TEACHERS' INTERPRETATIONS OF STUDENT STATEMENTS ABOUT SLOPE

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This paper describes seven in-service teachers' interpretations of student statements about slope. The teachers interpreted sample student work, conjectured about student contributions, assessed the students' understanding, and positioned the students' statements in the mathematics curriculum. The teachers' responses provide insight into their Knowledge of Content and Students (KCS) and Knowledge of Content and Curriculum (KCC). Results suggest these teachers value academic terminology related to slope, have limited perspectives on slope in real world contexts, and struggle to describe the extension of slope to precalculus.

Keywords: Algebra and Algebraic Thinking, Mathematical Knowledge for Teaching, Teacher Beliefs, Teacher Knowledge

Introduction

Ball, Thames, and Phelps (2008) introduced the Mathematical Knowledge for Teaching (MKT) Model based on Shulman's (1986) work as a means to consider the multifaceted knowledge that teachers need for their craft. MKT consists of two different types of knowledge: pedagogical content knowledge and subject matter knowledge. Pedagogical content knowledge has been outlined in terms of three domains: Knowledge of Content and Curriculum (KCC), Knowledge of Content and Students (KCS), and Knowledge of Content and Teaching (KCT). In this study, we focus on KCS, how students learn mathematics, and on KCC, where the mathematical topics students are learning fit in the curriculum. Subject matter knowledge also has been portioned into three domains. We will focus on Horizon Content Knowledge (HCK), which refers to understanding future mathematical topics and how the math at hand provides a foundation for those topics. We investigate these areas of MKT as related to the concept of slope, a key topic in the middle school mathematics curriculum upon which advanced mathematical (Moore-Russo, Connor, & Rugg, 2011) and statistical (Casey & Nagle, 2016; Nagle, Casey, & Moore-Russo, 2017) ideas are built. In addition to coverage across the curriculum, the multitude of ways in which students can reason about slope make it well suited for this study.

Slope Network

Nagle and Moore-Russo (2013a) proposed a network of five slope components, each with visual and non-visual as well as procedural and conceptual subcomponents (see Table 1).

The slope network outlines the multi-faceted nature of slope, but research has not described how these subcomponents may interrelate and be leveraged to help students develop a connected understanding of slope. In this study, we consider teachers' interpretations of student statements related to the various slope components (KCS) and their accounts for how they fit together across the secondary mathematics curriculum (KCC). Using this lens, we consider teachers' perspectives on the relative sequencing of these components and gain insight into their valuation of the components. In particular, we investigate the following research questions:

1. How do teachers interpret common student statements about slope? What notions of slope do teachers value in student thinking?

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2. What are teachers' perceptions of how the notion of slope is developed across the secondary curriculum?

Table 1: Slope Network (adapted from Nagle & Moore-Russo, 2013a)

Slope Component	Description	Subcomponents (shown as subscripts) v = visual, n = nonvisual, p = procedural, c = conceptual	
Ratio	Slope viewed as a ratio; extends to explain why linear behavior results in a constant ratio.	$R_{v,p}$: rise/run or vertical change over horizontal change	$R_{v,c}$: similarity of slope triangles yields a constant ratio of rise/run regardless of the position on the graph
		$R_{n,p}$: change in y over change in x; (y_2-y_1/x_2-x_1)	$R_{n,c}$: constant rate of change between two covarying quantities; equivalence class of ratios thus a function
Behavior Indicator	Relates slope to the increasing or decreasing behavior of a linear function or graph; links sign of the quantity m with the function's or graph's behavior.	$B_{v,p}$: increasing (or decreasing) lines have positive (or negative) slope	$B_{v,c}$: positive (or negative) rise corresponds to positive (or negative) run for an increasing (or decreasing) line, yielding a positive slope
		$B_{n,p}$: value of m in the equation for a linear function indicates whether f is an increasing (m>0) or decreasing (m<0) function.	$B_{n,c}$: If function f is increasing then $f(x_1)<f(x_2)$ for every $x_1<x_2$, so $[f(x_2)-f(x_1)]/(x_2-x_1)>0$; similar generalization for decreasing function
Trig. Conception	Describes slope in terms of the angle of inclination of a line with a horizontal; extends to relate steepness to the tangent of the angle of inclination.	$T_{v,p}$: steepness of a line; slope as the angle of inclination of the line with a horizontal	$T_{v,c}$: the angle of inclination determines the rise/run; a steeper line has a greater rise per given run than a less steep line
		$T_{n,p}$: slope is calculated as $\tan\theta$ where θ is the angle of inclination	$T_{n,c}$: the angle of inclination determines the ratio of (y_2-y_1/x_2-x_1), which is equivalent to $\tan\theta$.
Determining Property	Property that determines if lines are parallel or perpendicular; property can determine a line if a point on the line is also given.	$D_{v,p}$: parallel (perpendicular) lines have the same (negative reciprocal) slope; slope and point determine unique line	$D_{v,c}$: parallel lines have the same vertical change for a set horizontal change; may be seen in terms of congruent slope triangles
		$D_{n,p}$: y_2-y_1/x_2-x_1 is equal for parallel lines and results in negative reciprocals for perpendicular lines; slope and a point determine a unique linear equation	$D_{n,c}$: parallel lines have equivalent differences in y values for a set difference in x values, yielding equivalent slope ratios
Calculus Conception	Limit; derivative; a measure of instantaneous rate of change for any (even nonlinear) function; tangent line to a curve at a point	$C_{v,p}$: slope of a curve at a point is the slope of the tangent line to the curve at a given point	$C_{v,c}$: visual interpretation of secant lines approaching tangent line
		$C_{n,p}$: derivative f' is used to calculate slope of function f at a particular point	$C_{n,c}$: $f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$ as the average rate of change over increasingly small intervals

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Methodology

Participants

Participants included seven secondary mathematics teachers who elected to participate in a funded, year-long professional development cohort focused on promoting conceptual understanding. Of the seven participants, two had fewer than 5 years teaching experience, four teachers had between 5 and 10 years, and one had over 10 years of experience. All teachers reported experience teaching introductory algebra, including the concept of slope.

The Tasks

Prior to the first professional development meeting, each teacher submitted responses to a series of tasks related to slope PCK. The task analyzed in this study is provided below. All teachers in the cohort were emailed the task and given three weeks to complete it. The student statements below were generated by the researchers as typical responses noted in past research.

> *As an instructor, you asked each of the students in your class to make a statement about slope. For each student response* [in Table 2]*, please answer all of the following.*
>
> *a. Provide a visual representation (a graph, an equation, etc.) that you would expect each student could easily have created to accompany her statement about slope.*
>
> *b. If each student had been asked to contribute a problem to a study sheet on slope, provide an example of a problem that each would have been most likely to submit.*
>
> *c. Using the scale* [in Table 3]*, rate (and justify) each student's understanding of slope.*
>
> *d. By which level of schooling would you expect each student's response? Explain.*

Table 2: Student Statements Regarding Slope and Associated Slope Components

Slope Component	Student Statements Given to Teachers
Ratio	A: Slope is rise divided by run of a graph. B: Slope is found by taking the change in y values divided by the change in x values. D: Slope tells the rate of change between two variables, x and y. K: The slope of a line is constant regardless of which two points on the graph are chosen to calculate the value.
Behavior Indicator	J: Slope indicates if a line is increasing, decreasing, or constant.
Determining Property	I: Slope can be used to determine if lines are parallel or perpendicular.
Trigonometric	C: Slope describes the steepness of a line. F: Slope is related to a line's angle of inclination with respect to a horizontal line.
Calculus	G: The derivative function tells the slope of a function at a particular time.
Open – No specific component intended	E: Slope is represented by m in equations and formulas. H: Slope can be used in real world situations. L: Slope refers to the straightness of a line; the fact a line doesn't curve.

Table 3: Scale for Rating Each Student's Understanding of Slope

	1 - Strictly Procedural	*2- Procedural with Limited Conceptual*	*3- Emerging Conceptual*	*4- Robust Conceptual Understanding*
Description	*Demonstrates a strictly procedural focus on how to calculate slope through rote manipulation without any interpretation of the meaning of the concept*	*Demonstrates a primarily procedural focus on how to calculate slope with very limited attention to interpreting the meaning of the concept*	*Demonstrates an understanding of the meaning of slope in a particular situation or context*	*Demonstrates a flexible, deep understanding of slope that allows for understanding in multiple situations or contexts*

Data Analysis

Using the slope network (from Table 1), two researchers coded the teachers' responses to parts *a* and *b* of the task for the slope components and the visual or non-visual subcomponent evidenced. A number of responses did not provide enough detail to code the conceptual versus procedural subcomponents, so this coding was omitted. The researchers also recorded each teacher's rating of student understanding and recorded the level of schooling at which the teacher expected such a response. The schooling responses were categorized into PreAlgebra, AlgebraI/II, Geometry/Trig/Precalculus, and Calculus categories. The researchers completed all coding independently before meeting to compare codes. When discrepancies were found, a third researcher was brought in to discuss the coding until a consensus was reached. When all the data had been coded, all three researchers looked for trends within and across teachers' responses.

Results and Discussion

The teachers' responses to the student statements are summarized in Table 4. For each student statement (A-L), the first column indicates the slope component(s) and subcomponent(s) illustrated in the teachers' responses to parts *a* and *b* of the task. The data were combined for these parts of the task. Thus, only one slope component is recorded when the teacher used the same component for both the representation and example. When two slope components are listed, that means that the teacher included both slope components in both parts of the tasks or that the teacher included one component in part *a* and the other in part *b*.

Table 4: Teacher Responses to Students' Statements about Slope

Tchr	**Hypothetical Student Statements**																							
	A		**B**		**C**		**D**		**E**		**F**		**G**		**H**		**I**		**J**		**K**		**L**	
1	R_v	1 P	R_n	1 P	T_v	2 P	R_v	2 P	B_n	1 P	T_v	4 G	C_n	4 C	R_v	3 P	D_n	4 A	R_v	1 P	R_n	4 P	D_v R_n	2 A
2	R_n R_v	1 P	R_n R_v	2 P	T_v	2 P	R_v	3 A	R_n	1 P	T_v	3 ^	C_n	3 ^	R_n R_v	3 vr	D_n D_v	2 A	B_v	2 P	R_v	3 A	R_v	2 P
3	R_v	1 P	R_n	2 P	T_v	3 P	R_v	4 -	^	1 P	T_n T_v	3 G	C_n C_v	4 C	^	1 P	D_n D_v	2 A	B_n	3 P	R_v*	2 P	-	3 C
4	B_v R_v	1 P	R_n	1 P	T_v	1 P	R_n	1 P	R_n	1 A	T_v	3 A	C_n	1 C	^	1 A	D_n D_v	3 A	B_v	1 P	R_v	2 P	^	1 -
5	R_v	1 P	R_n	1 A	T_v	3 P	B_v* R_n	3 A	^	1 P	Tv	3 A	C_n	4 C	R_n	3 P	D_n	2 A	B_n	2 P	R_v	1 -	-	- -
6	R_v	1 P	R_n	2 A	T_v	1 P	R_n	3 A	^	1 P	R_v	3 A	C_n	1 C	R_n	3 A	D_n	2 A	B_n	1 P	R_n*	3 A	R_v	1 P
7	R_v	1 P	R_n	2 P	T_v	3 P	R_n	4 P	R_n	1 P	^	3 ^	Cn	? C	^	? G	D_n D_v	2 A	B_v	2 P	R_v	2 P	^	? A

Key: P=PreAlgebra, A=AlgI/II, G=Geometry/Trigonometry/Precalculus, C=Calculus, vr=varies

In a few instances, the researchers thought the response showed strong promise of indicating the conceptual subcomponent according to the slope network. In those cases, an asterisk is marked in the table. The second column reports teachers' responses to part *c* regarding the procedural versus

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conceptual rating (on the 1 to 4 scale). We distinguish between responses that did not align with any slope component (^), were left blank (-), or acknowledged uncertainty of how to interpret the given statement (?). Part *d* responses are below those to part *c* in Table 2. Consider the row 1 and column A intersection in the table. It reveals Teacher 1 responded to student statement A by providing a representation and example problem aligned with the *Ratio (visual)* component of slope, rated the statement as *1 (strictly procedural)* and placed the statement in PreAlgebra.

In the following sections, we report on teacher responses to the specific student statements in light of the anticipated slope components (from Table 2).

Results for Ratio Components

Statements A, B, D (*Ratio* component). All seven teachers' responses to these statements included the *Ratio* component. Furthermore, all teachers included a visual interpretation of statement A and a non-visual interpretation of statement B, as expected. Interesting trends emerge across the various *Ratio* components. Although statements A, B, and D all express that slope is a ratio, statement A describes it visually, statement B does so non-visually, and statement D describes it as a rate of change. Despite the statements' similarity, teachers interpreted them quite differently. All teachers rated statement A as strictly procedural and at the PreAlgebra level. Five of the seven teachers rated statement B (that had received R_n codes) as either more advanced in grade level or more conceptual (or both) than statement A (that had received R_v codes). Teachers 1 and 4 rated both statements as strictly procedural and at the PreAlgebra level. For the rest of these teachers, the visual approach seemed to be de-valued, as was apparent in many teachers' written explanations. Teacher 2 wrote, "B understands the idea of the slope as the change in the values, instead of just rise over run," and Teacher 7 wrote, "B is using academic vocabulary that suggests that she has a basic understanding of slope." Furthermore, six teachers reported that statement D was more conceptual or more advanced (i.e., grade level) than both the other ratio statements. Several teachers related the "rate of change" language of statement D to using slope in real world situations. Teacher 2 explained, "D has a firm grasp on how slope is applied in real life scenarios," and Teacher 3 justified her rating of this statement as robust conceptual understanding by stating, "the student understands the concept and can relate it to everyday solutions." These teachers are equating the phrase "rate of change" with slope applied to real world situations and conceptual understandings of slope.

Statement K (*Ratio* component). Responses to this statement varied from strictly procedural to robust conceptual. Teachers tended to put it at the PreAlgebra or Algebra level. Teachers 3 and 6 provided sample problems that the researchers felt showed promise of relating to the conceptual *Ratio* subcomponent, with Teacher 3 doing so with a visual emphasis while Teacher 6 did so in a strictly non-visual manner.

Results for Other Components

Statements C, F (*Trigonometric* component). Teachers' responses to statement C were quite consistent. Despite the researchers' interpretation of this statement as being open to visual and non-visual sub-components, every teacher's response emphasized a visual interpretation. These often included graphs of several lines with varying slopes, indicating that the line got steeper as the absolute value of slope increased. Interestingly, four teachers provided a real world context comparing two or more roads or roofs and making reference to steepness. Despite the potential to link steepness to the slope in these contexts, none of these teachers did so in a meaningful way that involved reference to the angle of inclination nor described steepness in terms of a ratio. All seven teachers saw statement C as a PreAlgebra interpretation of slope, with some variation in whether it was more procedural or more conceptual. Statement F was seen as emerging or conceptually robust by all teachers, and was categorized at the Algebra I/II level or later. Most teachers' responses emphasized a visual interpretation. Teachers' 6 and 7 responses suggested their inabilities to interpret

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this statement or place it in the curriculum.

Statement J (*Behavior Indicator* component). All teachers saw this as a PreAlgebra interpretation of slope, and six of the seven teachers' responses illustrated the *Behavior Indicator* component split equally between visual and non-visual interpretations. Visual interpretations tended to show graphs of increasing, decreasing, and horizontal lines with positive, negative, and zero slopes labeled accordingly. Non-visual representations tended to give the equation of a linear relationship and described the relationship in terms of the parameter *m* in the equation. Teacher 1 provided a graph of a line and asked whether it was increasing, decreasing, or constant but never linked this with slope. Thus, this response could not be linked with any slope component. Teacher 1's sample problem presented the graph of a horizontal line and asked what the graph represents. The research team interpreted this as asking for the equation of the line—which would not require use of the *Behavior Indicator* component. All but one teacher saw this statement as more procedural than conceptual (1 and 2 ratings).

Statement I (*Determining Property* component). Teachers' responses consistently evidenced the intended component. Non-visual representations generally presented two linear equations and asked whether the lines were parallel, perpendicular, or neither. Four teachers incorporated both visual and non-visual representations in their responses. Responses incorporating both representations included equations and graphs of the lines—showing how the relationship between the slopes was displayed graphically via lines that never intersected, intersected in right angles, or intersected in some other way. There was very little variation in the example problems and representations presented. All seven teachers agreed this notion of slope would appear in Algebra I/II, and most teachers rated this as a 2 (mostly procedural understanding), with one 3 and one 4 rating. Overall, the teachers were in agreement with where this fits in the curriculum.

Statement G (*Calculus* component). It is interesting that with this open statement, only one teacher linked this to a visual representation of a function's graph with tangent lines drawn at various points. Most teachers included $f'(x)$ notation and provided an example involving finding the derivative of a polynomial. Six of the seven teachers unsurprisingly placed this conception as occurring in Calculus. There was, however, great variation in whether teachers viewed this as procedural or conceptual in nature. Two teachers rated this as strictly procedural and three teachers rated it as robust conceptual understanding, highlighting a very distinct mismatch. Teacher 7 indicated that she was not sure how to rate this problem.

Statement E (*open* - no component). Three teachers' responses to statement E did not link any understanding of slope to the statement. Each gave a problem or representation that provided an equation in slope-intercept form and then labeled *m* in the equation as the slope with no indication of what *m* meant for the equation or its graph. Three of the remaining teachers linked this statement with R_n, acknowledging *m* in the equation $y=mx+b$ and writing $m=(y_2-y_1)/(x_2-x_1)$. It is interesting that these teachers viewed these algebraic representations as related, especially since none showed how one formula could be manipulated to achieve the other. All teachers viewed this understanding as strictly procedural, and all but one placed it in PreAlgebra.

Statement H (*open* - no component). The researchers expected this statement to elicit a variety of slope components in teachers' responses, but the teachers' responses were relatively uniform. Four of the teachers linked this statement to the *Ratio* component of slope, with two teachers focusing on non-visual aspects, one on visual aspects, and one on both. The link with the *Ratio* component was made via an equation or graph labeled with real world variables and a description of the slope in terms of the problem context. The final three teachers provided responses that could not be coded as indicating any slope understanding. For instance, Teacher 3 sketched a picture of a car driving up what appeared to be a hill with no indication of how slope was demonstrated. The others acknowledged that the statement itself did not indicate much about the student's understanding.

Teacher 4 wrote: "H does not show much with this statement. Sure it can be used in real world situations but how? If she knows how, then we are getting somewhere." Thus, this code does not mean that this teacher misinterpreted this student's understanding, but acknowledged the lack of clarity in the statement itself. In terms of responses, the most interesting result may be the absence of the *Trigonometric* component. One of the fundamental uses of slope in real world situations is to consider steepness of physical objects (e.g., ramps). In the one instance where such a connection was hinted at, the connection stopped short of showing how slope was demonstrated. The ratings and grade levels for this statement varied greatly. Interestingly, the three teachers who did not attach this statement to any particular conception of slope ranked it as strictly procedural. The remaining four teachers, who had interpreted this statement as being linked with the *Ratio* component, all rated the statement as mostly conceptual. For those teachers who linked this to a *Ratio* component, they seemed to value the use of *Ratio* in a real world context as indicating a more robust understanding of slope.

Statement L (*open* - no component). Statement L proved to be surprisingly difficult for teachers to interpret. Only three teachers provided codable responses, with two stating that they did not understand L's statement and the remaining two providing vague responses that couldn't be coded (e.g., a graph of a line and the graph of a curve with no mention or indication of slope on the graph). Of the three who did provide codable responses, two interpreted it using visual aspects by providing the graph of a line and describing in words or denoting on the graph that every time "you move right one unit on the graph, the corresponding vertical change on the graph is constant." This was accompanied by statements such as "therefore the function will be a line." Teacher 1 linked this statement with the *Determining Property* by asking how many lines can be drawn through a given point with a specified slope. She also asked whether three points lie on the same line, linking to R_n. Teachers' responses regarding grade-level and knowledge rankings varied greatly, adding to evidence of their overall uncertainty about this statement.

Implications

The results reveal important insight into the teachers' PCK in terms of their KCC and KCS. In particular, teachers' responses revealed (1) their valuation of academic language, (2) the nature of real world problems for slope and (3) their views of slope beyond the algebra curriculum.

Value of Academic Language

The responses to statements A, B, and D suggest that teachers value student use of academic terminology. Although attending to precision and using correct mathematical terminology is a key part of the mathematics curriculum (NGA & CCSSO, 2010), these results raise a red flag that teachers may equate academic terminology with conceptual understanding. Teachers' responses to statements A and B suggest that the teachers may devalue visual thinking by equating it with non-mathematical terminology. Likewise, responses to statement D suggest teachers valued the academic language of "rate of change" even though that expression could be used as a mnemonic just as "rise over run" often is. Together, these results highlight two important aspects of teachers' KCS: (1) distinguishing between students' use of terminology and their understanding of the terminology and (2) encouraging students to connect multiple representations to integrate academic terminology with visual reasoning.

Rate of Change and Real World Situations

Teachers' responses also revealed some interesting trends related to the role of real world situations in students' learning about slope. The real world situations provided by teachers either demonstrated the *Ratio* component within the context of a functional situation (e.g., time worked versus dollars earned) or the *Trigonometric* component within the context of physical situations (e.g.,

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steepness of roof). Furthermore, when physical situations were mentioned, they were done so trivially without explicit attention to how slope was related to steepness. These results suggest teachers may miss valuable opportunities to help students connect the *Ratio* and *Trigonometric* components of slope through real world situations (KCS). As a result, their students may fail to connect the ideas of slope and steepness (Nagle & Moore-Russo, 2013b).

Role of Slope in Advanced Mathematics

The results raise questions about how the teachers view slope as informing students' work with non-linear functions. Nagle and Moore-Russo (2014) describe the CCSSM's high school focus on extending the notion of a constant rate of change of linear functions to interpret and understand non-linear functional relationships. Recall that teachers generally were not sure how to interpret statement L that "slope refers to the straightness factor of line," a statement that links naturally to CCSSM's focus on moving from linear to non-linear relationships by understanding variable rate of change. Furthermore, other than the *Calculus* component of slope, the teachers tended to provide algebraic interpretations of slope, even when statements were open to more trigonometric or geometric interpretations. Even teachers who do not teach beyond the Algebra I/II curriculum, should have sufficient knowledge of the curriculum (KCC) and how slope is foundational to more advanced concepts, such as the derivative (HCK), to include more advanced interpretations of slope.

Future Work

By analyzing teachers' interpretations of student statements, we have described the teachers' apparent values related to student thinking about slope. We have not investigated how these values are carried out through teachers' intended or enacted instruction on slope. Future work should investigate to what extent the tendencies described for teachers do or do not play out in their intended and enacted lessons on slope. Doing so will allow for confirmation of these valuations and for exploration of the manner and extent to which teachers' valuations of understanding inform their written and enacted lessons (KCT).

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