the multiplicative patterns that existed at the macro level, she is arriving at the square root via a previously known rule that $10^{.5}$ is $\sqrt{10}$. This allowed her to still preserve her linear ways of reasoning about the exponents, while at the same time explaining where the square root is coming from as the teacher asked. This made it so she did not have to distinguish between the two ways of reasoning. This analysis is consistent with her groupmates' comments. Kathy said, "[Nathan's way of reasoning is] the same thing, because if you're doing 10^2 times 10 to the square root that's the same thing as .5." Rachel concurred saying, "He just thought of it as square root instead of…one half." Kathy summarized by saying, "Yeah, it's the same thing, he just wrote it differently."

In the second group, the students also continued to focus on exponents. However, instead of engaging with Nathan's idea, they explicitly said they did not understand it and ignored it. Farah said, "Well, I don't understand what [Nathan] said, but this is how…I thought of it." She then continued with her own idea.

In these small group interactions, the students explicitly said Nathan's and Lacey's way of reasoning were equivalent. Even though the teacher prompted them to attend to the square root, an idea that was central to Nathan's idea and absent from Lacey's, the students treated this as simply a notational difference. Tanya began by asking "the exponent one half is the square root right?" Rachel echoed this connection when she said, "He just thought of it as square root instead of…one half." This interpretation may have allowed them see Nathan's idea as simply another expression of Lacey's ideas rather than a new idea worthy of examination.

Summary

Analysis of the discourse in the four episodes where students reflected on methods for subdivision provide evidence for the claim: *When speaking, students distinguished between linear and exponential ways of reasoning, but did not distinguish between reasoning linearly with the exponents and multiplicative ways of reasoning*. In the first episode, when students discussed the problem of the point representing the Renaissance moving, they distinguished between linear and exponential ways of reasoning. They talked about subdividing the segments as a linear process while the macro structure was exponential in nature. This contrast between linear and exponential came up again in the third episode when students contrasted halving the values on the line with halving the values of the exponents. While this contrast is important, it does not help to disambiguate between the two exponential ways of reasoning. Furthermore, when the students talked about Nathan's multiplicative way of reasoning in small groups, they referred to it as the same as Lacey's method, which relied on linear patterns in the exponents. Thus, it is possible that students participating in the class discussion could see the two exponential methods as the same, which leaves little intellectual encouragement for students who can reason successful by focusing on the exponents to adopt multiplicative ways of reasoning.

Discussion

This study focused on how the nature of the classroom discourse can help explain how students could participate in a classroom where multiplicative ways of reasoning were developed and

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accepted by the class community, but not adopt those ways of reasoning as individuals. Through discourse analysis I discovered that exponential and linear ways of reasoning were contrasted, but reasoning multiplicatively and reasoning linearly with the exponents were not. This may mean that students thought there were primarily two ways of reasoning, a wrong way and a right way—linear reasoning and exponential reasoning. Thus when students heard multiplicative explanations, they may have thought that what they were hearing was no different from reasoning linearly with the exponents, since both were exponential.

It is reasonable to think that strongly contrasting linear and exponential methods in the classroom discourse is a desirable outcome, since previous research on exponential reasoning suggests that making the transition from linear to exponential reasoning is difficult (Berezovski, 2004; Liang & Wood, 2005). However, tackling this issue in this classroom seemed to background the subtler difference that exists between the two methods for subdividing exponentially. Being able to distinguish between the two methods is key to developing conceptual understanding of the relationships among numbers on the exponential number line. To develop this understanding, students need to coordinate the two methods of reasoning linearly with the exponents and continuing the macro multiplicative pattern. If students think of these as the same, then they can simply reason linearly with the exponents to get the right answer, without thinking exponentially at all. This means that, ironically, since the only contrast between methods of subdivision that existed in the discourse were between exponential and linear methods, students were able to participate in classroom discourse about exponential subdivision while reasoning only linearly.

This work raises the question of how to encourage students to see the differences between two ways of reasoning, especially when both ways of reasoning yield the same answer, so that they can then explore their relationships. In this paper, I have argued that the two exponential ways of reasoning were not explicitly contrasted, which may have contributed to students seeing them as essentially the same strategy. As such, an implication for teachers of this specific unit is to consider asking students to explicitly name and contrast the two exponential strategies. This would make it problematic for students who thought of the two methods as the same to continue to participate in the discourse. This may encourage them to disambiguate between the two methods, positioning them well to explore the relationships between them.

More generally, teachers teaching any unit should think about various ways of reasoning that may arise in the class and if they should be named and contrasted. However, it should be noted that determining which methods should be contrasted can be difficult to predict. While the research presented here underscores the point that the two exponential methods of subdividing a number line should be contrasted, this was not obvious before instruction. While the teacher recognized the complexity of transitioning from linear to exponential ways of reasoning and appropriately orchestrated the discussion to contrast those two ways of reasoning, she seemed to underestimate the difficulty students would have with disambiguating and coordinating the two exponential strategies. This highlights that to some extent, familiarity with and competence in executing general discourse moves, such as those involved in orchestrating discussion in such a way that strategies are contrasted, only goes so far in teaching and even highly effective teachers need support garnered through research that illuminates the conceptual difficulties of particular topics and gives insights into how to teach those topics. This suggests that teacher education focused on discourse should be paired with professional development focused on understanding the cognitive difficulties students face as they learn specific topics.

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