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Interest in improving mathematics achievement has garnered increased attention across an array of diverse settings ranging from the individual classroom to the halls of Congress. While multiple reasons exist for the growing interest in enhancing the quality of mathematics instruction, one powerful driving force is the current level of mathematics achievement. In the most recent National Assessment of Educational Progress (National Center for Education Statistics, 2013), 58% of fourth-grade students were classified as at or below basic in terms of their mathematics achievement. If teachers and schools are to address the challenge of improving mathematics achievement, and in particular, the achievement of students who struggle in mathematics, a natural question is, when is the best time to start? Evidence is accumulating that the early elementary grades represent a pivotal time in which the investment of resources may maximize student outcomes. If teachers and schools are to address the challenge of improving mathematics achievement, and in particular, the achievement of students who struggle in mathematics, a natural question is, when is the best time to start? Evidence is accumulating that the early elementary grades represent a pivotal time in which the investment of resources may maximize student outcomes. Using a nationally representative sample from the Early Childhood Longitudinal Study–Kindergarten Cohort, Morgan, Farkas, and Wu (2009) documented that students who were in the lowest 10th percentile at entrance and exit from kindergarten had a 70% chance of remaining in the lowest 10th percentile 5 years later. Such a finding is striking and raises serious concern for educators in that it appears unlikely for students with high levels of risk in kindergarten to achieve substantive growth in the absence of intensely focused efforts on improving their understanding of mathematics. Thus, the early elementary grades represent a critical time window in which schools can intervene with a goal of preventing more serious difficulties, including learning disabilities, from developing by providing all students a solid foundation upon which to expand their understanding of mathematics.

This article provides a broad overview of key considerations for teachers and schools as they focus on improving mathematics achievement of kindergarten and first-grade students at risk in mathematics. To provide a framework, sections of the manuscript are aligned with the instructional tiers found in a response-to-intervention or multitier service delivery model. Because large numbers of districts and schools utilize a multitier service delivery model in the area of reading, this alignment should provide a familiar framework for educators who are considering implementing changes in how they think about and provide mathematics instruction.
Tier 1 Mathematics Instruction

Tier 1 or core mathematics instruction is essential to students’ mathematical learning (Fuchs & Vaughn, 2012). One way to think about core instruction within a multitiered model of mathematics instruction is that it is the primary prevention level or the safety net of instruction. For many students, including those with or at risk for math difficulty (MD), core instruction is often the sole basis of mathematics instruction. This is particularly true in the early elementary grades, where schools face logistical constraints in providing early math interventions. For example, in kindergarten, students may attend a half-day program that includes other mandated instructional time (e.g., a 90-min reading block), thus limiting the feasibility of providing additional instruction in mathematics. For this reason, core instruction is conceptualized as having two primary roles: (a) preventing MD from occurring and (b) reducing the deleterious toll of existing difficulties in mathematics. Realizing core instruction’s potential to prevent and decrease the prevalence of MD begins with the delivery of high-quality instruction in the general education setting. In this section, we focus on the implementation of validated principles of instructional design and delivery (Coyne, Kame’enui, & Carnine, 2011) that teachers can employ within Tier 1 to better ensure they meet the needs of all students including those at risk.

Validated Principles of Instructional Design and Delivery

A hallmark of meeting the instructional needs of all students, particularly students who struggle to develop proficiency in mathematics, is the fashion in which mathematics instruction is designed and delivered. Smith and Ragan (1993) described instructional design as “systematic process of translating principles of learning and instruction into plans for instructional materials and activities” (p. 2). The delivery of instruction, on the other hand, attends to way in which academic content is presented. To enhance Tier 1, teachers should attend to critical validated principles of instructional design and delivery: (a) engaging students’ prior understandings (National Research Council [NRC], 2005) and (b) scaffolding instructional interactions (Coyne et al., 2011). Recent research suggests that these design principles, when judiciously incorporated into core instruction, substantially increase the chances that struggling students will reach proficient levels of mathematics (Agodini & Harris, 2010).

Engage students’ prior understandings. A consistent finding of educational research is that struggling students have difficulty connecting new concepts with previously learned content (Gersten et al., 2009). For example, as younger students transition through kindergarten, they may struggle to connect their understanding of counting and cardinality to solving basic addition and subtraction problems. So, if teachers are to meet the needs of struggling students, they will have to take calculated steps to ensure that core math instruction adequately addresses students’ background knowledge (NRC, 2005). Specifically, teachers will have to identify and pre-teach requisite knowledge and collect performance data, such as the accuracy of student math verbalizations, that directly link to instructional planning.

Identifying and preteaching requisite knowledge and skills can be thought of in two ways. The first has teachers consider the background knowledge that students bring to the classroom. It is clear that students enter school with varying levels of informal math knowledge. Teachers who work with struggling students will have to explicitly teach the prerequisite skills necessary for learning more difficult content. Support for engaging students’ prior understandings can come in the form of simple warm-up exercises at the start of each math lesson. These activities will allow students to make the connection between previously learned content and new material. For example, a teacher might incorporate a brief activity that focuses on numbers 1 through 9 to prepare students for a later activity on “teen” numbers.

The second way considers how instructional examples are ordered within a particular lesson. Struggling students often become overwhelmed when instruction begins with complex teaching examples. Teaching examples that tend to cause difficulties typically include more than one math concept or require multiple steps to complete (National Mathematics Advisory Panel [NMAP], 2008). Teachers can help prevent these difficulties by beginning instruction with easier teaching examples and then slowly transitioning to more difficult ones. In this case, when presenting “change” word problems involving basic addition, teachers might use an introductory teaching example that involves a simple addition problem so that students can answer it correctly. In this case, the teacher would begin instruction by presenting an easier change problem, such as one with the result unknown (e.g., Two frogs were sitting in a pond and one more frog joined them. How many frogs are in the pond now?). Once students demonstrate understanding of the underlying structure of change problems (i.e., the problems involve actions) and the procedures to solve ones with the different actions (i.e., “add to” or “take from”), problems with greater complexity can be introduced. For example, a teacher might introduce students to change problems with the start unknown (e.g., Some girls were on park bench. Four more girls sat down on the bench. Now there are eight girls on the bench. How many girls were on the bench at the start of the story problem?). This way, rather than overwhelming students with a difficult addition problem, the teacher can have students focus on the underlying structures of the change problem (i.e., beginning, change, and ending).
Scaffold instructional interactions. Researchers have described instructional scaffolding as the cornerstone to providing struggling learners access to the core curriculum (Coyne et al., 2011). To Chard and Jungjohann (2006), developers of early mathematics curricula, scaffolding is a “process of gradually releasing responsibility for learning to a student or group of students” (p. 11). However, the extent to which teachers should scaffold instruction can be a vexing issue. Overscaffolding is likely to inhibit children from thinking on their own or gaining personal responsibility of the task at hand. Conversely, underscaffolding is likely to leave many students, in particular, struggling ones, unclear of the lesson’s purpose and objectives. To determine the amount of instructional scaffolding to provide during an instructional task, teachers should consider whether students have the background knowledge necessary for completing the task successfully. In situations where students are less prepared or the task is novel or complex, teachers will have to provide greater support to deeply engage students in key math content. Teachers can use both informal and formal means to gauge students’ prior understandings and determine the amount of instructional support that can result in student success. Informally, teachers can use simple warm-up activities to help prime students’ prerequisite knowledge. These activities can help students build a connection between new and previously learned material. For example, a teacher might use a multidigit addition activity with no regrouping to get students ready for solving addition problems that require regrouping with larger numbers. Data collected from mastery-based assessments can provide teachers with a roadmap for where to begin instruction. For example, if a teacher was planning to introduce two-step word problems, he or she might administer an assessment on basic number combinations and the underlying structures of one-step word problems. A brief (1 to 2 min), timed math facts measure could be used to assess students’ proficiencies with basic number combinations. The teacher could then utilize such data to determine the type of addends used when introducing one-step “put-together” word problems with the result unknown (e.g., Sue has eight pink marbles and three purple marbles. How many marbles does Sue have in all?). This way, if students have a firm understanding of these prerequisite skills, the teacher can derive a plan for supporting them in solving problems with complex features, such as difficult vocabulary, irrelevant information, and unique formats (Powell, 2011).

Math verbalizations. One way to have students interact with teachers around such content is to facilitate math verbalization opportunities. In core math instruction, math verbalizations are opportunities students receive to express their mathematical thinking and understanding. A recent meta-analysis involving 41 intervention studies found student verbalizations of math concepts and strategies had a large effect on student math achievement (Gersten et al., 2009). Math verbalizations are considered critical because they give students the opportunity to speak about math, use precise mathematical language, and convey their mathematical understanding and thinking, particularly before other modes of responding, such as independent written exercises, have become instructionally appropriate. In the early grades, two ways that teachers can manage math verbalizations are through group and individual responses. Research has shown that struggling students can benefit from both types of practice opportunities (Doabler et al., 2014; Gersten et al., 2009). Group responses, when managed well, are important practice opportunities because they permit all students to participate in the instructional task, including those students struggling with math. For example, a group response might entail 20 first-grade students stating in unison the place value of two-digit numbers. Individual responses are also used to engage students but focus on one student verbalizing his or her mathematical knowledge. Teachers can use these types of student verbalizations to address potential misconceptions and estimate whether specific individuals understand the target concepts. Particularly effective ways to manage student math verbalizations are for teachers to first model and verbalize aloud the target task and then to pose guided questions. For example, Figure 1, taken from the Early Learning in Mathematics curriculum (Davis & Jungjohann, 2009), illustrates how a teacher would explicitly demonstrate and define the attributes of squares and circles and then present opportunities for the entire class as well specific individuals to verbalize their math thinking. It is important to point out in this example how the lesson intersperses group and individual responses to engage students in productive math discourse. Moreover, to further student understanding of two-dimensional shapes and their attributes, the lesson helps teachers offer timely academic feedback.

Tier 2 and Tier 3: Supplementary Interventions

When implementing a multitiered model of academic support in mathematics, some students will struggle to achieve grade-level mathematics objectives with only Tier 1 instruction. To identify at-risk students in the early elementary grades, schools often utilize early numeracy screeners that focus on critical skills, including the ability to identify numbers, compare magnitudes, and engage in strategic counting. Typically, these 1-min timed screeners are given to all students, and those students who fall below a certain threshold of performance are identified as needing the intervention services provided in Tiers 2 and 3.

Although the number of Tier 2 and Tier 3 mathematics interventions available is increasing, very few have been
“Today, we’re going to learn about squares.” Hold up the square card. “This shape is a square. What shape, everybody?” (“A square.”) “Yes, this shape is called a square.”

“Let’s talk about how a square is different from a circle. Raise your hand if you have some ideas.” Accept reasonable responses.

Explain, “A square is different from a circle because a circle is round and doesn’t really have any sides; but a square has straight sides. Two things are special about squares: they have four straight sides AND all four of sides are the same size.”

“How many sides does a square have?” (“Four”) “Yes, four. Are the sides round or straight?” (“Straight.”) “Right, the sides are straight. Are all the sides the same size?” (“Yes.”) “Yes, a square has four straight sides that are all the same size.” Repeat until firm. Provide individual turns.

Measure each side to show how they are the same length.

Show the children a variety of different color and size squares. For each square, say, “This is a square. What is this?” (“A square.”) Say, “Yes, this is a square.”

“Now, I’m going to mix up the circles and the squares. Each time I hold up a shape, tell me the name of the shape and something special about it. Get ready.” Hold up a variety of circles and squares. As children respond, confirm by say, “Yes, this is a ______.”

Repeat the preceding step and provide individual turns.

**Figure 1.** An example activity in Early Learning in Mathematics for scaffolding math verbalizations.
evaluated employing rigorous research methods. The paucity of studies demonstrating the efficacy of elementary mathematics interventions for students with and at risk for MD underscores the challenges schools encounter as they seek to support implementation of evidence-based practices in schools. However, several major themes and research findings have been identified in mathematics in the last decade that can be utilized to guide the work of practitioners (NMAP, 2008).

First, the validated principles of instructional design reviewed in the section on Tier 1 are of even greater importance when considering Tier 2 and Tier 3 instruction and interventions. Besides ensuring that interventions adhere to the key instructional design principles, including engaging prior understanding, scaffolding, and math verbalizations, interventions in Tiers 2 and 3 are also designed to increase the instructional intensity of the student’s experience. This can be accomplished by increasing the duration and time spent in the intervention or by decreasing instructional group size (Clarke, Doabler, Fien, Baker, & Smolkowski, 2011) Another mechanism for increasing the intensity of instruction is to increase the level of individualization and/or extend instructional scaffolding (e.g., instructor modeling, student opportunities to practice with support, immediate and targeted academic feedback) over time, with more discrete but still deliberate attempts to gradually withdraw scaffolding to optimize student success and independent learning. Whether or not a three-tiered system is used, the idea of increasing instructional intensity to match the severity of student need serves as a valuable guideline for schools seeking to provide mathematics interventions.

Second, numerous experts and national panels (NMAP, 2008) have noted that the breadth of coverage in U.S. math curricula where multiple topics are addressed but none are developed to sufficient depth is problematic in ensuring that students develop a strong foundation for later mathematics learning. It is vital for Tier 2 and Tier 3 interventions to provide a rigorous and coherent content focus. In elementary school, this means a focus on number and number systems (i.e., the whole-number system in the early grades) and the properties and operations within this system (NMAP, 2008). For example, in first grade, this would include a focus on the Common Core State Standards (CCSS; CCSS Initiative, 2010) strands addressing Operations and Algebraic Thinking and Number and Operations in Base 10 and objectives and skills that fall within those strands (e.g., relate counting to addition and subtraction, understand the relationship between addition and subtraction, and use an understanding of place value and operations to add and subtract).

**Single Skill Versus Broad Content Coverage**

Although the NMAP (2008) report advocated for a focus on whole-number content in early mathematics interventions, there are a variety of requisite skills that comprise early numeracy, and whole-number content encompasses a range of subskills. Effective interventions can either target specific skill development or employ a broad focus on various whole-number concepts. Each of these intervention content approaches has advantages and disadvantages that require careful consideration to ensure maximal effectiveness in practice. Single-skill interventions intend to provide targeted instruction to solidify knowledge of distinct mathematical concepts and allow instructors to closely monitor and gauge mathematics skill development. Conversely, broad whole-number interventions intend to provide integrated mathematics instruction on a variety of skills and concepts and allow instructors to utilize interspersed practice to evaluate numeracy development and mathematics skill integration.

**Single-skill interventions.** In the early elementary grades, single-skill interventions typically focus on number and number systems and specific identities and operations within those systems. Single skills and target concepts include (a) developmental number sense skills, such as number identification, counting, quantity discrimination, number line tasks, and basic mathematical operations; (b) basic number combinations, including addition and subtraction facts within 20, fact families, number decomposition, and two-digit problems with regrouping; and (c) basic problem solving, including word problems and equations with missing values (see Krosbergen & Van Luit, 2003, for a comprehensive list of single-skill interventions research studies).

**Developmental number sense skill interventions.** Counting, seriating, quantity discrimination, and number identification interventions are often utilized as brief supplements to traditional instruction with young students to review and solidify basic number sense concepts. These interventions are also frequently utilized with students with mathematics disabilities in early elementary grades to firm up numeracy foundations. Additionally, developmental number sense skill interventions are commonly employed in special education settings with students with developmental and intellectual disabilities. In all cases, basic number sense skill interventions are utilized to build prerequisite skills and capacity for more complex mathematics tasks. The efficacy of these interventions is frequently tested in small-group settings with curriculum-based measurement screeners as the outcome measures, but these single-skill interventions can also be utilized in concert with other single-skill interventions. For example, Fuchs et al. (2010) evaluated the utility of a strategic counting intervention for improving number combination fluency in the context of a word problem-solving tutoring protocol. Additionally, these basic numeracy interventions lend themselves well to technology-based delivery (Butterworth & Laurillard,
2010) because students can receive individualized practice at a flexible pace.

**Basic number combinations interventions.** Interventions that target memorization and fluent recall of basic number combinations (i.e., addition and subtraction facts) are some of the most commonly implemented and researched single-skill interventions. These single-skill interventions can be easily linked to assessments and lend themselves to efficient progress monitoring. Thus, they tend to produce the strongest results (Kroesbergen & Van Luit, 2003). Advocates for fluency interventions contend that training students to become more fluent with basic mathematics facts is important because possessing mathematical fluency frees students’ cognitive resources for more complex tasks. If students struggle to automatically retrieve basic number combinations, they will work more slowly and make more errors when solving complex mathematics problems, whereas students who are fluent in basic number combination retrieval may be able to focus more on the mathematics concept being taught. Although these interventions employ a variety of strategies for building number combination fluency, meta-analytic reviews of various fact fluency interventions suggest that single-skill interventions that employ incremental rehearsal with modeling components are most effective with all learners (Codding, Burns, & Lukito, 2011; Joseph et al., 2012). For example, the Cover-Copy-Compare intervention (Poncy, Skiner, & O’Mara, 2006) provides students with model answers and scaffolds their learning through a set of prescribed steps: (a) Study basic number combinations listed on one side of a page, (b) cover the problems, (c) write the problems and answers on the other side of the page, (d) compare their answers to the correct answers, and (e) rewrite any incorrect number combinations. This method can be combined with incremental rehearsal strategies (i.e., interspersing 10% new, unknown number combinations with 90% previously mastered number combinations) for maximal effectiveness.

**Problem-solving interventions.** Word problem instruction that emphasizes the underlying structures of problems and includes the use of model representations and heuristics to promote generalizability of problem-solving skill across problem types is a critical component of early mathematics instruction. Providing strategies and scaffolds for mathematics problem solving assists students in creating representations and developing equations to solve word problems. Schema-based (or broadening) instruction (SBI) is a particularly effective problem-solving strategy that is used with a range of students across grade levels (Jitendra et al., 2007). In the early grades, teachers can use SBI to guide students to solve simple group, change, and compare problems involving addition and subtraction (see Whole Number Foundations, Level 1; Davis & Jungjohann, 2009). Given a basic word problem (e.g., There are 15 ducks. Then three more ducks swim over. How many ducks are there now?), SBI involves directing students to identify the problem type (i.e., this is an addition change problem), modeling the use of strip diagrams to represent the problem, and facilitating guided practice to solve the problem. The goal of SBI is for students to apply knowledge of taught schemas to more accurately and efficiently solve word problems encountered independently. Fuchs and colleagues (2010) researched the effectiveness of SBI for improving the second graders’ representations of conventional word problems and found that SBI supported representational skills and algebraic reasoning. Additionally, a review of word problem interventions utilizing schema instruction with second- and third-grade students who were at risk for mathematics difficulty found that SBI was effective in enhancing word problem-solving skills (Powell, 2011). It should be noted that although problem-solving interventions require students to utilize a variety of mathematics skills, they are considered a single-skill intervention here because they explicitly target problem-solving strategies and do not include instructional pieces to improve number sense skills or number combination fluency.

**Broad whole-number interventions.** Experts in the fields of mathematics and education (NMAP, 2008) have also emphasized that instruction should include multiple types of knowledge to support adequate understanding of mathematics content, and procedural knowledge and conceptual knowledge should be integrated to support mathematical proficiency (NMAP, 2008). Therefore, although single-skill interventions can be very effective in targeting critical early mathematics content and skills, the specificity of single-skill interventions should be tempered with instruction that emphasizes the coherence of numeracy within broad early mathematics content. Broad content interventions aim to integrate multiple mathematics skills and utilize strategic sequencing and scaffolding to build solid, diverse mathematics foundations. Proponents of these broad interventions argue that when based on sound instructional design and well aligned to Tier 1 curricula, these interventions can most effectively supplement mathematics learning. Various supplemental whole-number mathematics interventions have been developed and evaluated for use with at-risk learners in early elementary grades and show promising impacts (Bryant et al., 2011; Dyson, Jordan, & Glutting 2011; Fuchs et al., 2005). A typical example is Number Rockets, a 63-lesson program covering 17 whole-number topics (three to six lessons on each topic) with topics advancing in complexity (e.g., from identifying and writing numbers to two-digit subtraction without regrouping) across lessons. Increasingly, interventions are developed to align with critical standards, like the CCSS. For example, ROOTS (Clarke et al., 2014), a 50-lesson intervention curriculum focused on
whole-number understanding, was designed to align with the three whole-number content strands (Counting and Cardinality, Operations and Algebraic Thinking, Number and Operations in Base 10) of the CCSS. See Figure 2 for an example of a Counting and Cardinality instructional scope and sequence. Interventions are delivered in small groups of between two and five students, thus enabling high degrees of teacher–student interactions. Advantages of this and other broad content-focused interventions are that they allow for interwoven practice, judicious review, explicit skill combination, and instructional strategies for intervening on a wide array of early mathematics tasks. Broad-based interventions can also include components specifically focused on single-skill proficiencies. For example, in the Number Rockets program (Fuchs et al., 2005), the last 10 min of each lesson was allocated to working on the computer with a program focused on building fluency with basic number combinations.

### Conclusion

The challenge of ensuring that all kindergarten and first-grade students develop a strong foundation of early mathematics...
knowledge to enable success in school is significant. In Tier 1, this entails utilizing core curricula with critical instructional design features targeting the learning needs of students at risk. In Tiers 2 and 3, a continued focus on instructional design principles delivered in more intensive ways coupled with a focus on critical whole-number content should provide the cornerstone of students’ experience as they advance into the increasingly more complex mathematics they will encounter in second grade and beyond. If schools focus on building a coherent system of support across instructional tiers, their efforts have the potential to positively impact student outcomes.

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