PROBLEM DRIFT: TEACHING CURRICULUM WITH(IN) A WORLD OF EMERGING SIGNIFICANCE

Nat Banting
University of Alberta
nbanting@ualberta.ca

Elaine Simmt
University of Alberta
esimmt@ualberta.ca

In this paper we frame our observations in enactivism, specifically problem posing, to propose the notion of problem drift as a method to analyze the curriculum generating actions of small group learning systems in relation to teacher interventions intended to trigger specific content goals. Teacher attentiveness to problem drift is suggested to be valuable in advancing the content goals in small group work.

Keywords: Problem Solving, Instructional Activities and Practices, Cognition

Small group problem solving has become a stalwart for classroom teachers attempting to occasion vibrant communities of mathematical communication and reasoning. Placing students in problem solving groups creates simultaneous short-range interactions, and results in a classroom ecology dense with opportunity for the teacher to curate productive mathematical insight. However, observing teaching and learning from an enactivist stance means a teacher cannot assume that the group will interact with the content goals of a lesson along a predictable pathway. Instead, cognition is a process of continually posing problems relevant in the moment (Varela, Thompson, & Rosch, 1991). A teacher may assign a task with specific content goals in mind, but, through their action with the task, groups bring forth multiple worlds of significance (Kieren & Simmt, 2009), each of which the teacher is then required to assess for relation to the original curriculum goals. After all, the ultimate responsibility of teaching any program of study is to deliberately impact learners. This leaves the teacher with the job of monitoring how a small group encounters targeted content outcomes within mathematically rich spaces during their course of interactivity as well as coupling with that interactivity with the intention of triggering interaction with targeted content outcomes.

Here, we suggest that the observation of the problem around which the group organizes their interactivity (the problem that is posed by the learners as relevant to addressing the task as currently understood by the group) can inform a teacher's intentional attempts to impact a group's mathematical action (Proulx & Simmt, 2016). This problem posing activity of a group signals the character of their knowing, of their world of mathematical significance. The ongoing re-posing of this relevant problem is termed *problem drift* (Banting, 2017), and can be thought of as a way of observing the emerging character of a group's curricular attention. We add to Proulx and Simmt's work by analyzing problem drift and its relation to the targeted content outcomes of a lesson. This frames our observations of the world of significance brought forth by a learning system (learners, teachers, and environment inclusive) in specific relation to the intended curricular outcomes. Doing so results in a pragmatic stance with(in) the learning system that includes more than just the interactions of the group surrounding content goals, but also the patterns of action in relation to the teacher interventions offered with content goals in mind.

In this paper we explore the notion of problem drift as a method for analyzing the dynamic bringing forth of meaning among members of small groups when given a mathematical task. We explore how problem drift can provide a focus for teachers (and researchers) whose goal it is to understand the nature of mathematical meaning and how they might be purposeful in their influence of it.

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Establishing Problem Drift

For the enactivist, knowing is doing (Maturana & Varela, 1987); the process of knowing and its products are one in the same thing (Pirie & Kieren, 1994). The mathematical knowledge of a group (its knowing) is not a source through which the mathematical action (its doing) is resourced or initiated. Rather, knowing/doing emerges through interaction between a subject and their environment (Proulx & Simmt, 2016). Reciprocally, through these interactions from which knowing emerges, the environment is continually shaped (hence co-emerges), and, in return, triggers further possibilities for action. It is through this process that meaning is brought forth. Therefore, knowing emerges out of context. The coordination of learner and environment is not fully in the agent nor the environment, but emerging from the interaction between the two, constituting an emerging world of significance (Kieren & Simmt, 2009); this is the image of knowing and learning with(in) an environment. The learners do not interpret their context in multiple different ways, which would imply that the environment remains static as the learner constructs an impression of its character. Rather, through the mutual triggering of environment and agent, the learners' knowing—that is to say, doing—brings forth distinct worlds of significance. It is through this lens that content outcomes are recast in active terms. In other words, claiming that a specific skill (like "creating equivalent fractions") is *known* means that it must emerge as relevant; it must be enacted.

The mutual specification of problem (environment) and problem solver (learner) means that "we do not choose or take problems as if they were lying out there, independent of our actions, but we bring them forth" (Proulx, 2013). Action of a learner is not dictated or prescribed by the environment, but a learner's structure allows certain features of the environment to become problematic, curious, or interesting. These worlds are maintained through the ability of the learner, in this case, the learning group, to pose problems relevant to its needs at that moment (Varela et al., 1991), where that relevance is contextually and structurally determined (Maturana & Varela, 1987) and the action triggers a furtherance in the posing. In other words, for the enactivist, problems are not given (by the teacher); problems are posed (by the learner). It is through the posing of the problem deemed relevant in the moment that the problem environment and the learning action of the problem solver co-dependently arise. This evolution of the problem posing constitutes the problem drift of the learning system. Because problems are not ready-made, determining what problem has been posed as relevant provides the context in which the group is acting—what meaning they have brought forth. Problem drift details the relevant problem posed by a learning system in order to analyze its knowing action, a sort of trace of mathematical knowing.

It seems that we arrive at a fundamental tension between the responsibility (for the teacher) to provide problems pertaining to a specific set of outcomes as delineated by a curriculum document and the enactivist notion of problem posing—the recognition that learners enact the nature of the task by entering into interaction with it. It problematizes the role of problems to prescribe content outcomes. Rather, teachers design prompts in anticipation that the structure of the task will trigger action that is observed to be mathematical. Teachers then become fully complicit in bringing forth the world(s) of significance by participating in the meaning making (Proulx, 2010). In other words, the teacher does not stand aside and perturb the world of significance of the learners, the teacher participates in the becoming as a fully coupled agent. Conceptualizing knowing in this fashion co-implicates the teacher in the generation of significance. In this sense, the enactivist notion of problem posing does not consider the process of learning as helter-skelter and unbridled where the teacher has little-to-no influence. Such an image would be unapologetic to the project of schooling. Rather, enactivist cognition heightens the role of teacher as one who participates fully in the action—provoking, triggering, orienting, and influencing the learning system. The task of teaching becomes the tethering of the emergent problem posing the group undertakes to bring forth a world of significance to the anticipated content goals built into the task.

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Problem drift sits at the crossroads between the enactment of a problem and the mathematical products required by a curriculum; it provides a method to analyze the curriculum a group brings forth. It allows an observer snapshots in the action that can be analyzed for the mathematical processes that were used to address the relevant issues that emerged through their interaction with the task. In short, problem drift allows us to observe the curricular outcomes emerge to address the problem(s) posed as relevant (Banting, 2017).

Methodology

The teacher participants for this study were recruited through previous professional relationships. Using a design research approach (Prediger, Gravemeijer, & Confrey, 2015) with specific attention on the tenants of enactivist methodology (Reid & Mgombelo, 2015), the daily instruction in two classrooms (belonging to two different teachers) was designed around small group tasks completed by students working in randomly-created groups of three. During the study, video data from the workstations of three randomly-chosen small groups was recorded on five separate occasions for a total of of fifteen accounts of group problem solving.

Three adults—a researcher, a classroom teacher, and a pre-service teacher partnered with the classroom teacher for the semester—took on the role of teacher during the classroom episode under analysis here. After each classroom session, a debriefing was audio recorded that included all three teachers. In it, they discussed what they observed in regards to the character of the groups, the interventions they offered, and the reasons behind their choices.

Portions of the action of Brock, Ria, and Sharla (pseudonyms used) is detailed as they worked together on the Tile Design task in their grade nine mathematics course which contained twenty-seven students, or nine groups of three. The Tile Design task asked students to create a series of shapes with colored square tiles to satisfy requirements given to them by a series of stage cards. The task was designed to provide occasions for students to work with two content outcomes:

- Creating equivalent fractions
- Comparing and reasoning about fractions in a part-whole model

Results

In what follows, we detail the action of Brock, Ria, and Sharla (along with the teachers) with interlacing dialogue, artefacts of their doing (images depicting the tile arrangements on their workspace), and description of teacher interventions with the intention of triggering interaction with the content outcomes of the task. For the sake of brevity, only a portion of their action (divided into three episodes) is provided.

In the first episode, the group was required to create a shape where one twentieth was yellow, one quarter was green, one half was blue, and the remaining was red. Dialogue begins after the group had arrived at an initial arrangement (Figure 1).

Teacher: Is that half blue?

After a quick glance of their arrangement, the group was unanimous that the shape was not half blue, and a student removed two blue tiles to leave the arrangement in Figure 2.

B	B	B	B	B	Y	G	G
B	B	B	B	B	G	G	G

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Figure 1. Initial arrangement.

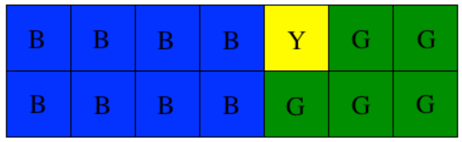

Figure 2. Altered arrangement.

Brock: This is only 14!
Ria: We need 20 tiles.
Sharla: Because the common denominator is 20?
Teacher: What were you going to say, Ria?
Ria: You need 20 tiles because the denominator, the common denominator is 20.
Brock: Yeah. It doesn't equal 20.
…
Ria: You add other blues.
Sharla: If you add, then you have to add in more. But then you have to add in another colour. Add another colour.
Brock: Wait, how many green do we have?
Sharla: We need to add in more green then if you added more blues.
Ria: Which means we have to add more yellow because it has to be equal.
…
Emma: Well, if blue is supposed to be one half, and green is supposed to be one-quarter, then green has to be half the size of blue because one half can be split up into two quarters and if this is one quarter it should be half the size of blue.

Shortly thereafter, a second episode was prompted when the group was provided with a new stage that required them to create a shape that was at least one half red, at least one quarter green, and no more than one sixth yellow.

Brock: At least means it has to at least be a half, right? Okay, so let's make it a common.
Ria: Let's just say this is half.
Brock: Let's do 12. Want to do 12?
Ria: Yeah, sure.
Brock: Okay. So half of 12 is 6. So we have to at least have 6 red.
Ria: Can I make the shape this time?
Brock: No more than one sixth is yellow. 2. And then at least one fourth is green.
Sharla: So that would be.
Ria: 3.
Brock: It has to be 3. Umm. Put one more yellow. It has to be like this. Make sure that's good.
Ria: So if its. We're using 12, then there can only be 2 yellow, right?
Brock: What?
Ria: We're using 12 and there can only be 2 yellow.
Brock: Yeah, but, at least. Oh yeah. K yeah, so that works.

The arrangement resulting from the second episode appears in Figure 3. The new stage did not trigger the group action away from the insistence of using a specific number of tiles, that equal to the lowest common denominator of the stage's requirements. In an attempt to trigger further action, a

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teacher removed a single green tile from the group's arrangement (Figure 4) prompting a third episode.

G	G	R	R	R	Y
G	G	R	R	R	Y

Figure 3. Initial arrangement.

	G	R	R	R	Y
G	G	R	R	R	Y

Figure 4. Teacher-altered arrangement.

Teacher: What if I did this? Does that still work?
Sharla: No, because he.
Ria: No, because this is less than a quarter now.
Sharla: No, it's not.
Brock: No, it's a quarter.
Ria: No, but it's still not a quarter of this.
Brock: But it doesn't equal 12.
Ria: Exactly, so you need that.

Analysis

The group's action was analyzed using the notion of problem drift. That is, we, as observers, interpreted the relevant problem that was posed by the group as they worked through the stages of the Tile Design task. (For a more detailed explanation regarding the method of analysis, see Banting, 2017.) During the action detailed here, three relevant problems were determined to emerge and compose the group's problem drift. In other words, we deciphered these episodes to each contain a unique problem posed as the group transformed—brought forth—mathematical meaning. The group's action in the episodes detailed above was organized around the following pathway of problem drift:

- How can we meet the requirements by using 20 tiles?
- How does the *at least* requirement affect our 12-tile solution?
- Can we satisfy the requirements without using 12 tiles?

We now turn our attention to the important connection for the teaching of mathematics that emerged when the action of the group was analyzed by characterizing its problem drift. Analyzing problem drift allowed us to observe the group's mathematical doing in relation to the teacher actions as they coupled with the learning system. Specifically, in the third episode detailed here, problem drift provides an explanation as to why a teacher intervention, while attempting to trigger mathematical action around a specific content goal, failed to do so.

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Problem Drift Informs Content Goals

The problem drift reveals the group to be operating in a productive space with regards to the intended curricular outcomes. That is, the creation of equivalent fractions and the comparison and reasoning about the sections of a part-whole model have emerged as processes to address the problems posed as relevant in their context. We see this knowing/doing in two regards. First, the group consistently computes targeted common denominators. Specifically, they identify the lowest common multiple of the denominators in the stage's requirements, and execute the algorithm for creating equivalent fractions. This is how the group arrives at the 16-tile arrangement in Figure 1. The group converted one half, one quarter, and one twentieth into ten twentieths, five twentieths, and one twentieth respectively, only to lose focus on the size of the whole and assume that ten blue, five green, and one yellow would constitute the desired solution. We see it again in the 12-tile arrangement in Figure 3. These actions show that, when it is deemed to address a relevant problem, the group can execute this process.

Second, problem drift allows us to analyze the knowing at a deeper level by interpreting what problem caused the creation of common denominators to become relevant. The group calculation of equivalent fractions emerges while acting with the problem, "How can we meet the requirements by using 20 tiles?" The algorithm to create equivalent fractions is brought forth through a need to establish how many tiles they should use to create their arrangement. Before they begin to reason about the size of each part, they need to first establish the size of the whole.

Through the analysis of problem drift, we interpret that the group knows that equivalent fractions can be used to establish a whole because their doing surrounds the establishment of that whole. That is, the group understands the problem as one where a specific size of the whole (first twentieths and then twelfths) needs to be established. Creating equivalent fractions accomplishes that goal. The problem drift allows us to observe the curriculum outcome in the context from which it emerged as relevant (in active terms) and not simply as a skill that can be executed. It provides context as to why the group feels equivalent fractions are suitable; we begin to see the place of equivalent fractions in their emerging world of significance.

In the second episode the group was left with eleven of twelve tiles assigned a definite color after computing common denominators. They then began to address the remaining tile's worth of "empty space" by posing, "How does the *at least* requirement affect our 12-tile solution?" Throughout their action, there is no wavering in their understanding that they must work from a well-established whole. They reason about the possibilities of adding the different colors (attempting yellow, rejecting yellow, and eventually settling on the addition of a green) within the frame of the posed problem. In this sense, their reasoning about the size of fractions emerges within the context of filling in empty space left by the firm establishment of the 12-tile whole. The problem drift allows us to observe that the group knows that there are certain restraints in adding colors to an arrangement with a fixed whole. Throughout the episode here, analyzing the problem drift of the group allowed us to observe their dynamic mathematical doing tethered directly to content outcomes. Not only do we observe the direct execution of mathematical processes, but we also observe the world of mathematical significance in which these processes emerged as relevant.

Problem Drift Informs Teacher Action

The problem drift of the group in reaction to the teacher's decision to trigger action by removing one of the green tiles reveals the world of significance brought forth by the group. In the post-session interview, the teacher explained that they observed the group creating equivalent fractions, but their action never required them to compare the size of two fractions. With an eye to this content goal, the teacher hoped that removing a single green tile would trigger the group to compare the new fractions (now with a denominator of eleven) to the requirements of the stage. Having witnessed the group

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create equivalent fractions previously (creating equivalent fractions was a part of their structure), they assumed the intervention would result in a furtherance of this skill.

The third episode details how the group interacts with the intervention. The group not only fails to reason about the size of fractions using common denominators, they dismiss the new arrangement as impossible and the intervention as borderline nonsensical. Analyzing the group action through their problem drift reveals a possibility as to why. While the teacher assumed that the group would be triggered into further creation of equivalent fractions, they failed to take into account the context from which the creating of equivalent fractions initially emerged as relevant for the group. The group knows common denominators as an important process for establishing the whole; they do not know them as a process to compare unfamiliar fractions. This knowledge of equivalent fractions was never a part of their world of significance. In their understanding, equivalent fractions are a process to create the whole, not to compare sizes of constituent parts. That is why an intervention attempting to trigger action around such a concept was foreign. The teacher treated the creation of equivalent fractions as a skill held by the group, and did not interpret their action as brought forth with(in) a specific context. Problem drift allows us to re-conceptualize the reaction of the group to the trigger. They did not ignore the invitation because they were unable to execute the creation of a common denominator. As detailed previously, they do exactly this several times throughout the episodes. They ignore the invitation because the trigger was foreign to the world of significance they had brought forth.

For them, in their world of significance, the problem was one of establishing a whole and then assigning the parts to meet the requirements. This is evident through their problem drift. It is the symbolic equivalent to determining a fixed denominator and then adjusting the numerators of each section until the requirements are met. By removing a tile, the intervention asked the group to alter the established whole—to adjust the denominator instead of the numerator. Their reaction does not indicate a lack of skill, but rather that the anticipated interaction was not relevant to the problem they posed. The teacher's anticipated vision was incompatible with the group's world of significance. In other words, analyzing problem drift reveals the group's preoccupation with establishing the whole, and why an intervention that suggests they do otherwise did not sponsor the desired action surrounding content outcomes. This does not suggest the intervention was an example of bad teaching (Towers & Proulx, 2013); it simply did not coordinate with their enactment of the problem—it was irrelevant.

Discussion

Problem drift is a method for analyzing the dynamic bringing forth of meaning. It provides a focus for teachers (and researchers) whose goal it is to understand the nature of mathematical meaning and how they might be purposeful in their influence of it. It focuses the search for emergent knowing on the nature of a learner's doing.

The teaching intervention of removing a green tile did not have the desired curricular effect, but that is not to dismiss it as having no effect. Through their intervention, the teacher acting with(in) the world of significance provided a possibility by suggesting that shapes of different sizes could meet all the requirements of the stage. The structure of the group, and their world of significance evidenced through their problem drift, did not allow that possibility to become problematic, and it was treated as foreign or nonsensical. In order to trigger the desired content outcomes, the possibility of different sizes of shapes would need to first become relevant to their structure before becoming enacted into the group's world of significance. Understanding this provides an avenue for the teacher to attempt further triggering.

We do not suggest that understanding problem drift will make the group response to a teacher intervention *predictable*. Instead, we suggest that identifying the problem that has focused action—

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that is, become relevant—makes teacher interventions *accessible*. Problem drift speaks to what the group knows because it analyzes what they have done—what problems they have posed as relevant and their relation to the intended curricular outcomes. It is a way of viewing emerging knowing/doing and assessing whether the desired content outcomes have become a part of that action. It is a way of attuning to the group's structure and situating teacher inter-actions therein. Here, we propose that attuning to problem drift is a critical piece to inform teacher interventions designed to advance the content goals of a lesson. It switches the orienting question of teaching from "*How did they solve the problem?*" to "*What problem are they solving?*" The former assumes the problem was engineered to meet certain curricular requirements, while the latter sits at the crossroads between emerging meaning and curricular mandates—that which has become relevant in the moment.

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