

Title: Schema-Based Instruction: Effects of Experienced and Novice Teacher Implementers on Seventh Grade Students' Proportional Problem Solving

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Abstract

This study examined the effects of a research-based intervention, schema-based instruction (SBI), implemented by experienced- (taught SBI in previous study; Jitendra et al., 2015) and novice-teacher implementers (taught SBI for the first time with professional development) on the mathematics outcomes of seventh-grade students. SBI is a multicomponent intervention that emphasizes the mathematical structure of problems through the use of schematic diagrams and incorporates problem solving and metacognitive strategy instruction. Results indicated that both experienced- and novice-teacher implementers delivered SBI with similar levels of fidelity; there was no SBI experience effect on the immediate and 10-week retention tests of proportional problem-solving, on a general measure of problem solving, or on the end of the year state mathematics achievement test. These results provide evidence that the effectiveness of SBI generalizes over time to different cohorts of teachers and that the impact of SBI on student mathematics outcomes is maintained over time without additional PD.

KEYWORDS: schema-based instruction, experienced and novice implementers, seventh-grade students, proportional problem solving

**Schema-Based Instruction: Effects of Experienced and Novice Teacher
Implementers on Seventh Grade Students' Proportional Problem Solving**

1. Introduction

Ratio and proportional relationships are of primary importance during the upper elementary and middle school grades. These relationships, along with the interrelated topics of fractions, decimals, and percent, provide a critical foundation for algebra. Proportionality involves the concept of ratio and is central to topics in mathematics such as linear functions, scale drawings, similarity, trigonometry, and probability. In the Common Core State Standards (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), instructional time focused on proportionality occurs in middle school when students “develop understanding of proportionality to solve single and multi-step problems ... solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease” (p. 46).

Researchers in mathematics education (e.g., Lamon, 2007; Lobato, Ellis, Charles, & Zbiek, 2010) note that ratio and proportional relationships are situated in the broader landscape of the multiplicative conceptual field (e.g., whole number multiplication and division, fractions, linear functions). While young students' difficulty with ratio and proportional relationships is related to their development of multiplicative versus additive reasoning (Lamon, 1995), secondary school students who learn to reason proportionally following instruction in various linear functions tend to apply the notion of linearity to most situations, even when it is not applicable (see Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2012; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005).

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Solving even simple proportion problems is challenging for many children and adolescents as they may not understand the problem situation or know when a solution strategy is applicable (Weinberg, 2002). Yet, only few intervention studies have focused on improving students' learning of ratios and proportions. Most studies were short-term and did not address the broad domain of ratios and proportional relationships (Adjigbe & Pluvinage, 2007; Fujimura, 2001; Miyakawa & Winslow, 2009) or used quasi-experimental research designs or teaching experiments, which limited causal inferences. Also, few studies have tested the effectiveness of a comprehensive curriculum package (e.g., Connected Mathematics Project; see Ben-Chaim, Fitzgerald, Benedetto, & Miller, 1998) or conducted randomized studies. The few randomized studies conducted have examined the efficacy of schema-based instruction (SBI), a multicomponent approach to teaching proportional problem solving (e.g., Jitendra, Star, Dupuis, & Rodriguez, 2013; Jitendra et al., 2015).

In this article, we report findings from the second year of a 2-year study of the efficacy and sustainability of SBI. In the first experimental study (Jitendra et al., 2015), teachers were randomly assigned to either the treatment condition (Cohort 1), in which they received professional development to implement SBI five days a week for approximately 6 weeks to teach problem solving involving ratio, proportion, and percent, or to the control condition (Cohort 2) to teach the same topics from their district-adopted mathematics textbook. Results indicated that students in the SBI classrooms outperformed students in the control classrooms on a proportional problem solving measure and maintained their improved performance nine weeks later. Scores on the Process and Application subtest of the Group Mathematics Assessment and Diagnostic Evaluation (GMADE) were equivalent for the two groups.

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Many teachers, even though highly motivated, may find it challenging to use new instructional materials to teach ratios and proportional relationships and effectively use visual representations to prime the underlying problem structure, foster problem solving and metacognitive strategy skills, and develop procedural flexibility (essential features of SBI implementation). Implementing a new instructional approach can pose a range of challenges even when teachers are provided with instructional materials (Obara & Sloan, 2010; Remillard, 2005). Factors such as initial mismatch between principles underlying the innovative approach and teachers' philosophy of teaching and learning, time to cover the rest of the curriculum, and the methods of assessment may restrict innovation that attempts to find a balance between (a) providing instruction to ensure meaningful learning and (b) having teachers "reflect on the mathematical value of the study of the topic" (Bennie & Newstead, 1999, p. 5). Despite these challenges, there is some research suggesting that instructional materials in conjunction with professional development can change teacher practices to align with the innovation (Cohen & Hill, 2000).

In response to the challenges of using new instructional materials, we invited all teachers who participated in the first study (Year 1; Jitendra et al., 2015) to continue in the study a second year to evaluate the effects of professional development (PD) and sustainability of SBI by examining both teacher implementation and student learning in classrooms of teachers who received PD and implemented SBI in the past year and possibly gained more expertise with this method (experienced-teacher implementers) and teachers implementing SBI for the first time with PD (novice-teacher implementers). Previous research on the long- and short-term effects of teachers' professional development is sparse, with one study (Fennema et al., 1996) documenting that when teachers participate in PD and develop expertise with instructional techniques for

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promoting students' mathematical thinking, greater gains are seen in subsequent cohorts of students than prior cohorts. Therefore, the purpose of this study was to examine whether teacher participation in PD along with more or less experience with SBI would differentially affect teacher implementation and influence their students' proportional and mathematical problem solving skills as well as mathematics achievement.

1.1 Theoretical Framework

1.1.1. Teacher learning in organized professional development. Research has shown that high quality professional development can not only affect teacher attitudes and classroom practice, but also influence student learning (Garet, Porter, Desimone, Birman, & Yoon, 2001). Several conceptual frameworks of professional development have been proposed for enhancing teacher and student learning (e.g., Borko, 2004; Desimone, 2009). One theory involves a link between the PD, teachers, and students such that teachers participate in PD, they implement the practices learned in PD in their classrooms, and these practices affect student achievement (Desimone, 2009). Features of high-quality PD that have been associated with changes in teacher practice and subsequent improvements in student performance include content focus, active learning, coherence, duration, and collective participation. A sustained focus on *content* appears to be a critical feature of high-quality PD (Blank & de las Alas, 2009). Content-focused PD activities designed to help teachers understand what they teach and how students learn the content can support teacher knowledge and practice to improve student learning (Sample McMeeking et al., 2012; Perry & Lewis, 2011; Saxe, Gearhart, & Nasir, 2001).

There is evidence that teachers engaged in *active learning* strategies through observation, discussion, practice, and reflection are particularly effective (Desimone, Porter, Garet, Yoon, & Birman, 2002). Such strategies may include implementing new approaches to teach familiar

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content that support teachers' thinking and reflecting on mathematical ideas or helping teachers understand students' difficulties in specific content by reviewing student work (Carpenter, Fennema, Peterson, & Loef, 1989; Cohen & Hill, 2000). One exemplary PD program that not only supports teachers' own knowledge of the relevant mathematics, but also enhances their understanding of children's mathematical understanding is Cognitively Guided Instruction (CGI). Research on CGI, which focused extensively on instructional practices, showed that the 4-week PD program led to changes in classroom teaching practices (e.g., fostered discussions of problem-solving strategies) and subsequent improvement in children's mathematical word problem solving performance. Students in CGI classrooms solved a variety of mathematics problems using multiple-solution strategies and demonstrated confidence in their mathematical ability compared to students in control classrooms. Furthermore, findings of a longitudinal study of CGI (Fennema et al., 1996) indicated that CGI teachers developed their own practices as they gained expertise with CGI approaches over time, which led to further improvements in subsequent cohorts of students.

Another core feature of effective PD programs is *coherence*, which refers to the extent to which the content taught aligns with state standards and assessments, is consistent with teacher goals for their professional development, and presents opportunities for communicating with other professionals (Garet et al. 2001; Penuel, Fishman, Yamaguchi, & Gallagher, 2007). Effective PD also requires considerable time that is carefully structured, organized, and focused on both content and pedagogy (Guskey & Yoon, 2009). Content-focused PD along with extended *duration* (more than 30 hours over a span of time) can lead to changes to teacher practice and student achievement (see Blank & de las Alas, 2009; Desimone et al., 2002; Guskey & Yoon, 2009; Yoon, Duncan, Lee, Scarloss, & Shapeley, 2007). Furthermore, *collective*

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participation of groups of teachers working together is more likely to influence teacher learning (see Desimone et al., 2002; Penuel et al., 2007; Penuel, Sun, Frank, & Gallagher 2012). Although there is some evidence that PD with all or most of these characteristics can support better curriculum implementation (Perry & Lewis, 2011) and enhanced student learning (Sample McMeeking et al., 2012; Perry & Lewis, 2011; Saxe et al., 2001), there is very limited causal evidence of the effects of teacher PD on improving student mathematics achievement (Gersten, Taylor, Keys, Rolfhus, & Newman-Gonchar, 2014).

Guided by findings from prior research on classroom practice and student learning, we designed our professional development program to focus on the SBI curriculum content on proportional reasoning as the basis for teachers' understanding of proportional reasoning and of the ways that students understand and learn. The PD was organized to provide teachers an opportunity to participate with other teachers in practices reflective of state standards (e.g., make sense of problems, look for and make use of structure, model with mathematics). The goal of the PD was to enhance effective teacher practices through observation, discussion, practice, and reflection. For example, an introductory activity that emphasized promoting deeper knowledge of the content of the curriculum required teachers to work in pairs or small groups to propose strategies for solving a non-routine problem that had insufficient information followed by the facilitator engaging teachers in reflection of ratios and proportional relationships and understanding students' efforts to solve such problems.

1.1.2. Schema-Based Instruction. SBI has its roots in schema theory of cognitive psychology, research on expert problem solvers, and is guided by cognitive models of mathematical problem solving. Consistent with schema theory, recognition of the semantic structure of the mathematical problem is critical to problem comprehension (Kalyuga, 2006) and

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is an essential feature of SBI. SBI integrates essential processes that expert problem solvers engage in when solving problems such as “perceiving rapidly and accurately the mathematical structure of problems and in generalizing across a wider range of mathematically similar problems” (Van Dooren, de Bock, Vleugels, & Verschaffel, 2010, p. 22). Providing students with problem categories and ways to place problems within categories improves problem solving (Chen, 1999; Quilici & Mayer, 1996).

In addition, SBI emphasizes knowledge of problem solving procedures (e.g., problem representation, planning) for a given class of problems (see Marshall, 1990; Mayer, 1999). Teachers are guided to use effective instructional practices (e.g., explicit modeling, guided questions to engage students) to help students recognize the problem type, represent the problem using a diagram, solve the problem, and check the solution. Another feature of SBI is the focus on procedural flexibility by having students use, share, compare, and contrast multiple solution methods for a given class of problems to improve procedural knowledge (Star & Rittle-Johnson, 2009).

Also integral to SBI is teaching students to display metacognition skills similar to expert problem solvers, who are able to plan, check, monitor, and evaluate their performance (e.g., Kramarski & Mevarech, 2003; Mevarech, & Kramarski, 2003). Guided by their teachers, students “think about what they are doing and why they are doing it, evaluate the steps they are taking to solve the problem, and connect new concepts to what they already know” (Woodward et al., 2012, p. 17).

1.1.3. Previous Research on SBI and Proportional Reasoning. Jitendra and colleagues focused on the broad landscape of proportional problem solving, which included a set of mathematical topics (e.g., ratio, proportion, percent, percent of change, simple interest, scale

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factor) whose boundaries are not easily identified. In the first study, Jitendra et al. (2009) randomly assigned seventh-grade classrooms to SBI or a “business as usual” control condition. Students in both conditions were taught the same topics (i.e., ratios, equivalent ratios, ratio word problems, rates, proportion word problems, scale drawing problems) for the same time (daily, 45 min sessions for two weeks). SBI students scored on average significantly higher than comparison students on proportional problem solving ($d = 0.45$) and maintained the effects on a 4-month retention test ($d = 0.56$).

Jitendra and colleagues conducted two randomized controlled studies to address several limitations of the Jitendra et al. (2009) study. These studies targeted additional topics (e.g., proportional problem solving involving percent), included more classrooms and schools, extended instructional time to 6 weeks, provided longer professional development (Jitendra et al., 2013; Jitendra, Star, Rodriguez, Lindell, & Someki et al., 2011), and reduced direct involvement of the research team in SBI implementation (Jitendra et al., 2013). Classrooms were randomly assigned to SBI intervention or control (business as usual) conditions, and teachers with multiple classrooms taught both SBI and control classrooms all of which were included in the studies. Results indicated that SBI students in both studies (Jitendra et al., 2011, 2013) outperformed comparison students on a measure of proportional problem solving at posttest (multilevel standardized effect sizes of 0.32 and 0.36). The effects of SBI were not maintained on the retention test given 4 weeks after the end of the intervention in Jitendra et al. (2011), which the authors attributed to a lack of power to detect significant differences given the modest number of classrooms ($n = 21$). Jitendra et al. (2013) included more classrooms ($n = 42$) and showed that SBI students maintained their problem solving skills at 6-week follow-up (multilevel standardized effect size = 0.29).

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Related specifically to the study reported in this manuscript is a previous randomized study that improved upon earlier study designs by randomly assigning teachers to SBI or control conditions and then randomly selecting one of their classrooms to participate in the study such that each teacher taught in a SBI or control classroom but not both (Jitendra et al., 2015). Furthermore, the study assessed whether the efficacy of SBI would hold when comparison classrooms used increasingly diverse mathematics programs and when we eliminated direct involvement of the research team to support SBI classroom implementation and test administration. Results indicated that students in SBI classrooms on average outperformed students in control classrooms on a measure of proportional problem solving at posttest (multilevel standardized effect size = 0.46) and maintained their problem solving skills at 9 weeks follow-up (multilevel standardized effect size = 0.32). While our previous SBI studies indicate the promise of our intervention when teachers implement SBI immediately after receiving training on its use, it is less clear whether teachers will persist in implementing SBI with fidelity in subsequent years. Based on two previous SBI studies (Jitendra et al., 2011, 2013), we found that teachers who are experienced SBI implementers (implemented SBI in both studies) gained expertise with SBI as a result of using our instructional materials to seamlessly implement SBI and continued to incorporate instructional practices from the PD training into their teaching (e.g., providing prompts that require students to think through the problem solving process and justify their responses). However, we did not test the relative effects pertaining to more or less experience with SBI on student learning.

1.3. The Current Study

The present study extends previous SBI research by including the same sample of teachers as in Jitendra et al. (2015) as a means to determine the relative effects of SBI with

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experienced-teacher implementers (Cohort 1 – treatment teachers participated in our professional development and implemented SBI in the previous study) and novice-teacher implementers (Cohort 2 – control teachers from the previous study participated in PD and implemented SBI for the first time in this study) on the proportional and mathematical problem solving performance of a new group of seventh-grade students. In addition, this study examined students' and teachers' attitudes in terms of acceptability and benefits of SBI. To our knowledge, the study reported here is the first research study to examine whether the effects of PD and teachers' expertise or experience with SBI leads to better implementation and subsequent improvement in student achievement compared to having no prior experience with SBI and implementing for the first time following PD proximal to treatment enactment. Specifically, we investigated two relationships: (a) between PD and teacher practice and (b) between teacher practice and student achievement. We considered the following research questions and hypotheses:

1. What is the effect of the PD training provided in this study for teachers who have prior experience with SBI vs. teachers with no prior experience with SBI on their classroom instructional practice in terms of fidelity of treatment implementation? Given the limited research on the short- and long-term effects of teachers' professional development, it is not clear a priori whether Cohort 1 teachers do a better job of implementing SBI in their second year of the study (despite not receiving any additional professional development in the second year) or whether the novice SBI teacher implementers, who have had a more proximal professional development experience, may implement SBI with a higher degree of fidelity than Cohort 1 teachers.

2. Did the students of teachers who participated in the PD training and had prior experience with SBI: (a) demonstrate higher levels of proportional problem solving performance

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on an immediate posttest and on a retention test (10 weeks later) than did the students of teachers who participated in the PD program and had no prior experience with SBI? (b) demonstrate higher levels of overall mathematical problem solving performance and mathematics achievement as measured by standardized assessments than did the students of teachers who participated in the PD program and had no prior experience with SBI? Similar to research question 1, it is not clear a priori whether students of Cohort 1 teachers would score higher than or comparably to students of Cohort 2 teachers on study outcomes. On the one hand, it may be the case that students of experienced SBI teacher implementers (translates to better SBI implementation) outperform students of novice SBI teacher implementers. On the other hand, the novice SBI teacher implementers, with more proximal professional development experience, may better implement SBI that would lead to greater student learning.

3. Did students and teachers who participated in the study report different levels of satisfaction with SBI based on their assignment to condition? On the basis of prior research on teacher participation in PD programs designed to help understand children's mathematical thinking (e.g., Carpenter et al. 1989) and previous research on SBI (e.g., Jitendra, Harwell, Dupuis, & Karl, in press), it is hypothesized that students' and teachers' beliefs about SBI program would be positive, but the direction of those effects is not clear.

A priori, it is not clear whether the students of Cohort 1 teachers score higher or similarly to students of Cohort 2 teachers on study outcomes. On the one hand, it may be the case that teachers do a better job of implementing SBI in their second year of the study (despite not receiving any additional professional development in the second year), in which case the students of experienced SBI teacher implementers may outperform the students of novice SBI teacher implementers. On the other hand, the novice SBI teacher implementers will have had a more

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proximal professional development experience and thus may implement SBI with a higher degree of fidelity, which in turn may lead to greater student learning.

2. Method

2.1. Participants and Sampling Procedures

2.1.1. Teachers. Sixty-nine seventh-grade public school teachers who participated in the SBI study in the previous year (Jitendra et al., 2015) returned in Year 2. A total of 13 teachers, nine from Cohort 1 (treatment) and four from Cohort 2 (control), did not continue after Year 1 due to changes in teaching assignments in that a teacher no longer taught a typical seventh-grade mathematics class and therefore was not eligible, or due to retirement or family circumstances. Because Cohort 2 teachers taught SBI for the first time during Year 2, we recruited four new teachers from the same schools to replace the four Cohort 2 teachers who did not continue. Thus, the current study included 73 teachers, 31 in Cohort 1 and 42 in Cohort 2.

Teachers' mean years of experience teaching mathematics was 12.2 years ($SD = 6.7$, range 1 to 34 years). The majority of teachers were White ($n = 66$), while four teachers were Hispanic, two teachers were Asian, and one teacher was American Indian. Sixty-three percent of teachers were female.

2.1.2. Students. The sample consisted of seventh-grade students in 48 school districts (36 rural, eight suburban, and four urban) in an upper Midwest state in the U.S. At the beginning of the school year each teacher identified the class periods during which typical math instruction would be delivered, and one of these classes was randomly selected to participate in the current study. All students in the participating class that were present at the pretest and posttest were included in the student sample. The original student sample consisted of 1,859 seventh-grade students; the majority of students were White (77%), with 8% Black, 8% Hispanic, 6% Asian,

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and 1% American Indian. Due to data analysis difficulties linked to small sample sizes American Indian students ($n = 14$) were removed; as such, the student sample used for the present analyses consisted of 1,845 students (834 in Cohort 1 and 1,011 in Cohort 2). The mean age of these students was 12 years 6 months ($SD = 4$ months). Approximately 40% of students were eligible for a free or reduced price lunch (FRL), 10% received special education services, and 6% were English language learners (ELL) (see Table 1 for teacher and student demographic information).

2.2. Research Design

Our focus on comparing student performances in Year 2 of the study for Cohort 1 and Cohort 2 prompted us to adopt a nonequivalent, quasi-experimental, cluster design (Cohort 1 vs. Cohort 2) in which students were treated as nested within clusters (teachers). The precise nature and magnitude of the selection bias associated with the quasi-experimental design is unclear. Following the What Works Clearinghouse (WWC, 2014) standards, control variables were used in the data analyses to take pre-existing differences among cohorts into account. This helped to ensure, but does not guarantee, credible inferences about the impact of Cohorts 1 and 2 on student proportional problem solving learning.

2.3. Procedures

2.3.1. Professional development. The 42 Cohort 2 teachers participated in professional development (PD) in the current study year that included training in SBI over 2 days (8 hr per day) on implementing the treatment practices. Cohort 1 teachers participated in a similar PD during the previous year and did not receive additional PD prior to implementing SBI for the second time during Year 2 of the study. As such, both cohorts of teachers received PD prior to implementing SBI for the first time.

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In both years of the study, the PD was designed to support teachers' learning and implementation of SBI. The PD focused on critical features of the SBI practices (e.g., recognizing problem types, generating estimates, applying multiple-solution strategies) to support student learning of ratio, proportion and percent, as well as how to teach students to apply these practices. Teachers were provided with SBI program materials for instruction and were given the opportunity to review the two curricular units (ratios/rates and percent), view multiple short video segments of other teachers implementing the SBI intervention, and discuss how to develop students' proportional reasoning skills. Although lessons were scripted to provide a detailed teaching procedure (i.e., questions to ask, examples to present), teachers were asked not to read the scripts verbatim. Video segments of SBI teachers in previous studies highlighted the importance of using their own explanations and elaborations to implement SBI.

2.3.2. Description of the SBI intervention. The SBI program consists of two units, the first focusing on Ratio/Proportion (e.g., equivalent ratios, rates, ratio and proportion word problem solving, scale drawings) and the second on Percent (e.g., percent increase or decrease, including those involving discounts, interest, taxes, tips). The program comprises 21 lessons, which cover the CCSS content standards of ratios and proportional relationships (For further details of the SBI instructional approach, see Jitendra et al., 2009, 2011, 2013, 2015). Teachers taught the two units using our SBI materials daily for 50 min a day for approximately six weeks during their regularly scheduled seventh-grade mathematics class.

The SBI intervention for solving proportional word problems consists of four instructional practices: (a) model problem solving and metacognitive strategies, (b) activate the mathematical structure of problems, (c) visually map information in the problem using schematic diagrams (see Appendix A for ratio, proportion, and percent of change diagrams), and (d)

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develop procedural flexibility. The problem solving and metacognitive strategy instruction components allow students to engage in applying learned content (e.g., ratios/rates, percent) in problem solving activities (i.e., recognizing the problem type, identifying and representing critical information in the problem using an appropriate diagram, connecting the problem to what is already known, estimating the answer, selecting a strategy to solve the problem, solving the problem and presenting the solution within the context of the problem, checking the reasonableness of the solution) and metacognitive activities (i.e., monitoring and reflecting on the problem-solving process). Teachers modeled these four practices through an extensive use of think-alouds with time for students to work with a peer and independently to apply each practice. Furthermore, teacher-student dialogues helped to clarify and refine student thinking as they solved problems.

Students were taught a problem solving strategy to solve proportion word problems. First, teachers provided opportunities for students to engage in whole class discussions to identify the type of problem (ratio, proportion, or percent) by reading, retelling, and examining information in the problem as well as thinking about how problems within and across types are similar or different. Second, students learned to connect the problem to a certain schematic diagram and represent the problem such that the visual mapping showed the relevant elements, relations and conditions embedded in the problem critical to solving the problem. Third, students were instructed to estimate an answer, make decisions about what solution method (equivalent fractions, unit rate, cross multiplication) to use (fourth practice), and solve the problem. The goal of this strategy is to develop procedural flexibility by having students become cognizant of the various methods and select the method that is most efficient based on the numbers in the

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problem. Finally, in checking the solution, students learned to use their estimated answer from the previous step in evaluating their work to determine whether the answer made sense.

Teachers used prompts or deep-level questions to support student learning and promote metacognition by having them monitor and reflect on the problem solving process. Each problem-solving step was accompanied by questions to ensure that students (a) understand and identify the problem type (e.g., Why is this a *proportion* problem? How is this problem similar to or different from one I already solved?), (b) identify and represent the critical information in the problem using an appropriate diagram (e.g., Which diagram is best to represent information in the problem?), (c) select a strategy to solve the problem (e.g., Which solution method would best help me solve this problem?), and (d) check the solution (e.g., Is the answer reasonable based on my estimate?).

2.3.3. Fidelity of implementation. We assessed procedural fidelity and adherence to the SBI intervention by coding classroom videos using a rubric. The rubric focused on procedural fidelity and generated scores ranging from 0 (did not implement) to 3 (highly aligned with SBI practices). For all teachers we coded videos of a lesson on proportion problem solving as a whole for seven features that represent essential problem solving activities in SBI: (a) identifies the problem type by focusing on the key problem features, (b) connects the new problem to previously solved problems, (c) represents critical information in the problem text using an appropriate diagram, (d) generates an estimate prior to solving the problem, (e) discusses multiple solution strategies, (f) solves the problem and presents the solution within the context of the problem, and (g) evaluates the solution.

Following coder training on use of the rubric, two raters independently assessed fidelity for each classroom video, producing a total of two raters x 73 teachers = 146 ratings (i.e., two per

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teacher). Disagreements in ratings were resolved through discussion and review of the videotapes. We estimated inter-rater reliability by computing intra-class correlations for the ratings, which averaged 0.84 (range 0.64 to 0.96).

2.4. Measures

Teachers administered the proportional problem-solving (PPS) test and Group Mathematics Assessment and Diagnostic Evaluation (GMADE) test prior to and immediately following the SBI intervention, with the PPS test also administered 10 weeks following treatment. We collected students' sixth- and seventh-grade scores on the Mathematics Minnesota Comprehensive Assessment-Series III (MCA-III), which is a state-mandated assessment (Minnesota Department of Education, 2014), as well as student race, sex, special education, and ELL and FRL status. Students and teachers also completed surveys about their satisfaction with the treatment at the end of the study.

2.5.1. PPS test. This assessment, used in prior studies (e.g., Jitendra et al., 2015), consisted of 22 multiple-choice questions and four short-response items testing students' proportional problem solving performance (see Appendix A for a sample short answer item). Each multiple-choice question was dichotomously scored, and all multiple-choice items were machine scored. The research team was blind to the study conditions and scored the four short-response items using a rubric (e.g., using sample student responses taken from a previous study), which emphasized correct reasoning. Responses were scored on a 0-to-2 point scale (Appendix B presents the scoring rubric for the sample short answer item). Inter-rater reliability for the short answer items was estimated using an intra-class correlation and averaged 0.92, 0.93, and 0.90 at pretest, posttest, and delayed posttest, respectively.

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We calculated students' scores on the PPS assessment by taking the sum of their total points earned (total possible points equaled 30), which means that the short-response and multiple-choice items contributed unequally to the overall score. To assess the internal consistency of the PPS we used the jMetrik software (Version 2.1.0; Meyer, 2011) to fit a congeneric model assuming a single continuous latent factor underlies the dichotomous- and trichotomously-scored PPS items (McDonald, 1999). The coefficient omega (Dunn, Baguley, & Brunsten, 2013) values for the PPS pretest, posttest, and delayed posttest of 0.75, 0.81, and 0.83, respectively, represent reliabilities estimated as the ratio of true score variance to observed score variance (Dunn et al., 2013; Revelle & Zinberg, 2009).

2.5.2. *GMADE*. The Process and Applications subtest of the GMADE (Pearson, 2004), a standardized assessment assessed students' ability to comprehend mathematical language and concepts and apply relevant operations to solve word problems across multiple content areas (e.g., algebra, geometry, number and operations), was used as a general measure of problem solving. The Process and Applications subtest consists of 30 multiple-choice questions, including multiple-step problems and Process Problems. Each question is worth one point. The coefficient omega reliabilities for our sample were 0.67 for the pretest and 0.74 for the posttest.

2.5.3. *Mathematics MCA-III*. This assessment measures students' knowledge of concepts and skills in four strands: (a) number and operation, (b) algebra, (c) geometry and measurement, and (d) data analysis and probability. At seventh grade, the MCA is reported to have a marginal reliability (Dimitrov, 2003) of 0.93 for the online test and 0.91 for the paper test. Each subscale has an average internal consistency correlation with the total scaled score of 0.88 (Minnesota Department of Education, 2014). Reliabilities for this test for our sample were not computed because the responses of students to individual items were not available. Scaled mathematics

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scores obtained in Spring 2013 served as a pretest and scaled scores for this test obtained in Spring 2014 served as a posttest.

2.5.4. Treatment Acceptability Rating Scale-Revised (TARF-R, Reimers & Wacker, 1988). Students rated four items related to diagrams (e.g., helped to organize information and understand how to solve problems) and five items related to problem solving procedures (e.g., helpful in solving ratio, proportion, and percent word problems; enjoyed solving word problems using the different strategies – cross multiplication, unit rate, equivalent fractions) on a 1-to-4 scale (4 = strongly agree to 1 = strongly disagree). Coefficient alpha was 0.84.

2.5.5. Teacher Satisfaction Survey. The instrument included 22 items that focused on the benefits of: (a) SBI (i.e., the SBI curriculum will produce a lasting improvement in most students' problem solving skills; the materials are helpful in implementing SBI), (b) diagrams (e.g., diagrams are effective in highlighting the underlying mathematical structure of problems and mediating problem solution), and (c) problem solving procedures (e.g., the focus on multiple solution strategies was effective in enhancing problem solving for most students). The items were evaluated on the same 0-to-4 scale as the TARF-R. For this sample, coefficient alpha was 0.94. In addition, the instrument included space for teachers to write additional comments.

2.6. Data Analysis

Several multilevel (i.e. two-level, students within clusters) models with covariates at both levels were fitted using the HLM 6 software (Raudenbush, Bryk, & Congdon, 2004). These models were used to determine if there were significant differences in the outcomes between classrooms of Cohort 1 and Cohort 2-teachers. Outcome variables were the PPS posttest, PPS delayed posttest, GMADE posttest, and the MCA mathematics posttest and each was analyzed separately to answer our research questions. All analyses were based on cross-sectional data.

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For each outcome, the Level 1 (student) model contained four covariates: sex (0 = male, 1 = female), and race (Black = 1, Hispanic = 1, Asian = 1, with White = 0 serving as the reference group), and for all outcomes except the MCA the pretest also served as a student-level control variable. All Level 1 variables were grand-mean centered.

Level 2 (teacher/classroom) variables included the treatment variable (1 = SBI-experienced, 0 = SBI-novice) and variables capturing the percentage of students per classroom eligible for FRL and students receiving special education services. The variable percentage of ELL students was problematic because of the large number of classrooms with minimal number of students qualifying as ELLs: 62% of classrooms did not have any ELL students and another 18% of classrooms had 10% or less students qualifying as ELL. This variable was not included in the multilevel models. Moreover, the coarse and discontinuous nature of the distributions of classroom percentages for the FRL and special education variables prompted us to rescale these variables to better capture the pattern in these data. Accordingly, these two variables were transformed to quartiles (i.e., 0 = lowest concentration of FRL students in a classroom, 3 = highest concentration; 0 = lowest concentration of special education students in a classroom, 3 = highest). The rescaled versions of these variables were used as classroom covariates.

Slopes capturing the impact of the student background variables of sex and race on the outcome variables were found to be statistically equal across classrooms (i.e., variance = 0). Thus our analyses of the impact of cohort on student proportional problem solving are based on intercept-only models. Residuals from the fitted models were examined for evidence of violations to model assumptions of normality, homoscedasticity, and linearity and no major violations were found for any of the models. To control for compounding of Type I error rates we used the Dunn-Bonferroni correction in which an overall error rate (e.g., $\alpha = .15$) was divided

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among all tests of fixed effects linked to each outcome variable with no requirement that the error rate be divided equally. Accordingly, we assigned .05 to the test of the cohort effect because this was the most important effect in the model and divided the remaining .10 among tests of the remaining effects producing $\alpha = \frac{.10}{8} = .0125$ per test. For the analyses reported below all available data were used meaning that sample sizes varied across analyses depending upon missing data patterns.

3. Results

A series of descriptive analyses were initially performed that included examining the correlations between all measures (see Table 2) as well as pre-existing differences between the SBI-experienced and SBI-novice teachers' classrooms. Bivariate correlations between the PPS pretest, posttest, and delayed posttest were 0.67, 0.68, and 0.78, respectively. The correlation between the pretest and posttest on the GMADE was 0.63, and the correlation between sixth-grade and seventh-grade mathematics MCA scores was 0.94. Correlations ranged from 0.57 to 0.68 across time points between the PPS and the GMADE tests and from 0.26 to 0.31 across time points between the PPS and the MCA tests.

Table 3 provides descriptive statistics for the outcome measures by cohort. Differences between the Cohort groups on the PPS and GMADE pretests were $d = 0.14$ *SD* and $d = 0.19$, are statistically significant at $\alpha = .05$ and favor students in Cohort 1 classrooms. It is not clear what the sources of these differences are, so we included these variables as covariates in the multilevel analyses to statistically adjust the outcomes for these differences and to establish baseline equivalence (see WWC, 2014).

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3.1. *Effects of PD*

To answer the first research question two-sample *t*-tests were conducted to test cohort differences on the fidelity of implementation data. The Dunn-Bonferroni correction was used to control for compounding of Type 1 error by dividing an overall error rate ($\alpha = .15$) among all statistical tests linked to each outcome variable. The mean total score across the seven items on the fidelity checklist was 14.06 ($SD = 3.05$) for Cohort 1 and 14.45 ($SD = 3.26$) for Cohort 2 out of a possible 21 points (higher scores are consistent with greater fidelity). Results indicated no statistically significant differences between cohorts on the total score and all individual items (Table 4).

3.2. *Cohort Effects*

Tables 5-8 report the results of fitting a multilevel model to the PPS posttest, PPS delayed posttest, GMADE, and MCA-III data, respectively that speak to the second research question. Results indicated that Cohort was not a significant predictor of PPS posttest scores, suggesting that students on average performed the same on the outcome regardless of whether their teacher was part of Cohort 1 or Cohort 2. The only statistically significant predictor among the student covariates was PPS pretest, with unit increases in this variable associated with an expected increase in PPS posttest of .73 (standardized effect = .06 *SDs*). The same pattern emerged for the PPS delayed posttest with the only significant predictor being the PPS pretest (.76, .65 *SDs*).

Multilevel results for the GMADE posttest show that there were only two significant predictors. One indicated whether a student was Black (-1.05, -.06 *SDs*) meaning that, with other predictors held constant, Black students tended to score 1.05 points lower on the GMADE posttest than White students. The significant pretest predictor (.62, .56 *SDs*) indicates that each

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one unit increase on the GMADE pretest was associated with a .62 expected increase on the GMADE posttest.

Multilevel analyses of the MCA outcome also indicated that the only significant predictor was a student's 6th grade MCA mathematics score (.81, .89 *SDs*), meaning that each one unit increase in MCA 6th grade mathematics scores was associated with a .81 point expected increase in posttest scores.

3.3. Student and Teacher Acceptability of SBI

To answer our third research question, students' and teachers' attitudes regarding SBI were compared (see Appendix B for a table of mean scores and standard deviations for student and teacher satisfaction data). The analyses (two-sample *t*-tests) of the TARF-R identified that student attitudes regarding SBI were significantly different ($t = 4.01$, $df = 1791$, $p < .001$) across cohorts. On a scale of 1 to 4, students in Cohort 1 classrooms had a mean score of 2.92 ($SD = 0.58$) and students in Cohort 2 classrooms had a mean score of 2.81 ($SD = 0.61$).

With regard to teachers' attitudes, results were not significantly different ($t = .043$, $df = 71$, $p > .05$). On a scale of 1 to 4, Cohort 1 teachers had a mean score of 3.30 ($SD = 0.46$) and Cohort 2 teachers had a mean score of 3.29 ($SD = 0.41$). Although Cohort 1 and Cohort 2 did not differ statistically in their attitudes toward SBI, descriptive evidence suggested that Cohort 2 teachers were much more positive in their comments about the training, SBI program materials and intervention features than Cohort 1 teachers. Many Cohort 2 teachers commented that they liked the training and resources and that the SBI program was fun and effective in developing strong reasoning skills. These teachers especially liked the diagrams as they were helpful in organizing and explaining ideas. Also, they noted that the mathematics vocabulary (e.g., ratios as part-to-part and part-to-whole) as well as the problem solving procedure (i.e., identify the

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problem type, using an appropriate diagram to represent the problem, solving, and checking the answer) and strategies (e.g., estimation) were useful and they would use them in their future teaching.

Other features Cohort 2 teachers liked about the SBI program were the problem solving tasks, which were related to things that middle school students are interested in and the review lesson using a Jeopardy game format, wherein students read problems in different categories (e.g., sports, entertainment) and identified the problem type. In contrast, Cohort 1 teachers noted that they liked the SBI program because it focused on the quality of the problem over quantity. They found the percent of change diagram and the focus on multiple solution strategies were effective for most students. Furthermore, they noted that special education students struggled less with the SBI program as compared to their district adopted text.

4. Discussion

In the wake of the CCSS, fewer topics are explored and studied in depth at each grade level, such that in Grade 7, for example, proportionality is one topic that has merited focused instructional time. As school districts document whether students meet the new standards and monitor achievement trends, they have focused efforts on supporting teacher learning through selection of textbooks and associated instructional practices. Research underscores the complexities of implementing newly designed curriculum materials that support new standards (e.g., Obara & Sloan, 2009; Remillard, 2005). However, whether participating in a PD program and experience in implementing innovative practices influences student learning has not been empirically validated.

In this study, we examined the relative effects of SBI implementation by experienced- and novice-teacher implementers. With regard to the first research question about the effects of

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the PD training provided in this study for teachers who have prior experience with SBI vs. teachers with no prior experience with SBI on their classroom instructional practice (i.e., fidelity of treatment implementation), there was evidence of fidelity of implementation for both cohorts. This finding suggests that even though the amount of PD was significantly less than that recommended (30 to 100 hours) in the PD research (Blank & de las Alas, 2009; Yoon et al., 2007), the quality of the PD program may have had a substantial, positive influence on teachers' implementation of the PD practices in the classroom (see Guskey & Yoon, 2009). We designed the professional development to incorporate important design elements such as (a) aligning what teachers learned in professional development with what occurred in the classroom (Timperley, 2008), (b) focusing on subject matter knowledge of proportional reasoning that students are expected to learn (Garet et al., 2001), (c) promoting and extending research-based instructional materials (Hiebert & Grouws, 2007), and (d) emphasizing personal learning and reflectiveness (Guskey, 1997). Our study suggests that teachers can use well-designed curriculum materials, along with guidance and structure during PD on how to use those materials, to positively influence their instructional practice (Perry & Lewis, 2011; Remillard, 2005). We created materials (e.g., diagrams to highlight the mathematical structure of problems) to explicitly support teaching and learning, which may explain the better curriculum implementation by both groups (Cohort 1 and Cohort 2 teachers).

Regarding our second research question, namely, whether students of teachers who participated in the PD training and had prior experience with SBI demonstrated higher levels of proportional problem solving performance on an immediate posttest and on a retention test (10 weeks later) than students of teachers who participated in the PD program and had no prior experience with SBI, we found no evidence of differences in these outcomes between SBI

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experienced- or novice-teacher implementers classrooms. We also examined whether students of teachers who participated in the PD training and had prior experience with SBI demonstrated higher levels of overall mathematical problem solving performance and mathematics achievement as measured by standardized assessments than students of teachers who participated in the PD program and had no prior experience with SBI. Again, the results indicated no evidence to support advantaged outcomes for students in either experienced- or novice-teacher implementers' classrooms. However, a particularly noteworthy result from the multilevel analyses is that the concentration of special education students in a classroom was not a significant predictor of any outcome. This provides evidence that SBI has closed the gap between students with and without disabilities. Given the previous contributions of SBI in special education (e.g., Jitendra, DiPipi, & Perron-Jones, 2002; Xin, Jitendra, & Deatline-Buchman, 2005), our results are encouraging in that the benefits extend to improving the learning of students with learning disabilities in inclusive classrooms.

In sum, cohort was not a significant predictor of any of the outcomes, suggesting that student learning in proportional problem solving was on average the same regardless of whether a teacher was an SBI-experienced or SBI-novice implementer. Specifically, these findings suggest that teachers implementing SBI in their second year of the study did so in a fashion that did not enhance student learning of proportional problem solving relative to that of students in SBI-novice classrooms. On the other hand, these results may suggest the “holding power” of SBI in the sense that student learning did not drop off despite the SBI-experienced cohort not receiving any additional professional development in the second year. The results also suggest that the more proximal professional development experience of SBI novice teachers did not translate to greater student learning.

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In short, there was minimal variation in student gains from differences in teacher experience with SBI. We believe these central findings are important and extend previous research on the effects of PD and SBI (Jitendra et al., 2009, 2011, 2013, 2015). Unlike teachers in Fennema et al. (1996) who gained expertise as a result of participating in the 4-year longitudinal study, Cohort 1 teachers in the present study may not have gained the expertise with SBI to result in better implementation and student gains compared to Cohort 2 teachers. Alternately, teaching mathematics teachers (Cohort 1 and 2) how to facilitate active learning using instructional methods (e.g., SBI) that allow students to engage in problem solving in-depth and for longer periods of time (as in this study, which emphasized quality of problem solving over quantity), and to demonstrate understanding and explore multiple solution strategies translated to student proficiency in mathematics. Such practices align with quality PD that provides “explicit guidance or instruction in the models of teaching specified within curriculum materials” (Penuel, Gallagher, & Moorthy, 2011, p. 1000). Our results highlight the importance of quality professional development and carefully designed curriculum materials to support better curriculum implementation by both SBI- experienced and novice teacher implementers (e.g., Perry & Lewis, 2011) and similar effects on their students.

Regarding our third research question about whether teachers and students who participated in the study reported higher levels of satisfaction with the treatment, we found no evidence of differences in the outcome for Cohort 1 and Cohort 2 teachers. One possible explanation for this finding is that the high levels of satisfaction we found based on the PD also leads to increased instructional mastery, which supports the finding of Sample McMeeking et al. (2012). However, the finding that students in Cohort 1 classrooms reported higher levels of satisfaction with the intervention than students in Cohort 2 classrooms suggests that teachers

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who had experience with SBI may have a greater buy-in with SBI and motivated their students such that teacher experience with SBI does make a difference in enhancing student attitudes.

4.1. Conclusions

Our study is an attempt to contribute to the knowledge about what type of PD and how much experience with SBI is necessary for teachers to implement it with fidelity. Furthermore, this study deals with the importance of considering best practices such as using visual representations to highlight the underlying problem structure, problem solving and metacognition strategies, and developing procedural flexibility when solving proportion problems involving ratio, proportion, and percent. Such practices can help teachers all over the world to learn more about effectively supporting their students learning of ratios and proportional relationships. In this regard, we think the current study could possibly add three contributions to the extant knowledge base.

The first contribution of our work is the extension of previous research involving characterizations of PD mathematics programs that affect teacher practice and student achievement (e.g., Carpenter et al., 1989; Perry & Lewis, 2011; Sample McMeeking et al., 2012; Saxe et al., 2001). Our study suggests that our approach to integrate new knowledge for teachers around content and pedagogy (teaching a wide range of students) focused on proportional reasoning, along with time to work with other colleagues during PD in meaningful, guided ways might have provided an effective professional development experience to positively influence teacher practices as in previous studies (Carpenter et al., 1989; Perry & Lewis, 2011). Unlike the Carpenter et al. (1989) and Perry and Lewis (2011) studies that provided content-focused PD of an extended duration, the short PD (16 hr) in our study might not be a concern given that it

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focused on using a well specified intervention (less open-ended than reform-based strategies) and related set of curriculum materials for teaching a specific topic (proportional reasoning).

Based on the results reported in this study, the second contribution is the extension of previous research involving SBI. Our study suggests that the SBI intervention found to be effective in enhancing student proportional problem solving performance (see Jitendra et al., 2009, 2011, 2013, 2015) was also effective for students when teachers with different initial levels of experience participated in organized PD (see Sample McMeeking et al., 2012). Taking this into account, it seems clear that teacher-educators should support teachers by organizing content-focused PD on how to implement SBI that reflect opportunities to engage in problem solving at a deep level to promote understanding and differentiate instruction (i.e., a combination of explicit and open ended problem solving strategies) to support a broad range of learners (National Mathematics Advisory Panel, 2008).

A third contribution of our study is the international relevance of our findings. International assessments of mathematics (e.g., TIMSS) have shown that on average students scored lower on more complex proportional problems (e.g., percent of change) than routine proportional problems. Considering that the mathematical competence of U.S. students is below the international average and the share of low achievers is at similar levels as students in several other countries (e.g., Lithuania, Slovak Republic, Italy, Luxembourg, see OECD, 2014), it might be that students in these countries would also benefit from effective instructional practices focused on critical content area (e.g., ratios and proportional relationships). Further research should be done to address whether the benefits of SBI would hold in other countries.

Although this study can be considered another step toward understanding more about what is essential to successfully implement SBI (type and amount of PD, necessary expertise

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with SBI) to affect student achievement, our results have to be interpreted carefully given the limitations of the study design and the need for future research to address questions arising from our work. This study was designed as the second year of a 2-year efficacy and sustainability study in which treatment teachers who participated during Year 1 continued in the study as experienced SBI implementers for Year 2, and control teachers in Year 1 received professional development for the first time and participated as novice SBI implementers in Year 2. The combination of teacher reassignment and teacher attrition produced a nonequivalent, quasi-experimental, cluster design with repeated measures, which implies that strong causal inferences are not warranted. Although conducting such longitudinal studies is difficult due to yearly changes in assignment of school personnel, we first need to replicate this SBI experience and the possible impact of a novelty effect in future research and include a group that receives the SBI materials but is not provided with PD.

Second, it is important for future research to examine the sustainability effect of SBI when teachers implement SBI for longer than 2 years as in this study. A study in which teacher participants are randomly assigned to implement SBI for different durations (e.g., 2 years vs. 4 years) is needed to examine whether there is a sustainability effect favoring longer duration of SBI implementation. Furthermore, this study is limited by the absence of a no-treatment comparison group in the design. However, recall that results of previous SBI studies provide evidence supporting the efficacy of SBI. Last, it is unclear whether the Year 2 effects would hold in a different geographic location. Our future work will investigate the Year 1 and Year 2 effects with a new cohort of teachers and students in schools within the U.S. in urban districts that include substantial diversity in the student population.

Third, given that our focus on procedural flexibility is limited to task characteristics,

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future research should also address subject or context characteristics (see Torbeyns & Verschaffel, 2015). At the same time, we think it would be necessary to analyze whether the types of strategies students used to solve proportional word problems would be effective in assessing students' thinking, especially given that students in Cohort 1 classrooms reported higher levels of satisfaction with SBI than students in Cohort 2 classrooms. In order to understand to what extent our results could be due to the outcomes (correct answer to the problem) we used, further research that examines student outcomes in terms of strategy choices and use would be necessary. For example, a random sample of students in both cohorts could be individually interviewed as they solve some problems (a limitation of this study) to assess their strategy choice based on their individual mastery of different strategies as in Torbeyns and Verschaffel.

Fourth, it would be interesting to explore whether the results obtained may be affected by variables such as the topics being studied. Future research might evaluate to what extent teachers can take the SBI approach, following PD in proportion content, and create their own practices to teach students to identify and solve other problem types (e.g., algebra and statistics).

In sum, this study extends the literature on schema-based instruction and provides evidence that teachers with limited exposure to, and experience with, SBI can implement it with the same level of fidelity as SBI-experienced teacher implementers, and that students of SBI-novice teacher implementers perform as well as students of SBI-experienced teacher implementers. It suggests that when provided with guidance and structure, teachers can use well-designed instructional materials to strengthen student learning.

References

- Adjage, R., & Pluvinage, F. (2007). An experiment in teaching ratio and proportion. *Educational Studies in Mathematics*, 65, 149–175. doi: [10.1007/s10649-006-9049-x](https://doi.org/10.1007/s10649-006-9049-x)
- Ben-Chaim, D., Fey, J. T., Fitzgerald, W. M., Benedetto, C., & Miller, J. (1998). Proportional reasoning among 7th grade students with different curricular experiences. *Educational Studies in Mathematics*, 36, 247–273.
- Bennie, K. & Newstead, K. (1999). Obstacles to implementing a new curriculum. In M. J. Smit & A. S. Jordaan (Eds.), *Proceedings of the National Subject Didactics Symposium* (pp. 150-157). Stellenbosch: University of Stellenbosch.
- Blank, R.K., & de las Alas, N. (2009). *Effects of teacher professional development on gains in student achievement: How meta-analysis provides scientific evidence useful to education leaders*. Washington, DC: Council of Chief State School Officers.
- Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, 33, 3–15. doi: 10.3102/0013189X033008003
- Carpenter, T., Fennema, E., Peterson, P., Chiang, C-P, & Loeff, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal*, 26, 499–531.
- Chen, Z. (1999). Schema induction in children's analogical problem solving. *Journal of Educational Psychology*, 91, 703–715. doi:[10.1037/0022-0663.91.4.703](https://doi.org/10.1037/0022-0663.91.4.703)
- Cohen, D. K., & Hill, H. C. (2000). *State policy and classroom performance: The mathematics reform in California*. Paper presented at the annual meeting of the National Council of Teachers of Mathematics, Washington, DC.

PROPORTIONAL PROBLEM SOLVING

- Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, 38, 181–199. doi:10.3102/0013189X08331140
- Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. *Educational Evaluation and Policy Analysis*, 24(2), 81–112.
- Dimitrov, D. M. (2003). Marginal true-score measures and reliability for binary items as a function of their IRT parameters. *Applied Psychological Measurement*, 27, 440–458.
- Dunn, T. J., Baguley, T., & Brunsdon, V. (2013). From alpha to omega: A practical solution to the pervasive problem of internal consistency estimation. *British Journal of Psychology*, 105, 399–412. doi: [10.1111/bjop.12046](https://doi.org/10.1111/bjop.12046)
- Fennema, E., Carpenter, T. P., Franke, M. L., Levi, L., Jacobs, V. R., & Empson, S. B. (1996). *Journal for Research in Mathematics Education*, 27, 403–434. doi: 10.2307/749875
- Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., & Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education*, 27, 421–438. doi:[10.1007/s10212-011-0087-0](https://doi.org/10.1007/s10212-011-0087-0)
- Fujimura, N. (2001). Facilitating children's proportional reasoning: A model of reasoning processes and effects of intervention on strategy change. *Journal of Educational Psychology*, 93, 589–603. doi: [10.1037//0022-0663.93.3.589](https://doi.org/10.1037//0022-0663.93.3.589)
- Garet, M., Porter, A., Desimone, L., Birman, B., & Yoon, K. S. (2001). What makes professional development effective? Results from a national sample of teachers. *American Educational Research Journal*, 38, 915–945. doi:10.3102/00028312038004915

PROPORTIONAL PROBLEM SOLVING

- Gersten, R., Taylor, M. J., Keys, T. D., Rolhus, E., & Newman-Gonchar, R. (2014). *Summary of research on the effectiveness of math professional development approaches*. (REL 2014–010). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southeast. Retrieved from <http://ies.ed.gov/ncee/edlabs>.
- Guskey, T. R. (1997). Research needs to link professional development and student learning. *Journal of Staff Development*, 18(2), 36–40.
- Guskey, T. R., & Yoon, K. S. (2009). What works in professional development?. *Phi Delta Kappan*, 90, 495-500.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *The second handbook of research in mathematics education*. Reston, VA: New Age and National Council of Teachers of Mathematics.
- Jitendra, A. K., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of word problem-solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. *Journal of Special Education*, 36, 23-38.
- Jitendra, A. K., Harwell, M. R., Dupuis, D. N., & Karl, S. R. (in press). A randomized trial of the effects of schema-based instruction on proportional problem solving for students with mathematics problem-solving difficulties. *Journal of Learning Disabilities*. Advanced online publication doi: 10.1177/0022219416629646
- Jitendra, A. K., Harwell, M. R., Dupuis, D. N., Karl, S. R., Lein, A. E., Simonson, G. & Slater, S. C. (2015). Effects of a research-based mathematics intervention to improve seventh-

PROPORTIONAL PROBLEM SOLVING

- grade students' proportional problem solving: A cluster randomized trial. *Journal of Educational Psychology*, *107*, 1019-1034. doi: 10.1037/edu0000039
- Jitendra, A. K., Star, J. R., Dupuis, D. N., & Rodriguez, M. (2013). Effectiveness of schema-based instruction for improving seventh-grade students' proportional reasoning: A randomized experiment. *Journal of Research on Educational Effectiveness*, *6*, 114-136. doi: 10.1080/19345747.2012.725804
- Jitendra, A. K., Star, J. R., Rodriguez, M., Lindell, M., & Someki, F. (2011). Improving students' proportional thinking using schema-based instruction. *Learning and Instruction*, *21*, 731-745. doi: 10.1016/j.learninstruc.2011.04.002
- Jitendra, A. K., Star, J., Starosta, K., Leh, J., Sood, S., Caskie, G., Hughes, C., & Mack, T. (2009). Improving students' learning of ratio and proportion problem solving: The role of schema-based instruction. *Contemporary Educational Psychology*, *34*, 250-264. doi: 10.1016/j.cedpsych.2009.06.001
- Kalyuga, S. (2006). Rapid cognitive assessment of learners' knowledge structures. *Learning and Instruction*, *16*, 1-11. doi:10.1016/j.learninstruc.2005.12.002
- Kramarski, B., & Mevarech, Z. R. (2003). Enhancing mathematical reasoning in the classroom: The effects of cooperative learning and metacognitive training. *American Educational Research Journal*, *40*, 281-310. doi: [10.3102/00028312040001281](https://doi.org/10.3102/00028312040001281)
- Lamon, S. J. (1995). Ratio and proportion: Elementary didactical phenomenology. In J. T. Sowder & B. P. Schappelle (Eds.), *Providing a foundation for teaching mathematics in the middle grades* (pp. 167-198). Albany, NY: State University of New York Press.
- Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research. In F. K. Lester (Ed.), *Second handbook of research on*

PROPORTIONAL PROBLEM SOLVING

- mathematics teaching and learning* (pp. 629–667). Charlotte, NC: Information Age Publishing.
- Lobato, J., Ellis, A. B., Charles, R. I. & Zbiek, R. M. (2010). *Developing essential understanding of ratios, proportions & proportional reasoning*. Reston, VA: National Council of Teachers of Mathematics.
- Marshall, S. (1990). The assessment of schema knowledge for arithmetic story problems: A cognitive science perspective. In G. Kulm (Ed.), *Assessing higher order thinking in mathematics*. Washington, DC: American Association for the Advancement of Science.
- Mayer, R. E. (1999). *The promise of educational psychology Vol. I: Learning in the content areas*. Upper Saddle River, NJ: Merrill Prentice Hall.
- McDonald, R. P. (1999). Test theory: A unified treatment. Mahwah, NJ: Lawrence Erlbaum Associates. doi:10.1111/j.2044-8317.1981.tb00621.x
- Mevarech, Z. R., & Kramarski, B. (2003). The effects of metacognitive training versus worked-out examples on students' mathematical reasoning. *British Journal of Educational Psychology*, 73, 449–471. doi: [10.1348/000709903322591181](https://doi.org/10.1348/000709903322591181)
- Minnesota Department of Education (2014). Yearbook tables for Minnesota's title I and title III assessments. Retrieved from <http://education.state.mn.us>
- Miyakawa, T., & Winslow, C. (2009). Didactical designs for students' proportional reasoning: An open approach lesson and a fundamental situation. *Educational Studies in Mathematics*, 72, 199–218. doi: [10.1007/s10649-009-9188-y](https://doi.org/10.1007/s10649-009-9188-y)
- Meyer, J. P. (2011). jMetrik Version 2.1.0 [Computer software]. Retrieved from <http://itemanalysis.com>
- National Governors Association Center for Best Practices & Council of Chief State School

PROPORTIONAL PROBLEM SOLVING

- Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Authors.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, DC: U.S. Department of Education.
- OECD. (2014). *PISA 2012 results in focus: what 15-year-olds know and what they can do with what they know*. <http://www.oecd.org/pisa/keyfindings/pisa-2012-results.htm>
- Obara S, & Sloan M. (2010). Classroom Experiences with new curriculum materials during the implementation of performance standards in mathematics: A case study of teachers coping with change. *International Journal of Science and Mathematics Education [serial online]*, 8, 349–372. doi.10.1007/s10763-009-9176-9
- Pearson (2004). *Group Mathematics Assessment and Diagnostic Evaluation [GMADE] (Level M, Form A)*. San Antonio, TX: Author.
- Penuel, W. R., Fishman, B. J., Yamaguchi, R., & Gallagher, L. P. (2007). What makes professional development effective? Strategies that foster curriculum implementation. *American Educational Research Journal*, 44, 921–958. doi: 10.3102/0002831207308221.
- Penuel, W. R., Gallagher, L. P., & Moorthy, M. (2011). Preparing teachers to design sequences of instruction in earth science: A comparison of three professional development programs. *American Educational Research Journal*, 48, 996–1025.
- Penuel, W. R., Sun, M., Frank, K. A., & Gallagher, H. A. (2012). Using social network analysis to study how collegial interactions can augment teacher learning from external professional development. *American Journal of Education*, 119, 103–136.
- Perry, R. R., & Lewis, C. C. (2011). *Improving the mathematical content base of lesson study summary of results*. Retrieved December 13, 2015, from

PROPORTIONAL PROBLEM SOLVING

<http://www.lessonresearch.net/IESAbstract10.pdf>

- Quilici, J. L., & Mayer, R. E. (1996). Role of examples in how students learn to categorize statistics word problems. *Journal of Educational Psychology, 88*, 144–161. doi:[10.1037/0022-0663.88.1.144](https://doi.org/10.1037/0022-0663.88.1.144)
- Raudenbush, S. W., Bryk, A. S., & Congdon, R. (2004). HLM 6.02: Hierarchical linear and nonlinear modeling [Computer software]. Lincolnwood, IL: Scientific Software International.
- Reimers, T.M., & Wacker, D.P. (1988). Parents' ratings of behavioral treatment recommendations made in an outpatient clinic: A preliminary analysis of the influence of treatment effectiveness. *Behavior Disorders, 14*, 7–15.
- Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*, 211–246.
- Revelle, W., & Zinbarg, R. E. (2009). Coefficients alpha, beta, omega, and the glb: Comments on Sijtsma. *Psychometrika, 74*, 145–154. doi:[10.1007/s11336-008-9102-z](https://doi.org/10.1007/s11336-008-9102-z)
- Sample McMeeking, L., Orsi, R., & Cobb, R. B. (2012). Effects of a teacher professional development program on the mathematics achievement of middle school students. *Journal for Research in Mathematics Education, 43*, 159–181.
- Saxe, G. B., Gearhart, M., & Nasir, N. S. (2001). Enhancing students' understanding of mathematics: A study of three contrasting approaches to professional support. *Journal of Mathematics Teacher Education, 4*, 55–79.
- Star, J. R., & Rittle-Johnson, B. (2009). It pays to compare: An experimental study on computational estimation. *Journal of Experimental Child Psychology, 102*, 408–426. doi:[10.1016/j.jecp.2008.11.004](https://doi.org/10.1016/j.jecp.2008.11.004)

PROPORTIONAL PROBLEM SOLVING

- Torbeyns, J., & Verschaffel, L. (2015). Mental computation on standard algorithm? Children's strategy choices on multi-digit subtractions. *European Journal of Psychological Education*, Advanced online publication. doi:10.1007/s10212-015-0255-8
- Timperley, H. (2008). *Teacher professional learning and development*. Brussels, Belgium: International Academy of Education.
- Van Dooren, W., De Bock, D., Hessels, A., Janssens, D., & Verschaffel, L. (2005). Not everything is proportional: Effects of age and problem type on propensities for overgeneralization. *Cognition and Instruction*, 23, 57–86. doi:[10.1207/s1532690xci2301_3](https://doi.org/10.1207/s1532690xci2301_3)
- Van Dooren, W., De Bock, D., Vleugels, K., & Verschaffel, L. (2010). Just answering... or thinking? Contrasting pupils' solutions and classifications of missing-value word problems. *Mathematical Thinking and Learning: An International Journal*, 12, 20–35. doi:[10.1080/10986060903465806](https://doi.org/10.1080/10986060903465806)
- What Works Clearinghouse. (2014). WWC Procedures and Standards Handbook (version 3.0). Retrieved May 27, 2014 from http://ies.ed.gov/ncee/wwc/pdf/reference_resources/wwc_procedures_v2_1_standards_handbook.pdf
- Weinberg, S. L. (2002). Proportional reasoning: One problem, many solutions! In B. Litwiller (Ed.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 138–144). Reston, VA: National Council of Teachers of Mathematics.
- Woodward, J., Beckmann, S., Driscoll, M., Franke, M., Herzig, P., Jitendra, A. K., Koedinger, K. R., & Ogbuehi, P. (2012). *Improving mathematical problem solving in grades 4 through 8: A practice guide* (NCEE 2012–4055). Washington, DC: National Center for Education

PROPORTIONAL PROBLEM SOLVING

Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from <http://ies.ed.gov/ncee/wwc/PracticeGuide.aspx?sid=16>

Xin, Y. P., Jitendra, A. K., & Deatline-Buchman, A. (2005). Effects of mathematical word problem solving instruction on students with learning problems. *Journal of Special Education, 39*, 181–192.

Yoon, K. S., Duncan, T., Lee, S. W. Y., Scarloss, B., & Shapley, K. (2007). *Reviewing the evidence on how teacher professional development affects student achievement* (Issues & Answers Report, REL 2007–No. 033). Washington, DC: U.S. Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southwest. Retrieved from <http://ies.ed.gov/ncee/edlabs>

PROPORTIONAL PROBLEM SOLVING

Table 1

Participant Demographic Information by Cohort (SBI-Experienced, SBI-Novice)

		SBI – Experienced		SBI – Novice	
		<i>n</i>	%	<i>n</i>	%
<i>Teacher Information</i>					
Sex	Female	18	58.06	28	66.67
	Male	13	41.94	14	33.33
Race	Asian	1	3.23	1	2.38
	Am. Indian	1	3.23	0	0.00
	Hispanic	0	0.00	4	9.52
	White	29	93.55	37	88.10
Location	Rural	19	61.29	23	54.76
	Suburban	9	29.03	14	33.33
	Urban	3	9.68	5	11.90
Experience ^a	<i>M (SD)</i>	12.26	(6.51)	12.09	(6.96)
Methods Courses	<i>M (SD)</i>	4.32	(5.27)	2.86	(3.01)
Math courses	<i>M (SD)</i>	8.68	(3.86)	8.43	(5.06)
<i>Student Information</i>					
Age in years	<i>M (SD)</i>	12.54	(0.42)	12.59	(0.36)
Sex	Female	420	50.36	488	48.27
	Male	401	48.08	510	50.45
Race	Asian	42	5.04	71	7.02
	Black	69	8.27	73	7.22
	Hispanic	62	7.43	87	8.61
	White	648	77.70	767	75.87
FRL	Yes	300	35.97	433	42.83
	No	521	62.47	565	55.89
ELL	Yes	45	5.40	65	6.43
	No	776	93.05	933	92.28
SpEd	Yes	91	10.91	99	9.79
	No	730	87.53	899	88.92
Location	Rural	482	57.79	473	46.79
	Suburban	257	30.82	385	38.08
	Urban	95	11.39	153	15.13
Missing		13	1.56	13	1.29

Note. Teacher data are based on the number of teachers who responded. ^a = years experience teaching math; SBI = schema-based instruction; FRL = students eligible for free or reduced priced lunch; ELL = English language learner; SpEd = students receiving special education services; Student *n* = 1,845 but the total number of students varies slightly across variables because of missing data (all variables had missing data except Location).

PROPORTIONAL PROBLEM SOLVING

Table 2

Intercorrelations Among Student Variables

Variables	1	2	3	4	5	6	7
1. PPS Pretest	–						
2. PPS Posttest	.671	–					
3. PPS Delayed Posttest	.681	.778	–				
4. GMADE pretest	.598	.573	.585	–			
5. GMADE posttest	.596	.680	.667	.627	–		
6. 6 th Grade Math MCA	.293	.299	.305	.312	.286	–	
7. 7 th Grade Math MCA	.264	.306	.315	.290	.278	.940	–

Note. PPS = proportional problem solving; GMADE = Group Mathematics Assessment and Diagnostic Evaluation; MCA = Minnesota Comprehensive Assessments. All correlations significant, $p < .001$.

PROPORTIONAL PROBLEM SOLVING

Table 3

Descriptive Statistics for Student Measures by Cohort (SBI-Experienced, SBI-Novice)

	PPS pretest	PPS posttest	PPS delayed	GMADE pretest	GMADE posttest	6 th grade math MCA	7 th grade math MCA
SBI – Experienced							
<i>M</i>	14.45	18.32	18.07	14.79	16.57	644.31	744.84
<i>SD</i>	4.67	5.27	5.36	4.08	4.39	20.66	19.76
<i>N</i>	822	815	811	821	816	777	813
<i>Missing n</i>	12	19	23	13	18	57	21
SBI – Novice							
<i>M</i>	13.80	17.93	17.61	14.01	15.84	647.84	747.64
<i>SD</i>	4.68	5.67	5.61	4.12	4.67	15.55	12.9
<i>n</i>	994	986	954	990	973	869	895
<i>Missing n</i>	17	25	57	21	38	142	116
Total							
<i>M</i>	14.09	18.10	17.82	14.36	16.17	646.17	746.31
<i>SD</i>	4.69	5.49	5.50	4.12	4.56	18.22	16.58
<i>n</i>	1816	1801	1765	1811	1789	1646	1708
<i>Missing n</i>	29	44	80	34	56	199	137

Note. SBI = schema-based instruction; PPS = proportional problem solving; GMADE = Group Mathematics Assessment and Diagnostic Evaluation; MCA = Minnesota Comprehensive Assessments. All test statistics are based on the total number of items correct, except the MCA Mathematics test, which is a scaled score. Student $n = 1,845$ but sample size varies across variables because of missing data.

PROPORTIONAL PROBLEM SOLVING

Table 4

Fidelity of Implementation by Cohort (SBI-Experienced, SBI-Novice)

Measure	SBI – Experienced			SBI – Novice			<i>t</i>	<i>df</i>	<i>p</i>
	<i>n</i>	<i>M</i>	<i>SD</i>	<i>n</i>	<i>M</i>	<i>SD</i>			
<i>Fidelity of Implementation</i>									
Identifies problem type	31	2.03	0.98	42	2.05	0.85	-0.07	71	.943
Problem similar/different	31	1.23	1.09	42	0.98	1.18	0.92	71	.359
Represents key information	31	2.55	0.51	42	2.55	0.50	0.01	71	.995
Estimates solution	31	1.97	1.20	42	2.05	1.29	-0.27	71	.788
Multiple solution strategies	31	2.10	0.83	42	2.40	0.80	-1.60	71	.114
Provides complete solution	31	2.68	0.75	42	2.57	0.59	0.68	71	.501
Checks solution	31	1.52	0.85	42	1.86	0.90	-1.64	71	.106
Total score	31	14.06	3.05	42	14.45	3.26	-0.52	71	.607

Note. SBI = schema-based instruction. The means of the two cohorts on each variable were compared using a two-sample *t*-test none of which were significant using $\alpha = \frac{.15}{8} = .0188$ following the Dunn-Bonferonni procedure. *p* = *p*-value; *n* = number of teachers (classrooms)

PROPORTIONAL PROBLEM SOLVING

Table 5

HLM Results for PPS Posttest

Fixed Effects	<i>B</i>	<i>SE</i>	<i>t</i>	<i>df</i>	<i>p</i>
<i>Between-Student Model</i>					
Intercept	18.61	0.403	46.20	69	<.001
Sex	0.38	0.186	2.07	1745	.038
Asian	0.55	0.426	1.28	1745	.200
Black	-0.89	0.389	-2.28	1745	.023
Hispanic	-0.34	0.364	-0.93	1745	.354
Pretest	0.73	0.022	33.06	1745	<.001
<i>Between-Classroom Model</i>					
Cohort	-0.08	0.372	-0.22	69	.824
SpEd	-0.02	0.167	-0.15	69	.884
FRL	-0.27	0.174	-1.56	69	.124
Random Effects	<i>VC</i>	<i>SD</i>	χ^2	<i>df</i>	<i>p</i>
Classroom	1.71	1.309	276.64	69	<.001
Student	14.76	3.841			

Note. FRL = percentage of students eligible for a free or reduced price lunch expressed in quartiles; SpEd = percentage of students receiving special education services expressed in quartiles; VC = variance component; student $n = 1,754$ nested within 73 teachers/classrooms.

PROPORTIONAL PROBLEM SOLVING

Table 6

HLM Results for PPS Delayed Posttest

Fixed Effects	<i>B</i>	<i>SE</i>	<i>t</i>	<i>df</i>	<i>p</i>
<i>Between-Student Model</i>					
Intercept	18.55	0.361	51.33	68	<.001
Sex	0.20	0.187	1.05	1712	.296
Asian	0.29	0.426	0.68	1712	.495
Black	-0.20	0.401	-0.50	1712	.614
Hispanic	-0.45	0.370	-1.21	1712	.229
Pretest	0.76	0.022	34.58	1712	<.001
<i>Between-Classroom Model</i>					
Cohort	-0.01	0.334	-0.04	68	.966
SpEd	-0.20	0.150	-1.33	68	.189
FRL	-0.31	0.156	-2.00	68	.049
Random Effects	<i>VC</i>	<i>SD</i>	χ^2	<i>df</i>	<i>p</i>
Classroom	1.25	1.116	216.72	68	<.001
Student	14.66	3.829			

Note. FRL = percentage of students eligible for a free or reduced price lunch; SpEd = percentage of students receiving special education services expressed in quartiles; VC = variance component; student $n = 1,721$ nested within 72 classrooms/teachers.

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Table 7

HLM results for GMADE Posttest

Fixed Effects	<i>B</i>	<i>SE</i>	<i>t</i>	<i>df</i>	<i>p</i>
<i>Between-Student Model</i>					
Intercept	16.91	0.344	49.11	69	<.001
Sex	0.29	0.161	1.80	1729	.072
Asian	0.01	0.373	0.03	1729	.979
Black	-1.05	0.338	-3.12	1729	.002
Hispanic	-0.26	0.314	-0.83	1729	.408
Pretest	0.62	0.021	29.04	1729	<.001
<i>Between-Classroom Model</i>					
Cohort	0.15	0.318	0.47	69	.637
SpEd	-0.19	0.142	-1.37	69	.176
FRL	-0.35	0.148	-2.33	69	.023
Random Effects	<i>VC</i>	<i>SD</i>	χ^2	<i>df</i>	<i>p</i>
Classroom	1.24	1.114	266.19	69	<.001
Student	10.98	3.314			

Note. FRL = percentage of students eligible for a free or reduced price lunch expressed in quartiles; SpEd = percentage of students receiving special education services; VC = variance component; student $n = 1,738$ nested within 73 teachers/classrooms.

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Table 8

HLM results for MCA Mathematics

Fixed Effects	<i>B</i>	<i>SE</i>	<i>t</i>	<i>df</i>	<i>p</i>
<i>Between-Student Model</i>					
Intercept	746.78	0.789	946.27	66	<.001
Sex	0.47	0.257	1.83	1613	.066
Asian	1.30	0.601	2.17	1613	.030
Black	-0.29	0.544	-0.54	1613	.591
Hispanic	-0.23	0.519	-0.45	1613	.653
6 th Grade MCA	0.81	0.011	74.91	1613	<.001
<i>Between-Classroom Model</i>					
Cohort	-0.45	0.727	-0.61	66	.542
SpEd	0.56	0.328	1.70	66	.094
FRL	-0.81	0.339	-2.40	66	.019
Random Effects	<i>VC</i>	<i>SD</i>	χ^2	<i>df</i>	<i>p</i>
Classroom	7.53	2.743	638.39	66	<.001
Student	25.93	5.092			

Note. FRL = percentage of eligible for a free or reduced price lunch expressed in quartiles; SpEd = percentage of students receiving special education services expressed in quartiles; VC = variance component; student $n = 1,622$ nested within 70 teachers/classrooms.