# Does Inquiry Based Learning Affect Students’ Beliefs and Attitudes Towards Mathematics?

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Ill-structured tasks presented in an inquiry learning environment have the potential to affect students’ beliefs and attitudes towards mathematics. This empirical research followed a Design Experiment approach to explore how aspects of using ill-structured tasks may have affected students’ beliefs and attitudes. Results showed this task type and learning environment created situations that exposed and challenged students’ beliefs and attitudes and required them to defend their position. Insights regarding factors that may influence students’ beliefs and attitudes are discussed.

Dewey (1938) suggested that in order to learn, the learner must first experience a sense of doubt. Traditional mathematics classrooms and curriculum could be argued to reduce the occurrence of doubt by providing structure with guided, repetitious skill practice and certainty through the ‘one correct answer’ type questioning. To challenge this learning model and in order to generate doubt, one method that has been employed by researchers and educators is inquiry based learning (Goodchild, Fuglestad & Jaworski, 2013).

Within this body of research, the use of ill-structured tasks are a tool utilised to initiate and support the creation of doubt in an attempt to generate real learning experiences in mathematics (Lodewyk, Winne, & Jamieson-Noel, 2009; Spector, 2006). Ill-structured tasks in mathematics are generally considered to be open ended, context based questions that have a deliberate ‘messiness’ in the question. This question ‘messiness’ varies from study to study but can include vagueness, having key factors missing, using data that is not uniformly collected or sorted and not having specific steps that lead to the answer. Simon (1978, p. 286) defined ill-structured tasks as satisfying the following three criteria:

1. The criterion that determines whether the goal has been attained is both more complex and less definite.

2. The information needed to solve the problem is not entirely contained in the problem instructions, and indeed, the boundaries of the relevant information are themselves very vague.

3. There is no simple ‘legal move generator’ for finding all of the alternative possibilities at each step.

Students’ beliefs and attitudes towards learning mathematics have been linked to their outcomes in mathematics and is also argued to predict a student’s propensity to use it in non-educational settings (Goldin, Epstein, Schorr, & Warner, 2011; Maass, 2010). Most mathematics curricula worldwide incorporate an affective domain that aspires to instil beliefs and attitudes that allow the learner to recognise the beauty and power of mathematics, but this is rarely tested or measured. To assist with identifying beliefs that are specific to learning mathematics they are defined for this report as being able to be categorised into four main types:

- the nature of mathematics in general;

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- self-efficacy for solving mathematical problems;
- how mathematics should be taught;
- the way society views mathematics (McLeod, 1992, p. 578).

Using mathematical inquiry and ill-structured tasks has been shown to have a positive impact on students' beliefs and attitudes about mathematics (Boaler 1998; Taylor, 2009). Schwab (1959) suggested that a proportion of our curriculum should be spent using inquiry methods to ensure students are engaged and involved in developing affective aspects of learning mathematics. Few empirical studies have investigated how using mathematical inquiry and ill-structured tasks actually brings about change to students' beliefs and attitudes towards mathematics.

## Theoretical Framework

Bandura (1986) argues that a student's self-perception of their ability in a subject like mathematics is not stagnant and relies on a variety of contributing factors that interplay, such as emotions, motivations, attitudes and beliefs. According to Bandura's theories, students' beliefs and attitudes are socially constructed and can therefore be influenced through classroom interactions. This indicates that deliberately exposing students' beliefs and attitudes during mathematical learning may instigate situations for change to occur.

Goodykoontz (2008) categorised particular factors that can affect change in a student's beliefs and attitudes in mathematics. She described these factors as being either external or internal from the student's perspective. The range of external factors include; teacher characteristics, teaching characteristics, assessment and achievements and classroom characteristics. Internal factors identified by Goodykoontz's (2008) are; family background, challenge and frustration levels and their level of understanding of the work. Goodykoontz found links between a student's positive beliefs and attitudes towards mathematics and their ability to collaborate with peers in class in an inquiry style environment.

The combination of these theoretical positions suggests that deliberately altering factors related to the way students experience learning may create situations that enable socially constructed change to students' beliefs and attitudes towards mathematics. This suggests a research approach that will now be discussed.

## Context, Design and Method

As students grow older, their attitudes and beliefs are regarded as becoming more defined and more stable (Goodykoontz, 2008). High school mathematics classes where inquiry based learning experiences are regularly utilised are rare. Combined with the logistics of finding a class willing to be involved in the study, this made the traditional, observational research approach untenable. Design experiments allow the researcher to deliberately create the desired intervention characteristics in order to observe the impact it may or may not have on the variables (Cobb, Confrey, diSessa, Lehrer & Schauble, 2003; Sloane & Gorard, 2003).

Design experiments are generally iterative in nature and cater for research settings where the environment consists of a web of interactions between tasks, objects and people and studies the interplay between those elements. This approach also allows for adjustments to the research process in order to alter situations to bring about the desired intervention characteristics (Cobb et. al., 2003).

One class of 26 students studying mathematics have been exposed to inquiry based style learning that incorporated an ill-structured task. These students were all aged between 15-17 years. The students in this research were enrolled in a subject called Maths A. Maths A is a university preparation course that has no calculus and is generally considered of medium difficulty for students of this age group. Students were asked one ill-structured question; “As the owner of a casino, what would you pay someone who wins the game of 7s?” Students were shown the guided inquiry model to assist with their understanding of the guided inquiry learning process. The students had roughly two weeks of learning time devoted to answering this question and this totalled approximately five and a half hours of class time.

Students were observed in a number of ways, including via video recording of classes, targeted interviews with four pairs of students after the unit was completed and collection of workbook materials from their classes. Data from video and voice recordings from interviews were sorted and examined using a whole-to-part inductive approach as suggested by Erickson (2006) and this was cross checked with questionnaire results and students’ workbooks.

These results were thematically summarised and mapped against McLeod’s (1992) categories of beliefs in mathematics using Goodykoontz’s (2008) theories of factors that can impact on students’ beliefs and attitudes. This was done using graphic organisers displaying McLeod’s four categories of beliefs and mapping partial video transcripts and interview transcripts to these categories. Further categorisation using Goodykoontz’s notions of external and internal factors was used to then group within the categories of beliefs. The results were examined to search for trends, interactions and key junctures that might give insights to when student’s beliefs and attitudes towards mathematics were being activated, discussed or challenged.

## Results and Discussion

The objectives of the mathematics unit were to expose students to extended concepts in probability such as compound events and their sample space. In the ill structured task studied for this report, the context involved student’s acting as a casino owner who is introducing a new game to their casino. The game itself was modelled off ‘typical,’ textbook-type, probability questions that involve the rolling of two dice and adding the total score of the dice.  Aspects of the way student’s beliefs and attitudes were exposed and challenged have been outlined below using discussion of excerpts from video, voice recording and student workbooks to explore the ways in which this learning technique may impact students’ beliefs and attitudes towards mathematics.

### *The Nature of Mathematics*

Aspects of the ill-structured question appear to have challenged these beliefs. For example, one group was expressing some difficulties with the way the question was presented to them. The following exchange occurred (pseudonyms have been used throughout this paper):

> Researcher: What are you finding difficult about this question?
>
> Ronald: It feels more independent. Kind of the way to go about it, not just do these questions from the book, the examples are here, it’s more like investigating and stuff like that.

Brian: Yeh, with the questions in the book, you usually just have to think about one thing, with this you have to think about multiple things and it's all put into one [question].

Students also expressed that the open ended aspect of the question challenged their views about the nature of mathematics.

Rachel: I found it difficult because there was not really a correct answer.

Julie: Yeh, there was no wrong answer, we could come to different conclusions but that's because of how different people think. With maths it's usually like one plus one equals two, that's it. With these problems it's like one plus one doesn't necessarily have to equal two. It can equal three.

Other external aspects of classroom behaviour, as observed on the video, also gave insight to how the ill-structured question may have evoked different working styles. For example, students were not instructed to get into groups but they all did form into groups. Within the groups there was vibrant, on task dialogue, as evidenced by comments such as "OK we are all talking at once, we need to take turns to answer."

This collaborative nature is a central aspect of inquiry style learning in mathematics (Boaler, 1998). It would seem from observing individuals in this class that the collaborative answering of the ill-structured question is linked to students' understanding that there is no single correct answer (e.g. Julie's voice above) and they therefore seek assurance from others of their conclusions. Students made reference to this collective agreement on a 'correct' answer on numerous occasions with comments within their group such as, "Should we write that down?"

### *Self-Efficacy for Solving Mathematical Problems*

External aspects viewed and recorded during class suggest that students worked as a group to solve impasses. Groups all appeared to follow similar patterns of working through the ill-structured task. This approach pattern can be summarised by saying that initially groups would engage in discussion that focused on interpreting aspects of the question. Once an agreement was made on definitions, students would generally then work individually on the booklet. As another impasse was reached, they discussed the issues and overcame them through argument, divergence and by reaching consensus.

Student's self-efficacy was most obvious during the initial phase of task interpretation and appears to have affected their development of a pathway for solution. The initial approach of the groups has been summarised below to assist with illustrating this point (see Table 1).

As the lesson progressed, it became obvious that no group had correctly listed the sample space for this compound event. This is in spite of having recently completed a unit that focused on this particular skill. All groups realised this at some point and subsequently requested assistance from the teacher. When speaking about the impasse of not being able to list the sample space, one group responded:

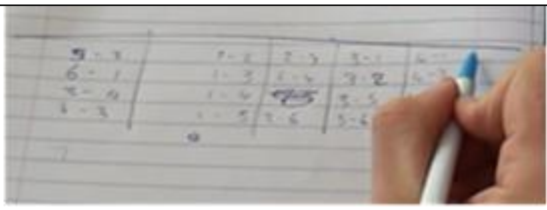
Beccy: I found it difficult cause I'm not very bright.

Wendy: I have been playing games like this since I was eight thanks to my dad, but I do understand the way the casino always wins and how they do out the odds and all that, so I did understand it.

Beccy: Counting and rolling the dice was good, I was good at that bit.

While these students had almost identical outcomes for the task as they had worked together, their expressed beliefs about the impasse shows a different perception of the final outcome. This supports Goodykoontz (2008) view that a both internal (family experiences) and external (achievements) factors affect students' beliefs and attitudes. Bandura's (1986) work would suggest that this type of dialogue between students may stimulate a socially generated change in their beliefs and attitudes.

Table 1
*Initial Approach to Ill-Structured Task about Compound Sample Space*

| Group/s | Strategy | Example | Impasse |
|---|---|---|---|
| A | Practical | Decided to roll dice 100 times to determine sample space | Could not defend 'lucky roller' question |
| B | Straight to answer | "Let's say 1 is to 4 payout." | When asked to defend it, students could not answer any questions |
| C,D | List randomly sample space |  | Groups missed at least one possible combination using this method |
| E,F,G | List all outcomes (not possibilities) | 2,3,4,5,6,7,8,9,10,11,12 | Could not defend question of odds of 1/11 for getting a '7' as rolling a '2' was less likely than rolling a '7'. |

Insights into how students' self-efficacy affected their initial interpretation and approach were gathered by asking students to verbalise a defence for their final outcomes. For example, as mentioned above (see Table 1), one group decided to roll the dice 100 times and use this as the basis for making further decisions regarding payouts. When asked why they took this approach, one group member commented:

> Michael: We handled the sample space fairly easily, we just did it practically. We were probably more real than statistical.

While this group could not readily defend their outcomes and conclusions against the idea of a 'lucky roller' or someone who plays more than 100 games, they held their 'practical' findings as valid right through until the end of the unit.

In this ill-structured problem, the sample space impasse put students in a situation where they then relied on their belief in their ability to justify how they approached solving the question. In some cases, this meant students expressed a defence for their choices. When these beliefs and attitudes were being verbalised, they were then subject to change due to the collaborative nature of the task.

### How Mathematics Should Be Taught

The way the class operated as an inquiry based learning environment seems to have presented challenges to students. Eight students were asked follow up questions in post unit interviews about whether they felt the class was different to their normal classes. In line with Goodykoontz's (2008) suggestion that the level of understanding is a significant internal factor affecting beliefs and attitudes, these students were also asked if they felt they understood the content of the unit of work they were engaged in. Some of the responses are listed below:

> Robert: It feels a lot different to how she normally teaches. I felt like she [the teacher] had to interact with us more because we weren't just doing questions on the sheet. She had to come to us individually.

> Alex: Easier understanding because we were given the question, so we had to solve it for how we think to do it, not how the teacher says we should do it, we had to use our own understanding to figure out the answer. We had to listen to her and then figure it out. Usually we just read it in the book and do it.

> Rachel: It was just one question, it wasn't something I could practice and practice again. You're not given the information even for that one question.

It appears from the reflections above that students found the style of classroom different from their normal classroom, regardless of their perception of the value of the experience. Further insights into challenges to students' beliefs and attitudes about how we should teach and learn mathematics came from the group that 'jumped straight to the answer.' Initially the members of this group expressed resistance to changing their position, but rather sought to discredit the question and process.

> Mandy: I kind of just had the answer but then you had to really explain why you had it. It was a bit tedious.

> Rachel: Yeh, we were not given the income of the casino, so how were we supposed to find out the income?

> Mandy: I suppose this would help with justifying in a modelling and problem solving question, it's almost humanities, like our assignments in maths.

The final comment from Mandy suggests she may have assigned some value to the learning style by relating it to a summative assessment type that contributes to her final results. This link between the perceived usefulness of this type of question type and her overall achievement may, if reinforced, contribute to the development a more sophisticated view of how mathematics should be experienced.

### The Way Society Views Mathematics

The nature of the ill-structured task meant that students were involved in developing a shared understanding of the task and needed to put themselves in context to properly interpret the question. This can be argued to improve the relevance to them as individuals and society members, regardless of their perceived need for mathematics in their future careers.

This socially constructed position was articulated by students throughout the class, for example on one student workbook a student wrote:

> Billy: How much should casino pay? A lot because they can afford it!

This inquiry learning environment allowed students to discuss social aspects that arose as a result of their context. The group exchange below took place towards the end of the unit.

> Callum: This casino won't get any profit, unless we change it so they can win. If someone was really lucky and someone else was unlucky, then wouldn't it even out?
>
> Brian: Yeh, talking to Miss about it, and everyone expects it to be a rip off, it's a casino.
>
> Paul: This is why you don't gamble. You go in to a casino with an amount of money and you only spend that, anything you win you put it aside and its ok to lose that. Otherwise you end up broke and spending $2000 like ...'s dad.

Other groups reflected that their initial approach may have left them in less defensible positions for their final outcome. The comment below came from the group that approached the question from a practical perspective by rolling the dice 100 times to determine the necessary probabilities to continue with the questions.

> Brad: I think at first I wasn't approaching it mathematically enough, I was just thinking about how it looks, from a marketing perspective

This student seems to be acknowledging that a casino owner would be thinking mathematically when making decisions about the odds for a game. This suggests a challenge may have occurred to the student's thinking or beliefs about how mathematics is used in practical ways in society.

## Conclusions

The intention of this study was to gather insights into specific aspects of inquiry based learning that may be responsible for influencing students to take on more sophisticated belief systems in regards to mathematics. Students in this class articulated that the inquiry based learning environment was notably different to their normal learning experiences. The use of the ill-structured task and inquiry based learning appears to have created situations that have exposed and challenged students' beliefs and attitudes towards mathematics.

The use of Bandura's (1986) and Goodykoontz's (2008) theories assisted with exploring the categories of beliefs as outlined by McLeod (1992). There were examples of student actions and interactions that were able to be interpreted and described using these frameworks. Further understanding how this learning environment can impact students' beliefs and attitudes may offer more detail to educators seeking specific classroom practices that can be used to enhance students' mathematical learning and build the affective domain.

Future iterations of this design experiment will occur with this class over the next year. These students will complete post research surveys and interviews that may give insight into their change in position and further support the notion that inquiry based learning experiences may have lasting impact on students' beliefs and attitudes towards mathematics.

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