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Developing Concepts and Generalizations to Build Algebraic Thinking: The Reversibility, Flexibility, and Generalization Approach

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Abstract

Many students with learning disabilities (LD) in mathematics receive their mathematics education in general education inclusive classes; therefore, these students must be capable of learning algebraic concepts, including developing algebraic thinking abilities, that are part of the general education curriculum. To help students develop algebraic thinking, teachers should ask questions in different ways to promote the ability to think algebraically. This article describes three types of questions—reversibility, flexibility, and generalizations—which support the acquisition of broader concepts leading to algebraic thinking. Examples of the question types within the contexts of rational numbers and integers are provided to assist teachers in creating similar questions for teaching mathematics to students with LD.

Keywords

algebra readiness, generalizations, reversibility, flexibility, learning disabilities, mathematics learning disabilities, mathematics difficulties

Algebra has long been thought of as a gatekeeper in secondary schools (Moses & Cobb 2001) because students' success determines whether or not they are considered "ready" for particular course pathways or careers. Although this applies to all students, those with learning disabilities (LD) are at considerable risk of failing. Considering that many students with LD in mathematics receive their mathematics education in general education inclusive classes; therefore, these students must be capable of learning algebraic concepts, including developing algebraic thinking abilities, that are part of the general education curriculum. Yet, findings from the most recent National Assessment of Education Progress (2013) of mathematics indicated that students with disabilities, including LD, continue to lag significantly behind their typically achieving peers. Thus, this poor performance is cause for determining why students with LD continue to struggle in mathematics.

There are many factors that can contribute to the high failure rate of students with LD in mathematics, but one factor, in particular, is that they may not possess the necessary prerequisite numerical concepts and skills to be successful with more complex mathematics, such as algebra (Bryant, Bryant, & Hammill, 2000; Mazzocco & Devlin, 2008). A concern shared by many educators teaching students with LD is the continual need to reteach mathematical skills (Kortering, deBettencourt, & Braziel, 2005). Because these students do not retain algorithms (step-by-step processes) for long periods of time, teachers may need to not just remind students of steps, but they may need to devote class time to reteaching in a significant way. It is a dilemma for both students and teachers as high-stakes academic outcomes increase and teachers are held responsible for closing gaps that occur as students acquire the skills and concepts indicated by state standards.

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Reteaching claims class time that could be used to develop new knowledge in deeper and significant ways.

Although being able to replicate and perform algorithms for noncontextual mathematics problems, such as 8.75×0.34, in efficient ways is important and needed if students are to be successful in algebra, it is even more critical that students know when to apply these algorithms to application and contextual problems or novel situations. The ability to know how the structure of such problems or situations relates to the appropriate algorithm gives evidence that students have gained an understanding of not only the algorithm but also the actions or relationships that can be associated with them. It shows that students with LD have gone beyond the memorization aspects of algorithms and can use them to solve more complex or sophisticated problems.

Ideally, new skills build from previously learned skills. The ability to make connections between new and previously learned skills helps students notice common characteristics and use those to build more robust understandings. For example, when students begin to work with decimal numbers, their grounding in place value provides a foundation. With regard to place value, by Grade 5, students should have noted that each place-value position to the left of any given number is 10 times larger. Similarly, each place-value position to the right of any given one is 1/10th as large. Thus, the understanding of relationships across the place-value positions of decimal numbers develops from whole-number relationships.

Equally important is the opportunity to teach some topics simultaneously rather than sequentially. For example, in middle grades, it is often the case that learning to solve one-step equations, like $8 + x = 11$, is taught separately from inequalities, such as $8 + x > 11$. Unfortunately, students with LD in mathematics often see no relationship between the two types of problems, even though they are closely related. The relationships within and across problems are “big ideas” that students can use to help them remember skills and generalize or apply the skills in contexts other than the ones in which they were learned.

Seeing relationships across concepts and skills gives students mathematical prowess, the power to attack problems, and the confidence to engage in the solution process in the first place. The purpose of this article is to share ways to build mathematical prowess in ways that prepare students to think algebraically and to focus on big ideas rather than isolated or fragmented skills. In particular, this article focuses on rational-number and integer concepts and skills needed by middle-grades students in order to be successful in algebra.

What Concepts and Skills Are Needed to Be Successful in Algebra?

Although little research is available to guide the identification of prerequisite concepts and skills needed to be successful in algebra (Star & Rittle-Johnson, 2009), educators can heed the recommendations from national efforts to focus on strong algebra readiness. For example, the National Mathematics Advisory Panel (2008) described three areas that it felt were pertinent to success in algebra: (a) fluent computational operations, (b) strong rational-number understandings, and (c) measurement. More specifically, the National Research Council (Kilpatrick, Swafford, & Findell, 2001) noted that operations with rational numbers—including fractions, decimals, and percentages—and integers are two areas in which many students need additional support as they enter high school.

Even though students have much experience with whole-number computations, Stavy and Tirosh (2000) pointed out that success with whole numbers does not imply that students will be successful with other types of numbers. This is particularly true when students begin working with rational numbers and integers in middle grades. Part of the difficulty of these two number systems is that when students have learned strict rules for computing with whole numbers, they are not able to apply those same rules to these number systems. For example, students believe that when adding or multiplying whole numbers, one “make[s] numbers bigger” (Karp, Bush, & Dougherty, 2014, p. 21). However, when one adds two negative numbers, such as $-1 + (-4)$, the sum (-5) is smaller than either of the two addends. Similarly, when one multiplies 2/3 by 3/4, the product (1/2) is smaller than either factor. Either students have created these rules about how numbers interact with each other or they were given the rules as a means of simplifying the mathematics. However, when the rules break down, students suddenly find themselves faced with solutions that do not fit their perception of what the answer should be.

In addition to the numerical aspects, limited algebraic thinking by students with LD is another factor that impedes success (Witzel, Mercer, & Miller, 2003). Development of algebraic thinking in earlier grades is a necessary component of mathematical experiences that would support success in algebra. Algebraic thinking can be considered as the ability to think about underlying mathematical structures (Cai & Knuth 2005) beyond performing only algorithms or computations. More specifically, algebraic thinking is being able to analyze quantitative relationships, generalize, model, justify or prove, predict, problem solve, and notice structure (Kieran, 2004). It is clear that these capabilities are not developed through extensive practice with algorithms. They must be developed through a conscious effort to focus students’ attention on particular problem aspects and elicit their understandings through discourse.

How Can Algebraic Thinking Be Developed and Reteaching Diminished for Students With LD in Mathematics?

Typically, students’ mathematical understandings have been developed through an approach based on example and

Table 1. A Framework of Three Questions to Promote Algebraic Thinking.

Type of Question	Fractions	Integers
Standard type of question	$\frac{1}{2} \times \frac{3}{4}$	−3 + −8
Reversibility question	What are two fractions whose product is $\frac{3}{8}$?	What are two integers whose sum is −11?
Flexibility question	$\frac{1}{2} \times \frac{3}{4}$ $\frac{1}{2} \times \frac{2}{4}$ $\frac{1}{2} \times \frac{1}{4}$ How are these problems alike?	−3 + (−8) −4 + (−8) −5 + (−8) How are these problems alike?
Generalization question	If the factors of a multiplication problem are between 0 and 1, what can you predict about the size of the product?	What are two negative integers whose sum is negative? What are a positive integer and a negative integer whose sum is negative? What are two positive integers whose sum is negative? What do you notice about the integers that you found?

practice. The teacher models how to solve multiple problems with specific algorithms, and students then practice these algorithms. In some cases, the practice is repeated over and over until students have reached a particular level of proficiency. However, the proficiency level observed may be short-lived because retention of isolated and fragmented skills is not robust and students often forget the entire algorithm or specific steps in it (Levav-Waynberg & Leikin, 2012). Even though this method is not highly successful, the use of worksheets to continually practice algorithms is one that is often used with students with LD (Swanson, Solis, Ciullo, & McKenna, 2012).

Worksheets and practice exercises typically focus on the memorized aspects of computations. Asking additional factual questions (What is the product of −2 times −4?), where one word or one number suffices as an answer (or solution), will not support deepening students' thinking. Students with LD need explicit questioning that will help them to focus on critical aspects of a problem or class of problems and make connections across them.

A framework consisting of three types of questions that can support deeper thinking and the development of generalizations is shown in Table 1. These types of questions (Dougherty, 2014; Krutetskii, 1976) motivate discussion in the classroom in ways that the use of only skill-based problems cannot. The three types of questions are reversibility, flexibility, and generalization. A discussion of each follows.

Reversibility questions are questions that change the direction of students' thinking. For many students, they think of mathematics as a series of sequential, linear steps. If the steps are followed, a correct answer is forthcoming. However, when a problem is changed slightly, it is often a teacher's experience that students' hands go up, indicating they are not sure what to do next. Their ability to see a problem from different perspectives is not well developed.

A reversibility question gives students the answer, and they create the problem. If, for example, students are practicing a page (or more) of fraction multiplication problems, the first step to determining a reversibility question is to look at the problems to see what they have in common. If all of the factors in the problems are between 0 and 1, select any fraction from 0 and 1 to call the product. Then, ask students to find one or more examples of a problem like that.

In Table 1, 3/8 was selected as the product. Students are asked to find two fractions whose product is 3/8. When they have been given some time to work, they can share multiple examples of two such fractions. Then, asking what students notice about the fractions leads to even further generalizations based on their examples.

Reversibility questions accomplish two major goals. First, there are an infinite number of solutions to the problem. That means that *all* students have access to the question and can respond. Second, having to construct the problem rather than working through a set of prescribed steps promotes a deeper thinking. Students have to work backward in a sense to create the problem. Questions of this type naturally push the level of thinking much deeper than asking the question in the more expected way, such as asking for the product of two given fractions. In this case, if students consider the inverse relationship between multiplication and division, they determine that they need only to select any fraction, then divide the given product by that fraction to find the other factor. To incorporate reversibility thinking into instruction, the following "generic" questions can be used:

- Give students answers to the types of problems being taught.
- What is/are the problem(s) you can identify for the answer?
- How can you model or show (through manipulatives, pictures, number lines) how to solve the problem to arrive at the given answer?
- Would it help to create a diagram? Make a table? Draw a picture? Make a model?

Flexibility questions are questions that support students' development of multiple ways of finding relationships among problems, their solutions, and solution methods. These questions cause students to expand their repertoire of strategies and broaden their perspectives about particular concepts and skills by seeing connections within and across problem types.

There are two types of flexibility questions. The first type asks students to identify how problems are alike and how they are different. For many students, especially students with LD, when they encounter a computational problem, they immediately begin solving the problem using some algorithm or process without thinking about the problem itself. Their primary goal is to solve the problem. However, when flexibility questions are routinely asked in the class, students begin to look first for a similar problem before starting to solve the problem. For example, in Table 1, students are asked to solve a series of integer addition problems. What they should notice when they finish all three of the problems is that the sum decreases by 1 each time because one of the addends decreased by 1. The relationship among addends and sums is an important one, and this specific generalization (if the addends decrease [increase] by some amount, the sum will decrease [increase] by the same amount) is one that helps students work flexibly with numbers. To incorporate flexible thinking into instruction about how problems are similar and different, the following "generic" questions can be used:

- Have we solved a problem that is similar (or different) to this one?
- How can you use what you know about this problem to solve the new problem?
- How is this solution or strategy method similar to . . . ?

The second type of flexibility questions asks students to solve a problem in multiple ways. This should be construed not as having students use multiple algorithms but rather prompting them to consider multiple alternatives for solving a problem. This gives students ways to access a problem when one algorithm, possibly the standard one, is not accessible to them. When asking students to solve a problem in multiple ways, students can consider options that might involve a manipulative, a diagram, or some other representation as well as other computational methods. Figure 1 shows a problem from a Tier 2 intervention lesson that illustrates how multiple methods can be integrated into instruction. In this case, a fraction problem is presented and students are asked to solve the problem using a method taught in one of the earlier lessons.

An example of a flexibility question is one where the students would first solve a problem like $-3 + (-8)$. Then, the teacher would ask students to find the sum in another way. In this problem, if students know the algorithm, they may use that for the first method. To find the sum another way, students might use a number line; a manipulative, such as two-color chips; or another type of diagram. After students share their strategies or methods, the teacher would have the opportunity to ask them how the methods are alike, how they are different, or how they are related. Focusing students on the structure of the solution methods builds their understanding of how different strategies can be used and creates additional tools for them should they not be able to retrieve the standard algorithm from memory. To incorporate flexible thinking into instruction for solving problems in different ways, the following "generic" questions can be used:

- Can you solve the problem in a different way using models we have worked on in class?
- Does anyone have the same answer but a different way to explain it?

Generalization questions aim to create statements about patterns observed within particular problem classes so that students can use them to predict answers or check the reasonableness of their responses. One can think about answers to generalization questions as formulating and producing statements about patterns and relationships and evaluating their reasonableness. Often, students finish their computations; then, to check their answers, they redo the computations. In some cases, they make the same mistake they made the first time they computed.

Generalization questions focus on specific patterns that are identified in classes of problems, or students are asked to create a specific example of a problem from a

Solving a Problem in Multiple Ways

Lesson Objective: Students compare and order fractions with like and unlike denominators using multiple methods.

Anna's mom told Anna that $\frac{9}{8}$ was less than $\frac{9}{4}$. Anna didn't believe her mom because Anna thought that 8 was greater than 4. Draw a model to show that Anna's mom is correct.

Have students with different models display their model and explain how they created it.

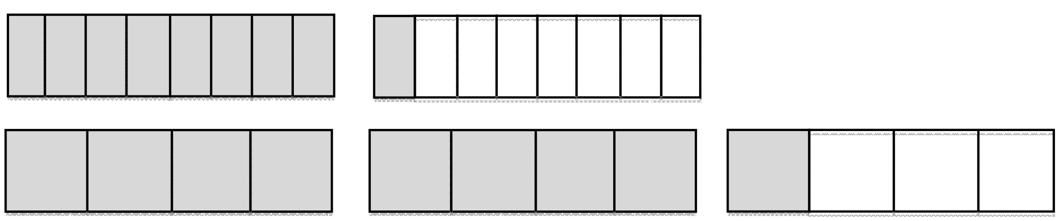
Ask the students:

What is the whole? *(Answer: One unit)*

How did you determine the number of parts to shade when using an area model or where each fraction was located on the number line? *(Answers: Counted the number of unit fractions in each given fraction. For example, count 9 one-eighth pieces or lengths for $\frac{9}{8}$.)*

Possible solutions:

Area Model:



Number Line Model:

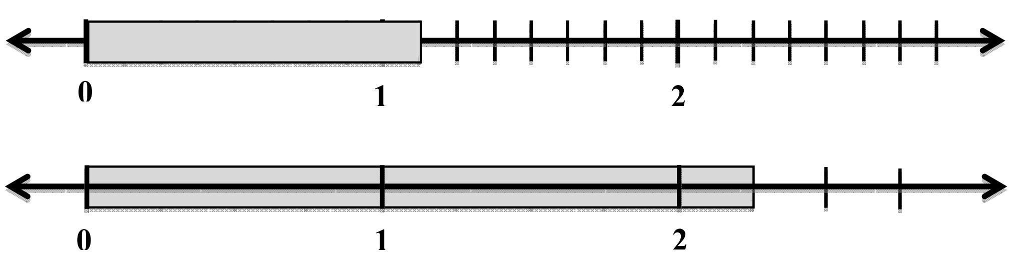


Figure 1. An example of flexibility in solving a problem.

generalization. These questions ask students what they notice, or they may present a series of problems that lead to asking students what they notice. For example, in Table 1, the fraction generalization question would be asked after students have completed practice problems on fraction multiplication. Students would be able to go back to their solutions and consider the size of the products in relation to the factors. What students should notice is that if they are multiplying two fractions between 0 and 1, the product should be between 0 and 1.

Likewise, in Table 1, the integer generalization question leads students to big ideas about integer addition.

They should notice that it is possible to find integers that fit the first two conditions but it is not possible to find two positive integers that sum to a negative number. They should also specifically note that adding any two negative integers would give a negative sum, but that is not the case when one negative integer and one positive integer are the two addends. The sign of the sum is dependent upon which of the two addends has the greater absolute value.

As generalizations are developed, students now have strategies that can be used to predict characteristics of answers before the problem is solved. For example, in the case of the integer generalization, the teacher can ask students to predict whether the sum will be positive or negative before they do the addition. Consistently asking a question like this establishes a routine for students with LD to use the generalizations to consider what type of answer they expect and then check their prediction when they have completed the problem. To incorporate generalization questions into instruction for solving problems, the following "generic" questions can be used:

- What patterns do you see?
- What conjectures can we make about our observation?
- Which of the representations we have used (teachers should name examples of relevant ones they have used) works for this problem?
- What is a strategy that you have learned that can be used to solve the problem?

How Can the Questioning Framework Be Used to Develop Assessment Tasks?

The questioning framework can be used to develop assessment tasks in addition to using them as part of the classroom routine. When questions of this type are integrated into the assessment program, teachers can better determine the depth of students' understandings. Although these types of questions are best answered with expanded, constructed responses, the framework can provide a way to develop multiple-choice items, if such items are necessary components of the assessment system.

Figure 2 gives examples of how multiple-choice items can be constructed using the questioning framework with fraction and integer topics. If reversibility, flexibility, and generalization questions are used routinely in the classroom, using the same framework to construct assessments provides a consistency between classroom instruction and assessment.

Even though items like this raise the level of thinking required by the student, they offer evidence upon which to base instructional decisions. Note that the item stem could be used as a self-constructed response item or an instructional task. By setting it up as a multiple-choice item, the teacher has the opportunity to use significant student misconceptions as the options. Based on student responses, it is possible to determine patterns or trends across students and thus plan instruction to address significant misconceptions that appear.

The three types of questions (i.e., reversibility, flexibility, and generalization) can help students with LD deepen their understanding of computational algorithms and other skills. When these questions become a regular part of the mathematics class routine, responses by students with LD can potentially become stronger and more robust as they are able to anticipate the types of questions that will be asked. In addition, these types of questions may help students with LD become more comfortable attempting new problems and applying these big ideas that stem from the generalizations.

How Are Reversibility, Flexibility, and Generalization Questions Implemented Into Mathematics Instruction?

When reversibility, flexibility, and generalization questions are first introduced in the classroom, teachers should anticipate that students are not going to readily respond. These are questions that are out of typical classroom norms and may be problematic for students with LD whose previous instruction may not have fostered deeper mathematical understanding. To promote the use of these questions, four considerations are offered to teachers who teach mathematics to students with LD (and all students for that matter): getting started, using think-pair-share, creating a safe environment, and incorporating consistency into the mathematics routine.

Getting Started

To begin, the following considerations are offered to incorporate more questioning into mathematics instruction:

- Explicitly teach how to respond to such questions by modeling and using "thinking aloud" to help students better understand how to approach answering these types of questions.
- Give students opportunities to talk with a partner or small group to formulate an answer (see Using Think-Pair-Share below).
- Provide guiding questions, like those presented earlier in this article to prompt thinking.
- Provide prompts for those students who get stuck, such as "What is the first step?" "What model can help you figure out the answer?" "What step of the strategy is causing you difficulty?"

FRACTION ASSESSMENT ITEMS

[Generalization item] Cassie added $\frac{2}{3} + \frac{3}{4}$. Her sum was $\frac{5}{7}$. What would you say to Cassie?

a. You are correct because you should add the numerators and the denominators.
b. You are correct because your sum should be close to 1.
c. You are incorrect because your sum should be more than 1.
d. You are incorrect because you should cross multiply to get $\frac{8}{9}$.

[Reversibility item] Sara found a fraction that could be rounded to $\frac{1}{2}$. Which fraction could Sara have found?

a. $\frac{2}{11}$
b. $\frac{7}{16}$
c. $\frac{6}{19}$
d. $\frac{11}{12}$

[Flexibility item] Ron said, "If I know the sum of $\frac{3}{4} + \frac{1}{16}$ is $\frac{13}{16}$, then I know the sum of $\frac{3}{4} + \frac{1}{8}$ without doing the addition." How would Ron do the addition?

a. Ron would use 8 as the denominator because it is the larger denominator. He would add the numerators. The sum is $\frac{4}{8}$.
b. Ron would use 16 as the denominator and add the numerators. The sum is $\frac{4}{16}$.
c. Ron would add $\frac{1}{16}$ to the sum because $\frac{1}{8}$ is $\frac{1}{16}$ more than $\frac{1}{16}$. The sum is $\frac{14}{16}$.
d. It is not possible for Ron to use the sum of $\frac{13}{16}$ to find the sum.

INTEGER ASSESSMENT ITEMS

[Generalization item] Jeri said, "Two negatives always make a positive." Do you agree with Jeri?

a. Yes, I agree with Jeri because –3 × –4 = +12.
b. Yes, I agree with Jeri because –3 + (–4) = +7.
c. No, I disagree with Jeri because –3 + (+4) = + 1.
d. No, I disagree with Jeri because –3 + (–4) = –7.

[Reversibility item] Alex found two negative integers that when subtracted had a positive difference. What integers could Alex have found?

a. –8 – (–7)
b. –8 – 7
c. –8 – (–11)
d. –8 – 11

[Flexibility item] Max added 8 + (–11). The sum was –3. Ella said, "Now I know the sum of 8 + (–15) without adding." How did Ella find the sum?

a. One addend is 4 less than Max's problem. The sum is –7.
b. One addend is 4 less than Max's problem. The sum is –3.
c. One addend is 4 more than Max's problem. The sum is –7.
d. It is not possible for Ella to find the sum without adding.

Figure 2. Fraction assessment items for three types of questions.

Using Think–Pair–Share

Consider using a think–pair–share strategy, which is most effective in motivating students to engage in these types of questions. It is important to recognize that reversibility, flexibility, and generalization questions require more time to respond. Having students first think independently, then share with a partner before sharing with the whole class, gives students time to think. Additionally, allowing students to share with someone else before their response is shared with the class gives students more confidence in a safe environment where the validity of the response can be assessed.

Using think–pair–share also establishes accountability for students. If students have the opportunity to talk with others, it should be expected, and explicitly stated to students when the question is presented, that all pairs (or small groups) will have a response. However, it is important for teachers to expect that not all pairs will have correct answers. Incorrect answers are as important as the correct ones because they bring misconceptions to the forefront and provide significant opportunities for discussion. Thus, allowing those responses to be shared is critical to help teachers understand the misconceptions students with LD may have about the mathematics and in moving students' thinking further along.

Creating a Safe Environment

The classroom environment has to be considered so that it is an inviting one for students with LD to feel comfortable sharing their ideas. For many students with LD, they have not participated in class discussions for a variety of reasons. In some cases, they are not able to solve a problem quickly enough, or they may have a fear of being incorrect. Regardless of the reason, students with LD need to have explicit directions on how to discuss the response and then how responses will be solicited from the class. When the question is asked, students should be told how long they have to think independently and/or talk with their partner. They should also be given the way in which their responses will be shared with the class. For example, each pair may be asked to give one response (or more responses) as the teacher records the responses, or each pair or group may put its response on chart paper and give it to other pairs to consider. Establishing a routine for the discussion helps students learn how to engage in these discussions of significant and complex questions. As their engagement grows, so does their confidence.

Being Consistent in the Daily Mathematics Routine

Consider consistency in using the three types of questions because it is key in getting high-quality responses. Reversibility, flexibility, and generalization questions should be included daily in mathematics lessons. They can be used as warm-up questions to introduce the lesson, based on the previous lesson. They can be asked during a lesson to extend student thinking about the topic. Additionally, these questions can be used as part of assessments, both formative and summative. With daily practice in responding to these questions, they become more familiar, and students understand the routine for engaging in and responding to them.

Summary

Helping middle-grade students with LD become prepared for algebra is more than presenting problems that are explicitly linked to algebraic concepts and skills. Changing the types of questions that are posed, which in turn increases the rigor of the mathematical tasks, can develop algebraic thinking. An increase in rigor brings about more robust learning and develops a repertoire of big ideas and generalizations that can be used to solve skill and novel problems alike. The three types of questions presented here can help students with LD deepen their understanding of computational algorithms and other skills. When these questions become a regular part of the mathematics class routine, students' responses become stronger and more robust as they are able to anticipate the types of questions that will be asked. With the increase in rigor comes more robust learning and repertoire of big ideas and generalizations, which can be used in skill and novel problems alike.

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