TOWARDS A HYPOTHETICAL LEARNING TRAJECTORY FOR QUESTIONING

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We report on efforts to better understand the questioning practices used by preservice elementary teachers (PSTs), including the range of preferred question types and the values they invoke when evaluating their questions. We sought to determine whether teachers exhibited consistent patterns in selecting questions with certain features, such as funneling students to a particular strategy or eliciting student thinking, across different instructional situations. We found that such patterns did exist; in this paper we use the patterns to propose a trajectory for developing the skill of asking questions that elicit and build on student thinking. The trajectory describes the beliefs, values, and questioning practices associated with PSTs at each stage.

Keywords: Learning Trajectories (or Progressions), Teacher Education-Preservice

Introduction

Asking questions is central to the work of teaching. Yet research (e.g., Franke et al., 2009) has demonstrated that some questions are more likely than others to provide opportunities for students to make their thinking explicit. More research is needed to better understand how teachers might improve the types of questions they ask in the classroom. This study was conducted to learn more about ways to support preservice teachers (PSTs) in developing the skill of asking questions that elicit or build on students' thinking.

Theoretical Framework and Literature Review

We view knowledge for teaching as situated in the context of teaching (Borko et al., 2000), which means that it should be developed through experiences that approximate, to some extent, the practice of teaching (Grossman, Hammerness, & McDonald, 2009). Approximations of practice provide opportunities to learn through the decomposition of teaching into components, which can then be studied and practiced (Baldinger, Selling, & Virmani, 2016). The learning experiences described in this paper use cartoon representations of teaching, developed using the online program Lesson*Sketch*, that provide opportunities for PSTs to choose from specific pedagogical actions (that is, questions), see their (pre-established) impact, and then reflect on those choices (Herbst, Chazan, Chen, Chieu, & Weiss, 2011).

Although this project focuses on developing knowledge about teaching through action and reflection (Ball & Forzani, 2009), we also acknowledge the role of beliefs and values in influencing teachers' practice. In particular, values, which involve teachers' views about what is *important*, have particularly strong impact on teachers' decisions (Bishop, 2012). Efforts to influence the teaching practice of novices must acknowledge and contend with the incoming values of PSTs. In this paper, we explore some interactions between PSTs questioning practices and their stated values with regard to questioning in mathematics teaching.

Features of Questions

The National Council of Teachers of Mathematics' Principles to Actions (2014) advocates teacher questions that "build on, but do not take over or funnel, student thinking," and those that "make mathematical thinking visible" (p. 41). Other productive questioning practices include pressing for mathematical justifications, asking students to make explicit connections between different strategies, and probing errors (Kazemi & Stipek, 2001). These features are in contrast to

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questions that invalidate students' thinking or impose a way of thinking onto the students. For example, funneling questions are a sequence of closed questions intended to direct students through a series of procedural steps until they obtain the correct answer (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). These questions reduce students' opportunities to build on their own understanding because the teacher ends up doing much of the cognitive work and the student merely answers with the expected response (Franke et al., 2009). Although these categories are helpful, more research is needed to articulate how novices improve their questioning practice and what their learning might look like as they transition from asking less productive to more productive questions.

Hypothetical Learning Trajectories

Hypothetical learning trajectories (HLTs) are constructed to represent a possible progression of student learning in a format that is useful for teachers and curriculum designers (Empson, 2011). While some researchers draw solely from existing literature to develop their HLT, others also use insights developed from the analysis of data collected in the first of two research phases (e.g., Meletiou-Mavrotheris & Paparistodemou, 2015). Our study aligns with the latter approach to developing HLTs. Once the HLT is developed, researchers then conduct multiple iterations of their experiment in order to refine their trajectory until it closely mirrors participants' actual progressions of learning (Cobb, Confrey, DiSessa, Lehrer, & Schauble, 2003). Complete HLTs consist of three main elements: "the learning goal, developmental progressions of thinking and learning, and sequence of instructional tasks." (Clements & Sarama, 2004, p. 84). In this study, we examined the patterns in features of questions selected by PSTs in response to student thinking across multiple Lesson*Sketch* experiences and sought to characterize these patterns in terms of a hypothetical learning trajectory. We will focus primarily on the first two elements of the HLT in this paper.

Methods

Participants were 86 elementary preservice teachers (PSTs) in their second of two method courses at a university in the Midwestern United States. Data consisted of PSTs' typed responses to prompts within five online Lesson*Sketch* experiences, two of which we classified as the pre and posttest. In this paper, we will focus on the last three experiences; namely, the Brandon and Cedric experiences and the posttest. In the pre and posttest, PSTs were initially presented with a mathematical task and one simulated student's solution to the provided task. PSTs composed and gave a rationale for a question they would like to ask the student (e.g., Brandon) and then selected all of the questions from a provided list they thought would be good questions to ask the student. In the remaining three Lesson*Sketch* experiences, PSTs were again presented with a mathematical task and one student's solution to the task; this time, they went through two rounds of selecting a question and seeing the student's (pre-determined) response, evaluating the question after each round. At the end of the experience, PSTs decided which of their two selected questions they thought was more effective and explained why. See Table 1 for examples of the questions used in the different experiences, classified according to their question feature. We developed the categories of question types drawing on the literature on effective questions described earlier.

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Table 1: Classifications of Question Types

Category	Examples
Suggest a specific alternate strategy (not eliciting)	• (Brandon experience) "Do you know what you need to do to the denominators before you can add fractions?" • (posttest) "Why don't you try dividing each sub into three parts?"
Specific to student's work, but invalidates and funnels	• (Cedric experience) "If it is 4 SQUARE yards, can you just multiply by 3?" • (posttest) "When you are comparing fractions, don't you need to use the same whole?"
Funnels	• (Brandon experience) "In the problem it says that there are three fourths and three sixths. Are fourths the same as sixths?" • (Cedric experience) "Are we talking about one-dimensional units or two-dimensions?" • (posttest) "Is it 1/3 of a sub, or 1/3 of half of a sub?"
Elicits student's thinking	• (Brandon experience) "Can you tell me more about where the fourths, the sixths and the tenths are in your picture?" • (Cedric experience) "Can you show me the square yards in your picture?" • (posttest) "Can you tell me more about how you were thinking about the 1/3?"
Help students build on their own thinking	• (posttest) "Let's look at your pictures for Car A and Car C. Based on the picture, who would get the most?"

Analysis

We began constructing the hypothetical learning trajectory by describing the learning goal, drawing on features of effective questions cited in literature, and hypothesizing which features of questions might be more difficult for students to adopt based on our findings from the Phase 1 data. Next, we looked for patterns among the types of questions PSTs selected and composed in response to student thinking on the posttest and then examined their responses in the earlier experiences to determine whether these groups of PSTs were more likely to select questions with similar features (e.g., funneling or eliciting) in the earlier experiences. As the patterns arose, we identified ways to categorize the different groups of PSTs and adjusted our learning trajectory as needed. During this process, we hypothesized what values and beliefs about teaching might be motivating different types of questions. For example, we conjectured that PSTs would be more likely to ask questions that reference students' work if they valued understanding students' current thinking. In order to gain insight into the PSTs' values, we analyzed their evaluations of selected questions using open codes, which we later condensed into the categories shown below:

- **Building on student thinking:** PST claims that the question provided an opportunity for Brandon to come to a new realization on his own
- **Understanding student thinking:** PST claims that the question helped the teacher to better understand Brandon's thinking or allowed the student to explain his thinking
- **Addressing misconceptions:** PST claims that the question helped the student understand, focused on a misconception, or failed to "fix" a misconception
- **Leading to correct answer:** PST claims that the question helped get the student to the correct procedure or answer

Finally, we investigated the links between PSTs' values (what PSTs believe is important) and practices by examining the relationships between the criteria they used to evaluate questions (i.e., the value codes just described) and the types of questions they tended to prefer (Table 1).

Findings

Patterns of Features of Questions PSTs Selected

After examining the questions PSTs composed and selected on the posttest, we formed groups of participants according to specific features of these questions. We initially classified PSTs as "funnelers" if they a) selected both of the funneling questions **or** b) composed a funneling question. We used similar criteria to create an initial "eliciters" group. These criteria yielded 51 funnelers and 54 eliciters, including 29 PSTs who were listed in both groups. We reclassified these 29 PSTs as "funnelers-eliciters." This resulted in three distinct groups: 22 funnelers, 29 funnelers-eliciters, and 25 eliciters.

A chi squared test of independence showed that in neither the Brandon experience (X^2 (2) = 7.08, p = .029) nor the Cedric experience (X^2 (2) = 5.95, p = .051) were the question types independent of the group. Overall, PSTs tended to select a question that funneled or directed the student in the Brandon experience and a question that elicited or built on student thinking in the Cedric experience (see Figure 1). However, an examination of the standardized residuals revealed that in the Brandon experience, *fewer* funnelers (std. res. = -1.92) and *more* eliciters (std. res. = 1.1) selected the question that elicited or built on student thinking than statistically expected. In the Cedric experience, the inverse was true: namely, *more* funnelers (std. res. = 1.45) and *fewer* eliciters (std. res. = -1.56) selected a question that funneled or directed the student than statistically expected. Figure 1 shows the percentage of PSTs within each group who selected the eliciting and funneling questions in the Brandon and Cedric experiences. Notice that the percent of funneling questions decreases and the percent of eliciting questions increases between each group in both experiences.

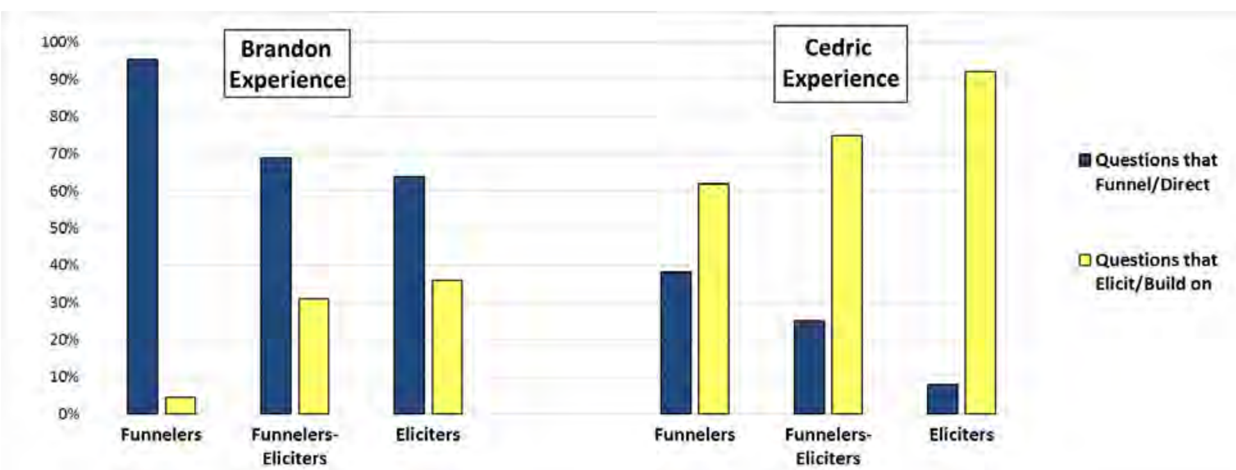

Figure 1. Percent of PSTs in each group who selected either a question that funneled/directed students or elicited/built on student thinking in the Brandon and Cedric experiences.

Criteria for Evaluating Selected Questions

Recall that after viewing the simulated student's responses to two questions they selected, PSTs indicated which question they preferred and why, which we analyzed in order to characterize the PSTs' values underlying their question selection. Over three-fourths of the PSTs in the funneler category gave justifications that focused on whether or not their question resolved Brandon's

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misconception, compared to less than half of the funneler-eliciters or eliciters (see Figure 2). One PST in the funneler group stated that she preferred the funnel question, "because it made [Brandon] realize that the pieces were not able to be added because they were not the same. He is not realizing it with [the eliciting question], he just keeps labeling his picture and justifying his original answer." Notice that the PST is evaluating both the funneling and eliciting questions based on whether or not they resolved Brandon's misconception. Additionally, her negative evaluation of the eliciting question suggests that the PST does not realize that Brandon could have discovered the error on his own in the process of justifying his original answer.

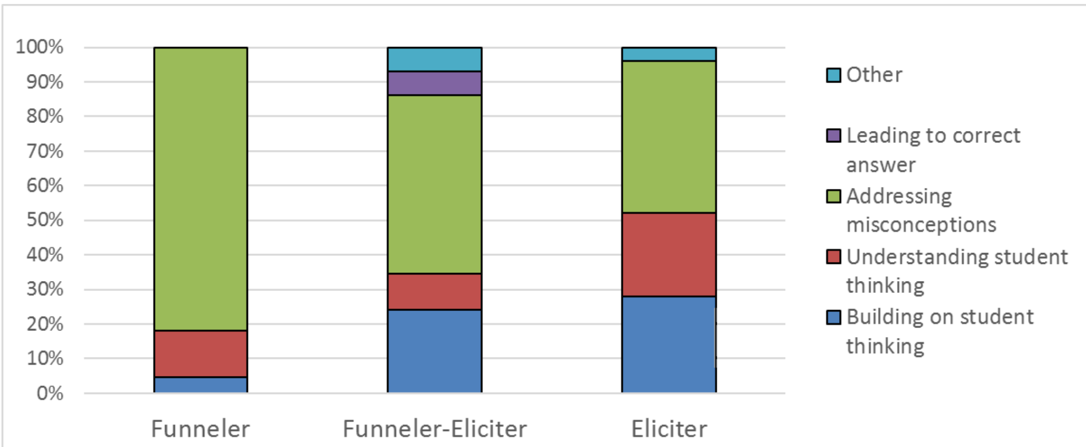

Figure 2. Percent of PSTs who focused on each criterion when evaluating questions in the Brandon experience.

PSTs in the eliciters group were more likely to evaluate their questions based on whether it allowed them to understand or build on the student's thinking. For example, one eliciter stated that she liked the eliciting question "because we actually get the chance to observe Brandon's thinking and strategies. He is able to explain his thought process for us. The other [funneling] question was more of the teacher telling Brandon what is right and what is wrong." Here, the PST appears to recognize the value in understanding Brandon's current thinking before seeking to move his thinking forward. Overall, patterns in the types of evaluations given suggest that not only were PSTs in different groups more likely to select different types of questions, but they also valued different things when asking Brandon a question. We looked for similar patterns in the Cedric experience, but in this case, the differences were not statistically significant. Despite this, the consistency individuals exhibited in the questions they selected and their evaluations in Brandon's experience lend support to our framework in the next section.

Hypothetical Learning Trajectory

Our trajectory is comprised of three layers – PSTs' beliefs and knowledge about mathematical understanding, their values about students' learning, and the features of questions they pose when asking a student about their mathematical work. We propose four main stages that PSTs progress through in the development of asking effective questions. In our descriptions of the stages below, we begin by talking about the features of questions PSTs in the given stage might prefer and then draw connections to the associated values and beliefs/knowledge.

We separated the proposed trajectory for PSTs' beliefs from values and practice in order to emphasize the distinctions between the components of the trajectory that were based in our data (values and practice) and the components were not explicitly measured, but were emphasized in the methods course and may be connected to the value/practice constructs. The dashed arrows represent the tentative nature of these proposed connections. Although our trajectory depicts four distinct stages, we acknowledge the potential overlap between categories and that some PSTs may not develop understanding in the linear path implied by the figure. Nonetheless, the trajectory serves as a model for understanding a general progression of PSTs' understanding.

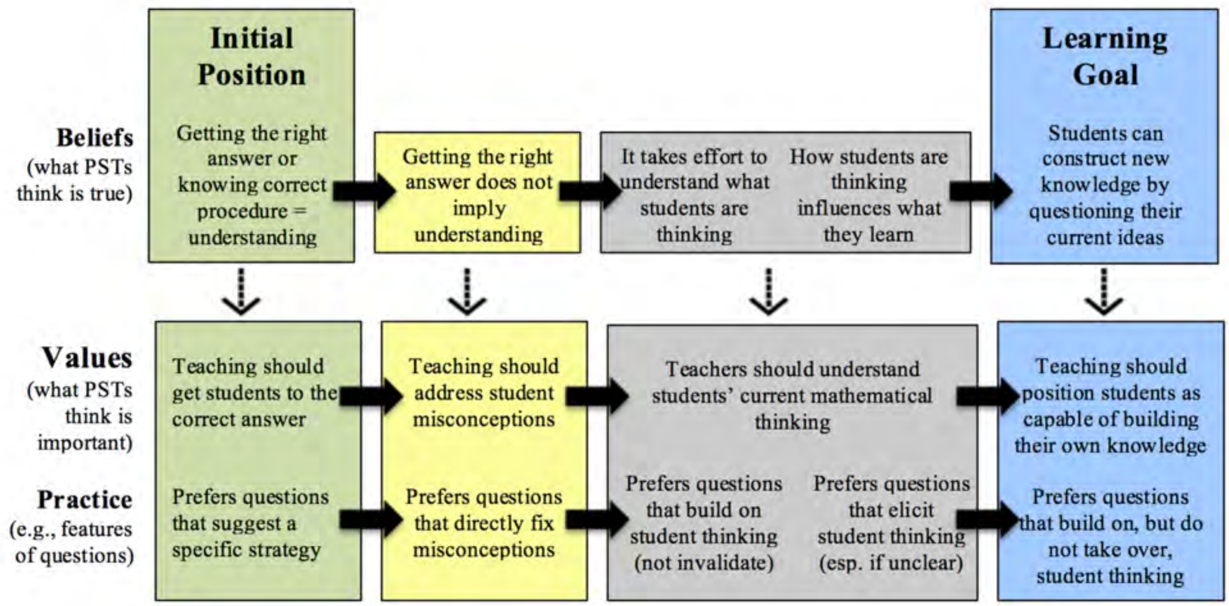

Figure 3. Hypothetical learning trajectory for asking effective questions.

Leading to correct answer (initial position). Based on literature and our prior experiences in methods classes, we began with the assumption that many PSTs enter into undergraduate programs with an unsophisticated view of teaching as telling, and default to using questions as a vehicle for directing students towards specific strategies. For example, one PST explained in the pretest why she thought the question, "why don't you try dividing each sub into 3 parts", was good by saying, "it might make more sense to the student if he divides each piece into thirds and can add them up more easily". Here, the PST projected her own strategy onto the student and assumed that her strategy might make it easier for the student to solve the problem than his current strategy. This reflection lends support to the idea that PSTs who ask questions designed to get students to the correct answer may equate answer-getting with understanding and may have a broad, vague idea of what they think is important for student learning.

Addressing misconceptions (yellow). At this stage, PSTs recognize the importance of asking questions that are specific to the student's work and begin preferring questions that directly confront the student's misconception. Funneling questions, such as "In the problem it says that there are three fourths and three sixths. Are fourths the same as sixths?" fit within this category as the question focuses the student's attention on their misconception without directly telling the student what to do. Of the 49 PSTs who initially selected this question, 34 preferred this question to the eliciting question they viewed subsequently. Nearly all (32/34) of the PSTs who preferred their initial funneling question gave justifications focused on whether they believed the question resolved Brandon's misconception. "My first question was better because he realized that fourths and sixths were not the same. The [eliciting question] just led him to point out where things are in his picture without realizing he was wrong." This response shows a PST who only considers whether the question helped Brandon “realize he was wrong” and failed to consider whether the question revealed more about Brandon’s thinking or positioned Brandon as capable of discovering his own error.

Eliciting student thinking (gray). PSTs in the third stage value understanding students’ current thinking in addition to helping them develop correct conceptual thinking. As a result, they tend to select questions that elicit the student’s thinking about a specific aspect of their work. For example, the question in the Brandon experience, “Can you tell me more about where the fourths, the sixths, and the tenths are in your picture?” focuses the student’s attention on how his answer of tenths relates to his drawing of fourths and sixths. This question also elicits his current understanding instead of directly pointing out his error. At the end of the Brandon experience, 19 of the 22 PSTs who preferred this eliciting question provided justifications that highlighted a desire to understand Brandon’s thinking or allow him to come to his own understanding. For example, one PST liked the eliciting question because “we actually get the chance to observe Brandon's thinking and strategies. He is able to explain his thought process for us.” As PSTs begin to value understanding students’ current mathematical thinking, we hypothesize that they begin to realize the effort involved in understanding student thinking and recognize that the student’s thinking influences what he/she learns.

Building on student thinking (learning goal). The final box represents the learning goal, where teachers pose questions that aim to build on, but not take over, the student’s thinking. Such questions often *do* elicit student thinking, but the description implies that there are additional ways to use questions to help students move forward in their thinking without reducing the cognitive demand or taking over the mathematical work. This corresponds to a value for teaching that not only draws out student thinking, but positions students as capable of developing, questioning, and refining their own ideas. For example, in the posttest, the simulated student Toby determined that sharing two sandwiches with three people equally would result in each person getting 5/6 of a sandwich. The question, “Can you show me the 5/6 of a sub that each person will get?”, asks Toby to pictorially represent his solution without indicating that his answer was incorrect. In doing so, Toby would have an opportunity to see that three shares of 5/6 of a sandwich would constitute more than two sandwiches—an unreasonable solution. Although 62 PSTs selected this question on the posttest, only 38 picked a similar question on the other posttest item. The PSTs who selected both of these questions did not exhibit clear patterns in the earlier experiences that suggested a consistent preference for questions that built on, but did not take over, student thinking. We interpret these findings to suggest that our sample did not include a sufficient number of PSTs who were at the final stage in the learning trajectory.

Discussion

Our data suggest that, across different instructional situations, PSTs show consistent patterns in the kinds of questions they select. PSTs who funneled in some situations tended to also funnel in others, and those who chose eliciting questions tended to be consistent in this choice as well. In addition, PSTs who selected both funneling and eliciting questions fell between the two groups in terms of preferences for questions that, on the one hand, take over student thinking, and, on the other, draw out and build on student thinking. These patterns suggest that PSTs were at different stages in their thinking about the questioning practice. We hypothesize that some PSTs (funnelers) prefer questions that lead students to correct answers and do not see value in questions that merely elicit student thinking. Indeed, several PSTs we placed at the beginning of the trajectory expressed dismay when questions seemingly left students still confused, even if those confusions were exactly the ones students needed to work through. At the next stage (funneler-eliciter), PSTs value questions that elicit students’ thinking, but they also continue to value questions that lead students to the correct answer or resolve their confusion. Finally, PSTs farther along the trajectory (eliciters) value questions that draw out and build on students thinking, and in this study were also less likely to select questions that imposed a teacher's idea. PSTs at this stage were more likely to criticize questions for being too leading and to value questions that prompted a student to figure something out "on his own."

Establishing these stages is an important first step to designing experiences that support movement along the trajectory. What does it take, for example, for PSTs who value resolving student confusion, to begin noticing and appreciating how questions can help them understand students' current mathematical thinking? Once PSTs value building on student thinking, what kinds of experiences might support them in moving towards a preference for asking eliciting questions? Answering these questions involve consideration of the questioning practices themselves, the PSTs' skills in enacting them, and their values in teaching. In our future work, we hope to further refine this trajectory and continue to develop interventions to help PSTs get closer to enacting the questioning practices in the ambitious goals endorsed by NCTM (2014).

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