Mathematics teacher education is faced with the challenge of preparing new teachers for ambitious instruction, but we have limited understandings of what happens within the courses where this preparation occurs. This paper draws on interview data from a larger investigation of novices’ enactment of ambitious instruction in elementary mathematics and language arts across six teacher preparation programs. Findings describe the application of the framework developed by Grossman et al. (2009) to opportunities to learn to teach in elementary mathematics methods courses and associated field experiences, focusing on the range of activities described and their relation to the framework.

Keywords: Teacher Education-Preservice

We frame our study of opportunities to learn in elementary mathematics methods courses using Grossman and colleagues’ (2009) framework that established three primary pedagogies for teaching relational practices, including teaching, to novices – representation, decomposition, and approximation. In this paper, we are focusing specifically on representation and approximation, both of which we see as also involving aspects of decomposition. Grossman and colleagues define representations as, “the different ways that practice is represented in professional education and what these various representations make visible to novices,” (Grossman et al., 2009, p. 2058). They noted that representations can vary not only in what they do and do not make visible for teacher candidates, but also in their “comprehensiveness and authenticity” (p. 2065). Approximations are defined by Grossman and colleagues as, “opportunities for novices to engage in practices that are more or less
proximal to the practices of a profession,” (p. 2058). They suggest that, similar to representations, approximations can vary along a number of dimensions, including the nature of the aspect of teaching practice being approximated, “how closely the activity approximates actual practice”, and “the role of the [teacher] educator” (p. 2079). This framework focuses attention not only on how novices learn to enact teaching practices, but also on how novices learn the knowledge and skills that underlie those practices.

More recent studies have taken up this framework to design and characterize pedagogical interventions in methods courses and to understand the relationship between these pedagogies and novices’ learning outcomes. For example, Amador and colleagues (2016) explored the differences in teacher candidate noticing of teacher practices in the context of a representation and an approximation of practice. Ghousseini and Herbst (2016) focused on the importance of sequences of representations and multiple approximations for teacher candidates’ opportunities to learn to lead classroom discussions. Here, rather than focusing on a specific activity or series of activities, we are investigating the range and sequences of activities across multiple teacher preparation programs, with a specific focus in this paper on those activities involving teacher educators’ enactment of representation and approximation pedagogies.

Methods

The findings reported here are part of a larger study of novices’ enactment of ambitious instructional practices in elementary mathematics and language arts. The larger study investigates (a) how a purposively sampled set of six teacher preparation programs in three states supports elementary teacher candidates to develop ambitious math and language arts instruction and (b) factors that are associated with how graduates of these programs enact math and language arts instruction as first- and second-year teachers. This investigation includes surveys of approximately 150 teacher candidates from the set of six teacher preparation programs during their final year of the program and their first two years of teaching. Additionally, we observe these graduates multiple times as they teach mathematics and language arts as first-and second-year teachers.

For this specific study, we focused on interviews with 12 elementary methods instructors and 9 program coordinators to better understand their perspectives on the opportunities to learn to teach provided in elementary mathematics methods courses and associated field experiences across the a subset of three of the six teacher preparation programs in the study. Each participant was interviewed once during 2015-2016 for approximately 45-60 minutes. Data was audio recorded and later transcribed. Interviews were semi-structured based on a protocol designed to solicit information about the backgrounds, instructional activities, and philosophies of method instructors and their respective programs. Questions asked included the following:

1. How would you characterize the overall approach to teaching that you seek to develop among the teacher candidates through the course?
2. What major instructional strategies do you want teacher candidates to learn and know how to enact? Why do you focus on these strategies?
3. How do you engage teacher candidates in learning these strategies? What kinds of activities do you use to help them learn about these strategies?

Analysis

Our analyses focused on making sense of the interview data from methods instructors and program coordinators. Our process involved iterative cycles of coding during which we both developed emergent codes from the data and built from theory. Specifically, we began by applying grounded theory techniques of initial coding (Saldana, 2015) to examine a broad sample of our interview data looking for common issues discussed across the group of methods instructors and


Articles published in the Proceedings are copyrighted by the authors.
program coordinators. Next, we refined our initial codes by looking for similarities and differences across the codes and comparing our emergent ideas to those categories present in existing theoretical frameworks (e.g., Darling-Hammond & Bransford, 2005; Grossman et al., 2009). Subsequently, we continued multiple rounds of this iterative process to clarify our codes. In particular, we sought out examples of interview excerpts that were not well captured by previous versions of codes in order to identify those features of opportunities to learn that our codes did not yet capture. Finally, we generated code definitions and selected representative examples.

Ultimately, our analyses resulted in a multi-leveled codebook that distinguishes: (a) what knowledge, practices, or content teacher candidates have opportunities to learn, (b) how those opportunities to learn are made available to teacher candidates, (c) who provides the opportunities to learn, (d) teacher preparation program capacity for opportunities to learn, (e) teacher preparation program structure, (e) program and teacher candidate evaluation, and (f) reasoning behind particular opportunities. Here we focus on a subset of the codes related to how opportunities to learn are made available to teacher candidates.

Findings
In the following sections we share initial findings from our early coding work focused on interviews with methods instructors and program coordinators. Broadly, we found the theoretical constructs of approximations and representations to be a useful starting point to interpret the opportunities teacher candidates have to learn during methods courses. Additionally, we identify a number of interesting dilemmas with regards to parsing the work of methods instructors into these discrete categories. Here we share our codes, summarized in Table 1, along with representative excerpts of interview data to illustrate these dilemmas and describe what we have learned.

Table 1: Codes for How Opportunities to Learn are Provided to Teacher Candidates

<table>
<thead>
<tr>
<th>Code Name</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Representations</td>
<td>Opportunities for teacher candidates to watch examples of the work of teaching</td>
</tr>
<tr>
<td>Approximations</td>
<td>Opportunities for teacher candidates to experience deliberate practice immersed in activities of actual teaching.</td>
</tr>
<tr>
<td>Do a math or literacy task</td>
<td>Opportunities for teacher candidates to engage with specific content knowledge through tasks</td>
</tr>
<tr>
<td>Learning to learn from teaching</td>
<td>Opportunities for teacher candidates to learn how to be reflective of their work as teachers and to use their teaching experiences as a means to grow as professionals</td>
</tr>
<tr>
<td>Reflection</td>
<td>Opportunities for teacher candidates to reflect about their teaching in writing or aloud</td>
</tr>
<tr>
<td>Formative feedback</td>
<td>Opportunities for teacher candidates to receive formal or informal feedback about their work</td>
</tr>
<tr>
<td>Problems of practice</td>
<td>Opportunities for teacher candidates to learn from challenges that arise during instruction.</td>
</tr>
<tr>
<td>Examining classroom artifacts</td>
<td>Opportunities to examine samples of student work or other classroom artifacts (generated or authentic) as a specific focal point for reflection.</td>
</tr>
<tr>
<td>Connect to other coursework/knowledge</td>
<td>Opportunities for teacher candidates to build on work from previous classes or to connect to ideas that will be the focus of courses later in the teacher preparation program</td>
</tr>
<tr>
<td>Field Experiences</td>
<td>Opportunities for teacher candidates to learn to teach through field experiences (e.g., practicum placements, student teaching)</td>
</tr>
</tbody>
</table>


Articles published in the Proceedings are copyrighted by the authors.
This paper explores only our findings related to the codes representations and approximations. In our efforts to understand how to organize and apply these two constructs, however, we identified a number of additional ways in which methods courses provide opportunities for teacher candidates to learn to teach. Thus, an important initial finding is that representations, approximations (and decomposition) as proposed by Grossman and colleagues (Grossman et al., 2009) are not sufficient to address all of the opportunities provided in methods courses. Although this paper focuses on applying and clarifying definitions of the constructs of representations and approximations, we note that there exists an important course of future research to explore the nuances of opportunities to learn to teach presented through our other codes in Table 1, especially learning to learn from teaching and field experiences.

Representations

Following Grossman et al. (2009), we define representations as opportunities for teacher candidates to observe examples of the work of teaching. A classic representation of teaching practice might be a video recording of a teacher teaching a lesson to a class of students. In our initial analyses we did find some discussion of instructors using videos of practicing teachers as representations of quality practice. Early coding revealed, however, that instructors more often discussed representations of practice other than those involving video, including teacher candidates observing their cooperating teachers in action and observing their methods instructors modeling particular instructional practices.

For example, the following excerpt, from an interview with a mathematics methods instructor, is representative of some of the issues we encountered related to identifying representations of teaching practice.

Instructor: Part of the modeling is that I model. If we’re going to do something, like for example, we have a giant number line that I made and they had to put decimals and fractions on it. And one of them facilitated it. But, I made the materials and then we said, how does this help to model?
Interviewer: Oh, to model a concept or something?
Instructor: Yes, model a concept… but modeling how you model a concept. [laughs]
Interviewer: Right
Instructor: And so that’s another thing that, in terms of conversation, a lot – I always have them sit in groups. And if there’s something that I think is important, I will either model it or I’ll set it up so someone else [a teacher candidate] can help to model it. […] So I try to set up experiences for them to experience things like modeling and then they talk about it. And I will facilitate their discussion or I will say to somebody else [a teacher candidate], could you please facilitate the discussion on… So, that they are doing as much as possible.

In this excerpt the methods instructor described her strategy of using her own instruction during the methods course to model the kinds of teaching activities and strategies she was presenting to the teacher candidates. That is, she used her own instruction as a representation of the kind of ambitious instruction that she would like her teaching candidates to learn.

This excerpt also highlights some of the complexity we found in determining the boundaries of representations in methods courses. It was not always clear when teacher candidates were engaging in opportunities to observe representation of practice as compared to other opportunities to learn, such as approximation of practice, or developing content knowledge. For example, in the excerpt above, the instructor described how she represented quality teaching practice while simultaneously engaging students with mathematical content (e.g., locating decimals and fractions on the number line) and providing opportunities for teaching candidates to try out pedagogical strategies (e.g.,

---


Articles published in the Proceedings are copyrighted by the authors.
facilitating discussion). This was typical across our data set, and thus a dilemma arose for us as to how to bound the idea of a representation of teaching practice. Could we consider any teaching the method instructor does (e.g., share PowerPoint lectures, ask thoughtful questions) a representation of teaching practice? Since not all interactions with teaching are necessarily supportive of learning to teach (e.g., Lortie 1975) nor intended to serve as representations of teaching, we limited considerations of representations to only those instances in which the interviewee explicitly discussed the pedagogical reasoning related to the representation. For example, to be considered a representation, a methods instructor would need to explicitly describe how their actions provided an opportunity for teacher candidates to learn about teaching practice. Additionally, in instances where teacher candidates had opportunities to observe the work of teaching in the field (e.g., practicum, student teaching) without explicit discussion of the pedagogical purpose, we coded those instances as opportunities to learn through field experience, not representations.

**Approximations**

Building from the work of Grossman and colleagues (2009), we define approximations as opportunities for candidates to experience deliberate practice (Ericsson, 2002) as they engage in activities of actual teaching practice. Through approximations, teacher candidates can experiment with their new teaching skills, knowledge, and ways of thinking. Unlike representations, which focus on observing practice, approximations necessarily involve teaching candidates in doing aspects of the work of teaching. As described by Grossman and colleagues (2009), approximations may be simplified or scaffolded versions of practice (e.g., only facilitating a small group discussion instead of the whole class, analyzing assessment results with extensive instructor support), or they might engage teacher candidates in more explicitly elaborated versions of practice (e.g., detailed unit plans). Since approximations are filtered version of reality, they typically involve intervention from instructors and/or cooperating teachers.

A typical example of an approximation we observed in the data was related to engaging teacher candidates in planning for a lesson. The following excerpt was from a mathematics methods instructor describing how she engaged her teacher candidates in learning teaching strategies. She explained,

> So, they will do lesson planning, and they will present a lesson, and they will create an assessment that they use with students in their field site and then reflect on that. So, you know, reflection is a big part of it. They do journal writings and the journal writings are specific to the students in their field sites, whether it be just observing what the teacher does and reflecting on that, or when you work with the students, what happens there.

The instructor described a characteristic methods course activity in which teacher candidates plan a lesson, teach that lesson to a small group of students, assess students’ learning, and then reflect on what they learned from the experience. This type of activity was pervasive throughout our interviews with methods instructors. Lesson planning could be considered a canonical approximation of teaching practice. It was simplified in that teacher candidates had to only prepare a single lesson plan, usually with extensive feedback from instructors, and often the lesson was taught to only a single class or a small group of students.

Methods instructors also described such lesson planning activities as involving more details and complexity than might otherwise occur in a regular classroom. For example, another mathematics methods instructor described the major assignment in her course in the following way,

> And even though in some ways it feels silly to have one lesson plan count so much, we really, I feel like we use that as a vehicle for learning all kinds of other things. Because we use it as a vehicle – it’s essentially an annotated lesson plan, with a lot more required than would be typical,
and I continue to tell them that. You will never write this much in a lesson plan as long as you live.

This quote highlights one of our findings that opportunities to learn through approximations, even typical lesson planning activities, do not always follow a clear trajectory of moving from simplified to more complex activities. Our interviews with methods instructors revealed that the purposes instructors have for engaging teacher candidates in an approximation affect how complex or simplified that approximation of practice might be. For example, the instructor in the preceding excerpt explained that she used the lesson planning approximation to support nearly all of the learning to teach activities that occurred throughout her course over an entire semester. Teacher candidates were expected to examine content and practice standards, consider student development, plan for thoughtful pedagogy and intentionally use instructional strategies. Subsequently, teacher candidates also taught the lesson to a group of students, video recorded their teaching, and then later reflected on the video recording of their instruction. This level of detail and time commitment to a single lesson would likely be impossible in the real world of a classroom teacher. The approximation of practice, however, allowed the methods instructor the space to support teacher candidates to deeply explore a range of components that go into a single lesson.

Grossman and colleagues (2009) noted “approximations may require more elaborated versions of practice than what novices will enact in their careers” (p. 2077) citing detailed unit plans as an example. In addition to more elaborated versions of planning activities, our initial analyses also revealed additional examples of approximations with added complexity, including elaborated investigations of students’ communities. For example, a mathematics methods instructor described an assignment that required teacher candidates to immerse themselves in the community of their teaching placement to better understand the specific challenges and resources of their placement location. The methods instructor explained,

[teacher candidates are] not just talking to their mentor teacher. They’re talking to parents. They’re talking to shop owners. They’re exploring the space around school and spending time in coffee shops and groceries stores and the community where their school’s situated, despite the fact that they might live in [another town].

Practicing teachers may not always immerse themselves in the community where their students live; however, an important part of ambitious instruction involves building on students’ knowledge and experiences. This example of an elaborated approximation of practice illustrates another way methods instructors provided opportunities for teacher candidates to learn about teaching in ways that might have been more complex than their counter-parts outside a methods course.

Another interesting complexity we found related to approximations was the ways they juxtaposed with representations of practice. The illustrative excerpt below was taken from an interview with a program coordinator, during which the coordinator described the opportunities to learn to teach provided to teacher candidates during their student teaching placements.

Mentors are trying to help them learn to teach, but I think that they are helping them learn to teach in the way that they teach their district teachers. So we do encourage them to do co-planning, co-teaching, but in terms of the philosophy of teaching, this year we talked about our own visions of teaching, how those match with their mentors. But I worry sometimes our interns go out and just try to imitate with what they are seeing the mentor doing and consider that good teaching or not good teaching.

The coordinator described opportunities for teacher candidates to learn from their mentor teachers, including opportunities to co-plan and co-teach.


Articles published in the Proceedings are copyrighted by the authors.
By engaging in co-planning and co-teaching, teacher candidates have the opportunity to actively participate in the work of teaching, engaging collaboratively alongside their mentor teachers. Thus, we would consider opportunities to co-plan and co-teach as approximations. This quote highlights, however, that participating in co-planning and co-teaching also provides opportunities for teacher candidates to observe a representation of practice, the practice of their mentor teacher. In this case, the coordinator emphasized the authenticity of this representation, in that mentor teachers teach in the ways supported by their district, not necessarily in the model suggested by the university teacher preparation program.

Although co-planning and co-teaching fit within the category of a scaffolded approximation of teaching practice, we found that the opportunities to learn provided to teacher candidates are not easily delineated to separate an approximation from a representation. When in the field, any interactions teacher candidates have with practicing teachers offer potential representations of practice. The ways in which teacher candidates interact and engage with those examples of practice offer potential approximations of practice. Methods instructors and mentor teachers make use of these opportunities to learn in specific and sometimes overlapping ways.

**Discussion and Implications**

We found that methods courses provide a range of opportunities for teacher candidates to learn to teach, and Grossman and colleagues’ (2009) framework provided a useful starting place to interpret and organize these opportunities. These findings reveal that applying the constructs of representations and approximations methods course data is not straightforward and that we needed to refine and supplement the definitions. Likewise, these findings build on literature that encourages interrelationships between pedagogies (e.g., Darling-Hammond & Bransford, 2005) by illustrating how methods instructors and mentor teachers use representations and approximations intertwined with one another.

Additionally, this paper highlights the need for a more detailed taxonomy of methods course activities as they fit within the broad pedagogies of practices. Thus, we seek to identify possible sequences of learning activities within methods courses. One model of using approximations to learn to teach might be that methods instructors move teacher candidates from more distal examples of practice, with more scaffolds, to more proximal examples of practice, dropping the scaffolds as they progress. For example, teacher candidates might move from facilitating a single small group discussion to facilitating a full class discussion. Alternatively, we found that approximations of practice do not necessarily progress along a trajectory of complexity, but rather the degree of complexity of an approximation may be tied to an instructor’s learning goals for a specific activity. For example, some methods instructors discussed how they used a variety of approximations within the course to support teachers’ candidates learning of specific instructional practices (e.g., asking higher-order thinking questions; learning about students’ communities), but did not discuss the progression of these approximations from one activity to the next, suggesting that they may be thinking more closely about the alignment of individual approximations to instructional goals than the progression of approximations over time. Given the similarities we observed across the methods courses, we wonder then if there are also underlying trajectories of particular pedagogical activities that support learning to teach, as found by Ghousseini and Herbst (2016) with respect to learning to teach through discussion. Relatedly, we seek to explore instructors’ rationales for using particular pedagogies with teacher candidates. Additional research is warranted into opportunities to learn to teach including further exploration of approximations and representations, as well as additional opportunities we identified beyond the scope of the Grossman et al. framework (2009).
Acknowledgements

This material is based upon work supported by the Spencer Foundation and the National Science Foundation under Grant No. DGE 1535024. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the funders.

References


