Constructing a Frame of Cube: Connecting 3D Shapes with Direction, Location and Movement

Andy Yeh
Queensland University of Technology
<a.yeh@qut.edu.au>

With the advancement of new technologies, this author has in 2010 started to engineer an online learning environment for investigating the nature and development of spatial abilities, and the teaching and learning of geometry. This paper documents how this new digital learning environment can afford the opportunity to integrate the learning about 3D shapes with direction, location and movement, and how young children can mentally and visually construct virtual 3D shapes using movements in both egocentric and fixed frames of reference (FOR). Findings suggest that year 4 (aged 9) children can develop the capacity to construct a cube using egocentric FOR only, fixed FOR only or a combination of both FOR. However, these young participants were unable to articulate the effect of individual or combined FOR movements. Directions for future research are proposed.

Human spatial ability is recognised as a factor of intelligence, and an important predictor of future career paths, especially in scientific research, engineering and the arts (Ivie & Embretson, 2010). Yilmaz (2009) reviewed research from the last three decades and summarised a range of spatial abilities such as spatial visualisation, spatial orientation, spatial relations, flexibility of closure, closure speed, perceptual speed, environmental ability, and spatiotemporal ability. However, in the mathematics curriculum (e.g., Australian National Curriculum: Mathematics), spatial abilities have not been subjects to be explicitly taught or developed. The closest branch of mathematics to spatial abilities is commonly referred to as geometry with its sub-topics about shape, location and transformation. Because of this, this author developed an online virtual reality (VR) learning environment named VRMath 2.0 (VRMath2) as a vehicle to investigate and develop children's spatial abilities while at the same time, integrate with the curriculum and classroom activities.

This paper reports on a learning episode under the big lens on examining human spatial abilities, and specifically looking at "how young children utilise 3D movements to construct a frame of a cube in VRMath2?" as the main research question in this paper.

Literature Review

Although there are a few spatial abilities found in the research literature (see Yilmaz, 2009), three major spatial abilities are commonly recognised (Lohman, 1988):

1. *Spatial visualisation* – the ability to mentally rotate, manipulate, and twist two-and three-dimensional stimulus objects.
2. *Spatial orientation* – the comprehension of the arrangement of elements within a visual stimulus pattern; the aptitude for remaining unconfused by the changing orientations in which a figure may be presented; the ability to determine spatial relation with respect to one's body.
3. *Spatial relations* - the ability to mentally transform (e.g., translate or rotate) objects with respect to an environmental frame of reference (e.g., a landmark or cardinal points) while one's egocentric reference frame does not change.

In V. Steinle, L. Ball & C. Bardini (Eds.), Mathematics education: Yesterday, today and tomorrow (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia). Melbourne, VIC: MERGA.

When describing the spatial relations, the construct of a "reference frame" is an important underlying concept. Sachter (1991) explained that the reference frame is a construct, which underlies projective and Euclidean space and is an important factor in the performance of various spatial and developmental tasks. A "reference frame" is a systematic representation of spatial relations among objects, which provides a set of coordinates for expressing transformations of such relations. There have been some classifications about the "frame of reference" (FOR). For example, Olson and Bialystok (1983) identified four general classes of referents: ego, observer, object, and environment. Darken's (1996) proposed three succinct reference frames: egocentric, fixed object, and coordinate systems. The egocentric FOR can only locate objects in the environment relative to the body. The fixed object reference frame locates objects relatively according to a particular object such as a tree, landmarks and cardinal points. The coordinate system of reference (e.g., the Cartesian coordinate system) is more abstract and locates objects to an absolute position not relevant to body or any fixed objects. Geometrical language can be associated with each FOR. For example, relative descriptors such as forward, back, turn left or right, above, below, beside, front and behind, and north, east, west, south etc. can be used in egocentric and fixed reference frames, while absolute terms such as x, y, and z axis etc. can be used in coordinate system of reference.

Yakimanskaya, Wilson and Davis (1991) found that a variety of frames of reference are used to solve graphic problems, and constant transition from one FOR to another enriches and influences each other. However, they pointed out that the predominant use of some one particular frame of reference (most often the human body) often impedes successful problem solving particularly in descriptive geometry. They suggested that it is necessary to use several frames of reference simultaneously. Sachter (1991) also pointed out that it is important to know how, why, and when a particular reference frame is chosen in problem solving.

Yeh and Nason (2004), and Yeh and Hallam (2011) explored problem solving activities involving the use of egocentric and fixed FOR to construct 2D and 3D shapes. They confirmed that young children could solve problems easier when they could refer to both egocentric and fixed FOR together. However, they also found that the combined use of egocentric and fixed FOR could generate unexpected results in some conditions. This was due to the fact that the fixed FOR usually involved the change of location only and no change in direction. For example, "to east 1 metre", the movement would simply slide to the east for one metre and therefore a change of direction is not required. When children were confronted with these unexpected results, they were presented with an opportunity to develop their understanding. Sachter (1991) suggested that it is important to know how, why, and when a particular reference frame is chosen in problem solving.

Based on the literature reviewed, this study employed purposefully designed problem solving tasks to investigate how young children utilise 3D movements of different FOR to construct 3D shapes and assess their spatial abilities.

The Learning Environment

VRMath2 (Figure 1) is an online application. It has an interactive 3D virtual reality computer graphics, in which users can navigate and build/construct microworlds in virtually unlimited space. It has an extended LOGO programming language to assist the geometrical and mathematical creations of microworlds. As an online application, it allows users to collaborate and share their creations.

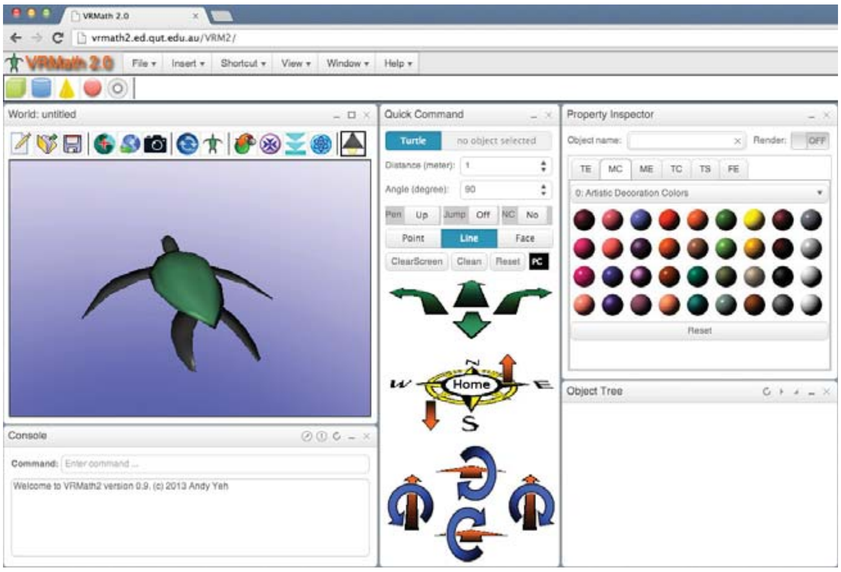


Figure 1. VRMath 2.0 (http://vrmath2.ed.qut.edu.au/VRM2).

In VRMath2, three main reference frames: egocentric, fixed object (fixed), and coordinate systems (Darken, 1996) have been adopted. Below it describes only the design of egocentric and fixed movements in VRMath2, as the coordinate systems movements were not utilised in this study.

There are eight egocentric movements designed in VRMath2. Two of the eight movements that will change location are FORWARD (FD) and BACKWARD (BACK or BK). The other six egocentric movements are turning or direction/orientation changing movements. They include LEFT (LT), RIGHT (RT), ROLLUP (RU), ROLLDOWN (RD), TILTLEFT (TL), and TILTRIGHT (TR). It is important to note that each egocentric movement can only change either location or direction.

There are seven fixed movements in VRMath2. They are the four compass points EAST, WEST, NORTH, SOUTH, and UP, DOWN, and HOME. All seven are location-changing movements. They will not change direction except for HOME, which moves the turtle back to coordinate (0, 0, 0) and facing north (-Z).

Method

A design-experiment (design-based research) methodology (Bereiter, 2002) is employed in this research study. There were twelve participants involved in this research. However, due to the amount of data, this paper only reports on three grade 4 (pseudonym Jack, Rita, Nina) participants. These students were from a government school in Brisbane. They were aged 9 years old. Before this research session, all participants had been introduced with VRMath2 and knew about the 8 egocentric and 7 fixed movements. They had tried and seen these commands (e.g., EAST, ROLLUP, FORWARD, RIGHT etc.) with the associated movements in VRMath2. Then, as a group of three, they were given a 45-minute session with the following tasks:

10. Discuss cubes and draw a cube on blank paper and grid paper.
11. Draw a cube in VRMath2. Write down a sequence of movement for an egocentric cube (using egocentric movements only), a fixed cube (using fixed movements only), and a combination cube (using both egocentric and fixed movements).
12. Test the three cube procedures (sequence of movement) with a starting tilt of 45°.

During the session, the discussions were audio-recorded. Participants' drawings and written procedures were collected and their interactions (e.g., using Quick Command and navigation) with VRMath 2.0 were automatically logged into an online database. Field notes were also taken if any development was observed.

All data were transcribed and arranged chronologically for cross-reference and analysis (e.g., conversations, written artefacts, drawings etc.). Participants' thinking and reasoning were then assessed and interpreted by two researchers to strengthen the validity of this qualitative report.

Results

The discussions about cubes among participants found that all participants knew about the properties of a cube such as six square faces, 12 equal length edges, 8 vertices, and 90° angles. Drawing of cubes on blank paper generally showed some inaccuracies in the drawing of the square and size from various perspectives (e.g., isometric and oblique). Drawing on grid paper increased the quality of cubes. For the purpose of this report, this step 1 served as a warm up activity. The participants' ideas and strategies about drawing cubes are not reported in this paper. Due to the large amount of data and purpose of this paper, the following report focuses on the results from tasks 2 and 3.

Draw a Cube in VRMath2

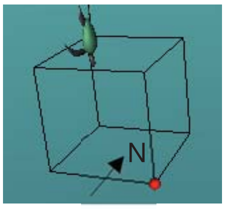
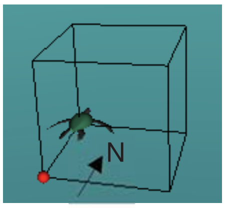
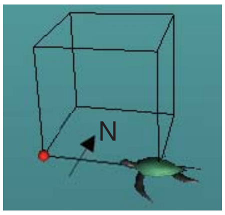
Draw a cube in VRMath2 is an interactive process, in which participants make a movement and receive visual feedback in the 3D turtle graphics. Participants were allowed many attempts (i.e., clear screen and restart) for each cube. The VRMath2 system will record each movement so when a cube is successfully built, the participants can then copy and write down the recorded movements. Due to the time constraint for this group of young participants, instead of getting each participant to create 3 cubes, the 3 participants were assigned 1 cube each for step 2. Then they came back together to complete step 3.

Jack designed a cube using only egocentric commands. Rita designed a cube using only fixed commands. And Nina used a combination of egocentric and fixed commands. According to the computer log, all three participants were able to complete a cube using assigned FOR movements. Table 1 below shows the sequences of the three cubes and the resulted cubes in VRMath2's 3D space.

As can be seen in Table 1, the three cubes all started from forward or north. This was due to the fact that north was the starting direction of the turtle. The three cubes also were constructed from the based then went up. This could be attributed to that the starting viewpoint was above the turtle. However, after first movement to north, the participants had the choice to move to the right or left. As can be seen, 1 cube developed to the left, 2 developed to the right. The three turtles also ended up with different location and direction.

The egocentric cube was completed with 43 movements, which demonstrated a more significant cognitive load than the other two cubes. From analysis, Jack's egocentric commands do make a cube. However, some commands are not necessary. For example, where he has turned the wrong way, he then turned the counter way twice. It however, should be noted that he has grasped an understanding of angles as on multiple times he has accurately simplified the steps by combining turns into one larger angle (e.g., he combined two ROLLDOWN 90 to just one ROLLDOWN 180 on the worksheet).

Table 1
Grade 4 Participants' Drawings of Cubes in VRMath2

Jack's egocentric cube		Rita's fixed cube	Nina's combination cube
FORWARD 1	FORWARD 1	NORTH 1	NORTH 1
LEFT 90	RIGHT 90	EAST 1	EAST 1
FORWARD 1	LEFT 90	SOUTH 1	SOUTH 1
LEFT 90	LEFT 90	WEST 1	WEST 1
FORWARD 1	FORWARD 1	UP 1	NORTH 1
LEFT 90	RIGHT 90	EAST 1	SOUTH 1
FORWARD 1	LEFT 90	DOWN 1	UP 1
ROLLUP 90	LEFT 90	NORTH 1	FORWARD 1
FORWARD 1	FORWARD 1	UP 1	RIGHT 90
ROLLUP 90	BACK 1	SOUTH 1	FORWARD 1
FORWARD 1	TILTRIGHT 90	NORTH 1	RIGHT 90
ROLLUP 90	ROLLUP 90	WEST 1	FORWARD 1
FORWARD 1	TILTRIGHT 90	SOUTH 1	RIGHT 90
ROLLDOWN 90	ROLLUP 90	NORTH 1	FORWARD 1
ROLLDOWN 90	ROLLUP 90	DOWN 1	NORTH 1
FORWARD 1	ROLLUP 90	(15 movements)	DOWN 1
ROLLUP 90	FORWARD 1		EAST 1
TILTRIGHT 90	ROLLDOWN 90		UP 1
TILTRIGHT 90	FORWARD 1		SOUTH 1
ROLLUP 90	ROLLUP 90		DOWN 1
TILTRIGHT 90	BACK 1		(20 movements)
ROLLUP 90	(43 movements)		
			

* The dot denotes the starting point of the turtle, and the turtle starts with facing north.

It could be said that Rita had the easiest cube to construct. By using fixed movements, Rita was able to complete the task in fewest commands (15 movements) and found it a fairly straightforward way to construct a cube. Nina was struggling with the combination cube. And since Rita has finished early with her fixed cube, they worked collaboratively to complete the combination cube. In writing out the steps on the worksheet, Nina missed out a RIGHT 90 and a FORWARD 1. She was not aware that NORTH 1 and SOUTH 1 are not necessary. Overall, the combination cube was still easier to construct than the egocentric cube.

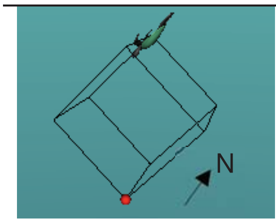
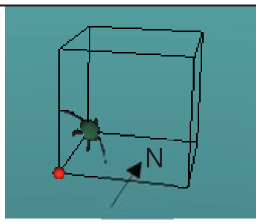
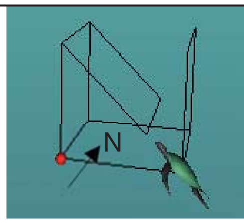
Test Cube Procedures Beginning with TILTRIGHT 45

After the three cubes' procedures were written down on worksheets, the three participants were gathered to complete this step 3 activity. This activity was to run through the three procedures but with the turtle at the home location and tilted right 45° to start with. Before trying in VRMath2, the participants were asked to reason from their procedures and predict what the results will be. In order to facilitate the presentation of this paper, the following transcriptions are numbered in square brackets so they can be easily referred to in the discussion section later.

[1] Researcher: What would you predict will happen?
[2] Rita: Um, I think that they all might be the same. As long as they've got the same distance. What's he doing?
[3] Researcher: He's doing all egocentric, so it's always talking about your direction.
[4] Rita: Actually, yeah they should all be the same.

All participants agreed that the three cubes should still be the same. They then tried their procedures on computers with a TILTRIGHT 45 to begin with. These three cubes are presented in the Table 2 below.

Table 2
Cubes with a Beginning Tilt of 45° in VRMath2

Jack's egocentric cube	Rita's fixed cube	Nina's combination cube
		

* The dot denotes the starting point of the turtle, and the turtle starts with facing north but tilted right 45°.

After the participants finished their cubes, they then gathered to discuss the results. Rita was the first one to complete, as she required fewer movements for her fixed cube. When finished, she started rotating around to examine her cube from different viewpoints.

[5] Rita: When you look at it like that it doesn't look like a cube.

[6] Researcher: What do you mean by that?

[7] Rita: Like, um, once we got onto this angle and it didn't look like a cube anymore. Like that doesn't look like a cube anymore, but it is.

Jack and Nina's procedures took longer to finish as they have more movements. After completed, the researcher questioned to confirm their results.

[8] Researcher: Jack, your cube is looking a bit different to before isn't it? Why?
[9] Jack: Yes, because it's tilted a bit. It's done the whole cube on a tilt.
[10] Researcher: What about yours, Nina? The combination cube?
[11] Nina: It's not a cube at all.
[12] Researcher: Did your cube change at all? (Ask to Rita).
[13] Rita: Not really.

Then, the researcher started probing the participants' thinking and reasoning.

[14] Researcher: The egocentric cube made a cube on a tilt, the compass cube on the tilt produced the same cube and having a combination didn't produce a cube anymore. Why do you think this happened?
[15] Jack: Well I think on the tilt it may have done something to it. Like, when you did forward and right it might have, like, changed the direction.
[16] Nina: It goes down. (She pointed at her combination cube).
[17] Jack: Yeah it goes down because it would have been on a tilt and then it would have gone like up and then right and it could go right that way, right that way, right that way, or right that way. It could go anyway.
[18] Researcher: So how come that didn't matter when you used compass directions? (Ask to Rita).
[19] Nina: Because you're like, um, it's just, like, going left and right rather than up and down because when you go up it tilts more than just the straight, up straight.
[20] Rita: What are you asking?

[21] Researcher: I'm asking why did your cube, the compass cube, still look exactly the same? How come the tilt had no effect on it?
[22] Rita: I don't know.
[23] Jack: I know why the combination cube didn't work. Because it may work with the other one but because um, egocentric it was tilted, which is an egocentric move and then all of the um, like, the north. Then she might have started with an egocentric move (TILTRIGHT 45) and that would have changed the north, south, east or west.
[24] Researcher: It would have changed north, south, east or west?
[25] Jack: Because they would have tilted it so it would be a different angle so to north. But with the forward, it was just forward that way and then you tilted again.
[26] Researcher: So if you're on a tilt, will north change?
[27] Jack: Yes. Like if you were standing up, upright, then north would be that way and if you tilted then north would be a different way.
[28] Researcher: So say north is this way, I'm not sure where it really is, if I go like this (TILTRIGHT 45), is north still that way? Or if I move this way (slide to east), is north now this way (east) or would north still that way (north)?
[29] Rita: North would still be that way.
[30] Nina: Yeah.
[31] Jack: But north would be on more of an angle.
[32] Researcher: So north would be on an angle?
[33] Jack: Like even though it's on a straight line, it would be on an angle.
[34] Nina: Yeah because you're tilted, it (north) would tilt as well.
[35] Jack: You're on an angle and when you go west, um, it would turn a different way because it would be tilted. And if you turned right it would be a different way. (West will not affect direction).

Discussion and Conclusion

The construction of a cube using movements is somewhat new and challenging to these grade 4 participants. However, the results suggested that with the visual feedback provided in VRMath2, these grade 4 children were able to construct a cube with egocentric movements only, fixed movements only, and a combination of both egocentric and fixed movements. The results in Table 1 seemed to confirm that the use of only one FOR, particularly egocentric, tends to impede successful problem solving (Yakimanskaya et al., 1991). In this study, the egocentric cube took more movements than fixed and combination cubes. This is in fact the nature of egocentric movements that in order to reach a new location, the egocentric FOR would normally require at least one turning (change direction) and one locating (change position) movements. However, more movements it may be, the egocentric movements can satisfy any starting location and direction to maintain the expected outcome (a cube tilted as the turtle's starting direction). A fixed cube would also maintain as a cube but it would not have any rotation effect. With more choices of movement, the combination cube also took fewer movements than the egocentric cube. However, more choices may not always be easier as Nina did struggle to select which movement to proceed.

The three cubes in Table 2 did puzzle the three participants. They were purposefully designed by this author to create a cognitive dissonance for learners, as suggested by Sachter (1991), to know how, why and when to use a different FOR. The results turned out to suggest that these young participants were unable to articulate why different FOR would result in different outcomes when there was a starting TILTRIGHT 45. Findings about children's thinking and reasoning from this include: (a) the tendency to consider distance only and ignore angle (see transcript [2]), (b) notice the beginning tilt would affect egocentric movements (see [15], [17] and [25]), (c) reasoning was influenced by the perspective in 3D space (see [19]), (d) unable to identify that fixed movements (compass) are not affected by the turtle's orientation (see from [21]-[23]). From discourse [26]-[35], Jack's idea about "north on an angle" when the turtle tilted was interesting. It was thought that Jack and Nina did not understand that compass directions are fixed, and compass movements will not affect the turtle's direction. However, Jack and Nina could simply mean that when the turtle turned an angle, there was an angle between north and the turtle's direction. Nevertheless, they were incorrect as after the beginning TILTRIGHT 45, the turtle is still facing north.

To conclude, this paper reported a learning environment VRMath2 that can integrate the teaching and learning about shapes and location, direction and movement. When children construct objects using movement language, seeing the creation process and navigate in the 3D virtual space, they develop their spatial abilities (see [7]). Future research in using VRMath2 should allow more opportunities and time for children to construct and express their thinking. Follow up research activities could include detailed analysis of construction sequence and more mental reasoning practice (e.g., mentally write down the complete procedure). Using VRMath2 as a research instrument, it is possible to chart many unexplored human spatial abilities. And with its creative power, VRMath2 would also be a pertinent vehicle to develop a wide range of spatial abilities.

Acknowledgement

The author wishes to acknowledge the financial support from QUT ECARD fund and thank to Mrs Jenifer Green for her contribution in transcribing and interpreting results.

References

Bereiter, C. (2002). Design Research for Sustained Innovation. *Cognitive Studies: Bulletin of the Japanese Cognitive Science Society, 9*(3), 321-327.

Darken, R. P. (1996). Wayfinding in large-scale virtual worlds. (Doctor of Science thesis), George Washington University, Washington, DC. Retrieved from http://www.movesinstitute.org/darken/publications/Dissertation.pdf

Ivie, J. L., & Embretson, S. E. (2010). Cognitive process modeling of spatial ability: The assembling objects task. *Intelligence, 38*(3), 324-335.

Lohman, D. (1988). Spatial abilities as traits, processes and knowledge. In R. J. Sternberg (Ed.), *Advances in the psychology of human intelligence* (Vol. 40, pp. 181-248). Hillsdale: LEA.

Olson, D. R., & Bialystok, E. (1983). *Spatial cognition: The structure and development of mental representations of spatial relations*. Hillsdale, N.J.: L. Erlbaum Associates.

Sachter, J. E. (1991). Different styles of exploration and construction of 3-D spatial knowledge in a 3-D computer graphics microworld. In I. Harel, S. Papert & Massachusetts Institute of Technology. Epistemology & Learning Research Group. (Eds.), *Constructionism : research reports and essays, 1985-1990* (pp. 335-364). Norwood, N.J.: Ablex Pub. Corp.

Yakimanskaya, I. S., Wilson, P. S., & Davis, E. J. (1991). *The development of spatial thinking in schoolchildren* (Vol. 3). Reston, VA: National Council of Teachers of Mathematics.

Yeh, A. J. , & Nason, R. A. (2004). Knowledge construction of 3D geometry concepts and processes within a virtual reality learning environment. In E. McWilliam, S. Danby & J. Knight (Eds.), *Performing educational research: theories, methods and practices* (pp. 249-264). Flaxton, Qld: Post Pressed.

Yeh, A. J., & Hallam, J. (2011). *Young children's understandings about "square" in 3D virtual reality microworlds*. Paper presented at the Mathematics : Traditions and [New] Practices : Proceedings of the 34th Annual Conference of the Mathematics Education Research Group of Australasia. http://eprints.qut.edu.au/46617/

Yilmaz, H. B. (2009). On the development and measurement of spatial ability. *International Electronic Journal of Elementary Education, 1*(2), 83-96.