Teacher Actions to Facilitate Early Algebraic Reasoning

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In recent years there has been an increased emphasis on integrating the teaching of arithmetic and algebra in primary school classrooms. This requires teachers to develop links between arithmetic and algebra and use pedagogical actions that facilitate algebraic reasoning. Drawing on findings from a classroom-based study, this paper provides an exemplar of one teacher's journey in shifting her practice to integrate early algebra into her everyday mathematics lessons. The findings highlight the importance of addressing different areas including algebraic content, task development and enactment, and the classroom and mathematical practices to facilitate algebraic reasoning.

Introduction

Significant changes have been proposed for Western mathematics classrooms of the 21st century in order to meet the needs of a *knowledge society*. One key focus has been on the learning of algebra as an essential type of thinking for "participation in a democratic society" (Mason, 2008, p. 79). Algebra takes an important role in ensuring access to both potential educational and employment opportunities (Knuth, Stephens, McNeil, & Alibabi, 2006). Given this position, there has been a growing consensus in both research (e.g., Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003) and policy documents (e.g., Department for Education and Employment, 1999; Ministry of Education, 2007) that algebra be introduced at a much younger age with a focus on the integration of arithmetic and algebra as a unified curricula strand.

To ensure links to early algebra are developed and maintained, teachers have a key role in developing and enacting tasks that integrate arithmetic and algebra and reforming classroom practice. However, many primary teachers have not had experience in how to develop links between arithmetic and algebra or in using pedagogical actions that facilitate algebraic reasoning (Blanton & Kaput, 2005). To meet the need for models of how early algebra can be integrated into the primary classrooms, this paper provides an exemplar of one teacher's journey in shifting her practice to integrate early algebra into her everyday mathematics lessons.

Many studies (e.g., Bastable & Schifter, 2008; Blanton & Kaput, 2005; Carpenter, Franke, & Levi, 2003) illustrate how teachers can develop aspects of algebraic reasoning in their classrooms. Key findings of these studies include the importance of content areas within the existing curriculum with which early algebra has connections, a focus on student thinking and reasoning, and the use of task design and implementation to promote algebraic reasoning. There are also many studies (e.g., Fosnot & Jacob, 2009; McCrone, 2005; Monaghan, 2005) that address productive classroom and mathematical practices in the mathematics classroom. However, there are few studies that specifically attend to algebraic content, task development and enactment, and the classroom and mathematical practices that facilitate primary students to engage in early algebraic reasoning. The present paper aims to address this gap in the literature by presenting a framework of teacher actions to facilitate early algebraic reasoning that addresses algebraic content, task development and enactment, and the classroom and mathematical practices.

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The theoretical framing of this paper draws on a socio-cultural perspective. In this view, individuals participate in the everyday activities within a classroom community of practice (Lave & Wenger, 1991) and through this participation learn the ways of thinking and acting that are valued by the community. Social participation facilitates the development both of a sense of what it means to be a member of a specific community and a sense of self in relation to the community.

Method

This paper reports on episodes drawn from a larger study (Hunter, 2014) that involved a year-long continuing professional development (PD) classroom-based intervention focused on developing early algebraic reasoning. The participants included two separate groups of primary teachers (from England and the British Isles) from schools that used the Mathematics Enhancement Programme (MEP) curriculum. The focus in this paper is on one teacher who was an experienced teacher interested in strengthening her ability to develop early algebraic reasoning within her classroom. Her class consisted of 25 Year Three students from a semi-rural primary school in the British Isles. The students were from predominantly middle socio-economic home environments and represented a range of ethnic backgrounds.

The model for PD used during the intervention initially drew on research literature. As the intervention progressed, the re-design of the PD drew on a range of sources including researcher observations from the classrooms, study group meetings, teacher interviews and discussions. The focus for professional learning comprised four separate but related components; understanding of early algebraic concepts; task development, modification, and enactment; classroom practices; and mathematical practices. Key aspects of the PD included the use of research articles to extend teachers' understanding of early algebra, to provide models of classrooms that would support early algebraic reasoning, and to promote reflection on current practice. Also central was a focus on the selection, design, and enactment of tasks. This included the teachers completing algebraic tasks themselves, analysing tasks from the MEP material to identify opportunities for algebraic reasoning and investigating ways of modifying existing tasks. In addition, the teachers engaged in activities where they both predicted and analysed student responses to algebraic tasks. A final key element of the PD was facilitating reflection on practice, including developing tools and skills for noticing relevant aspects of their own practice. To support this, the teachers were provided with an adapted framework from Hunter (2009) and also engaged in a series of lesson study cycles.

Data gathering included classroom observations prior to the beginning of the professional development and throughout the school year, video records of professional development meetings, audio recorded interviews with the teachers and students, detailed field notes, and classroom artefacts. On-going data analysis supported the revision of the model for professional development. Retrospective data analysis used NVivo 10 qualitative software programme (2012). The initial codes were developed from a variety of sources including research literature, the initial viewing of the video records, and the observational and reflective field notes. Repeated viewing of the videos and re-reading of the transcripts and field notes confirmed or refuted the initial hypotheses and codes and other hypotheses and codes were developed as necessary.

Results and Discussion

The results and discussion will present the *Framework of Teacher Actions to Facilitate Algebraic Reasoning.* This framework integrates four separate, interlinked components that the study identifies as key to the development of early algebraic reasoning. An analysis will be undertaken of the shifts across the three phases of the study.

Teacher Awareness of and a Purposeful Focus on Algebraic Concepts

Prior to the PD, the teacher demonstrated some awareness of the links between arithmetic and algebra. Instantiations of types of early algebra such as the commutative property, equivalence, inverse relationships were evident during the observed lessons. However, there was no explicit identification or examination of the properties of numbers or operations during lessons. This meant that for students, the properties remained implicit and they were not provided with opportunities to develop deep generalised understanding as advocated by researchers (e.g., Bastable & Schifter, 2008; Carpenter et al., 2003).

Central to each phase was a purposeful focus on algebraic concepts. This is not intended as an exhaustive list but consists of algebraic concepts that are identified as relevant to primary classrooms. The following sections of the findings and discussion will show the teacher's growing propensity to focus on these concepts and integrate exploration of these into her everyday mathematics lessons.

Table 1
Teacher awareness of and a purposeful focus on algebraic concepts

Phase One to Three	Address the following concepts: understand the equal sign as representing equivalence; relational reasoning including whole numbers and rational numbers; commutative property; inverse relationships; odd and even numbers; identity elements; distributive property; associative property; properties of rational numbers; using and solving equations; function

Teacher Actions to Develop and Modify Tasks and Enact Them in Ways That Facilitate Algebraic Reasoning

Prior to the initial PD, the teacher used tasks from the MEP curriculum and carefully guided students through the steps required to complete the task with an emphasis on a fast pace. Her questioning focused attention on computational approaches and was characterised as leading or funnelling students towards correct responses or teacher chosen solution strategies.

Developing new methods of task implementation was an important pedagogical strategy to facilitate algebraic reasoning. In the first phase, an immediate change involved the implementation of tasks as problem-solving opportunities. This included emphasising student effort to approach and complete cognitively challenging tasks. Enabling prompts such as described by Sullivan, Mousley, and Zevenbergen (2006) were used to scaffold all students to access the tasks, without lowering the cognitive demand. Another key change in the second phase related to task implementation involved shifting attention away from recording answers to focusing on patterns and relationships. Teacher questioning oriented students to use a structural focus. For example, in one lesson the teacher introduced a task involving a series of number sentences (100 – 10 =, 90 – 9 =, 80 – 8 = …). She said: "Look at those questions and see if there is a pattern, don't work out the answers yet, just look at it." She then drew attention to the patterns in the answers by asking: "As there is a pattern in the questions, do you think there might be a pattern in the answers?" Many researchers (e.g., Carpenter et al., 2003; Fosnot & Jacob, 2009) argue that the development of structural perspectives is an important aspect of algebraic reasoning.

Changes to lesson planning were important in integrating algebraic reasoning into the everyday mathematics lessons. In the first phase, the teacher began by examining the MEP lesson plans and selecting parts of tasks that focused attention on an algebraic concept. At this point, this did not extend to engagement in a deeper investigation of algebraic concepts. For example, one task involved an array and two number sentences with missing parts ($3 \times \_ = 6$, $6 \div \_ = 2$). Initially teacher questioning focused attention on the general relationship between multiplication and division:

> Three times two equals six and six divided by three equals two. With your partner, what do you notice about those please? A student responded: They're just the other way around… because the three is in the middle and the six is at the beginning and at the end.

After this response, the teacher shifted to ask students to examine related equations where the position of the numerals had changed. This limited opportunities for students to further explore the relationship between multiplication and division as the focus moved to specific equations.

Through the second phase, there was growth in the teacher's understanding of different types of algebraic reasoning. This meant that she was able to more readily modify tasks to include early algebra. It also led to her noticing when students provided responses related to algebraic reasoning. Later during this phase the teacher began to recognise and use spontaneous opportunities for algebra as tasks were enacted. In this phase, the shift in teacher actions also extended to structuring tasks to address misconceptions. For example, in one lesson, students were asked to solve $36 – 6 = \_\_ + 20$. Some students responded by writing 30. The teacher used this as an opportunity to engage the class in prolonged discussion focused on the equal sign.

In the final phase, the teacher consistently planned classroom activities in a way that focused on opportunities for early algebra. She described herself thinking as she planned about how to: "Draw out the commutative law from this one, or this could be a great discussion point for timesing by one, or dividing by zero, get them to come out with conjectures." Another point of difference in this phase was the teacher's propensity to engage in anticipating the outcomes of the task enactment. This supported her to develop her use of monitoring, noticing and sequencing student responses that could be used to spontaneously investigate algebraic concepts. For example during one lesson, the teacher asked her students to think about an efficient method to solve $26 – 8 =$. A student suggested breaking the eight into six and two. The teacher then used this as a spontaneous opportunity to investigate how numbers could be partitioned to solve subtraction tasks: "If you were doing 34 take away seven, with your partner can you just talk about how Misty would tackle that?" Blanton and Kaput (2005) note that spontaneously integrating algebraic reasoning opportunities into lessons is key to developing a classroom context that emphasises algebraic reasoning.

These changes resulted in a clear focus on algebraic reasoning integrated into lessons and included coverage of a broad range of algebraic concepts. In summary, the teacher actions are illustrated in Table Two.

Table 2
Teacher Actions to Develop and Modify Tasks and Enact them in Ways that Facilitate Algebraic Reasoning

Phase One	Implement tasks as problem-solving opportunities
	Emphasise student effort to approach and complete cognitively challenging tasks
	Extend or enact tasks to include opportunities for generalisation
	Interrogate tasks for opportunities to highlight structure and relationships
Phase Two	Adapt tasks to highlight structure and relationships. This includes changing numbers or extending to multiple solutions
	Structure tasks to address potential misconceptions
	Use enabling prompts to facilitate all students to access tasks
	Implement tasks by focusing attention on patterns and structure
	Use spontaneous opportunities for algebraic reasoning during task enactment
Phase Three	Recognise and use links to algebra in tasks across mathematical areas
	Implement tasks as open-ended problems
	Anticipate student responses that could provide opportunities for algebra
	Use spontaneous opportunities for algebraic reasoning from student responses

Teacher Actions to Develop Classroom Practices That Provide Opportunities for Engagement in Algebraic Reasoning

Prior to the initial PD, paired work was a feature of the classroom but rather than complete tasks collaboratively, the partnerships were used as a support mechanism when students were stuck. The discourse patterns in the classroom were dominated by the teacher. Students frequently gave answers with no mathematical reasoning and the teacher provided the majority of mathematical explanations.

In the first phase, to support student engagement in algebraic reasoning it was necessary to address the ways in which students worked collaboratively and the forms of talk used in the classroom. The teacher explicitly discussed with her students how to successfully talk together and facilitated them to generate rules for productive talk similar to what is described by Monaghan (2005). A key expectation was that students developed a shared understanding of a jointly constructed solution strategy. The teacher drew on student models to develop understanding of the new expectations and to affirm productive shared discourse norms. For example after observing small group work she said to the class: "Zanthe said to everybody 'do you get it?' And everyone nodded, but you didn't get it, did you? How did you know that Calvin hadn't got it?" This was followed by asking Zanthe to share with the class how she had known her group member, Calvin, was unsure by asking him to explain the jointly constructed solution strategy.

In the second phase, to advance all students' opportunities to engage in algebraic reasoning it was important to extend collaboration to whole class discussions. The teacher positioned students to listen actively to their peers' reasoning and explanations and make sense of these. During whole class discussions she intervened to provide space for other students to question or modelled how to ask a question herself. For example, in one lesson she asked the students to generate different two factor equations using the digits two, three and five. A student provided her group's solution strategy: "We think we should work out two times two first, then two times three and two times five." At this point the teacher provided a space for questions that led to a student question focused on clarification and justification: "If you were to do that, how would you be able to know whether you'd done the two and five, or two and three, or two and two, how would you know?"

In the final phase, a consistent expectation was established that students would work as a collaborative community. When students explained their strategy solutions during whole class discussions, the teacher emphasised that their partners or group needed to listen carefully and support them when necessary. She made the speaker aware of peer support and facilitated the rest of the class to listen to the explanation and make sense of it while supporting everyone in the class to understand it. This was similar to the pedagogical actions described by McCrone (2005). Although an emphasis was placed on developing a collaborative community, teacher continued to use pedagogical actions to ensure that students did not view this as always needing to agree with their peers. She emphasised mathematical argumentation when working with partners: "I was really impressed with the discussion that was going on when you didn't agree with your partner." This focus led to students attending both to their own thinking and the thinking of others and using mathematical reasoning to agree or disagree.

In summary, the teacher actions are illustrated in Table Three.

Table 3
Teacher Actions to Develop Classroom Practices that Provide Opportunities for Engagement in Algebraic Reasoning

Phase One	Lead explicit discussion about classroom and discourse practices Ask students to apply their own reasoning to the reasoning of someone else Require students working in pairs or small groups to develop a collaborative solution strategy that all can explain
Phase Two	Require that students indicate agreement/disagreement with part of an explanation or a whole explanation and provide mathematical reasons for this Lead explicit discussions about ways of reasoning Provide space for students to ask questions for clarification Request students to add on to a previous contribution Ask students to repeat previous contributions Use student reasoning as the basis of the lesson Facilitate students to examine solution strategies for similarities or differences
Phase Three	Lead explicit discussion about mathematical practices Sequence solution strategies to advance mathematical thinking and reasoning Provide space for students to question for justification

Teacher Actions to Develop Mathematical Practices That Support the Development of Algebraic Reasoning

Prior to the PD, key mathematical practices such as making conjectures, developing generalisations, justification and proof were not established within the classroom. The introduction of key mathematical practices associated with algebraic reasoning was important aspects to support student engagement with algebraic reasoning. In the first phase this included the new expectation that students would explain and clarify their ideas and reasoning. In the second phase of the study, a key shift for the teacher was her emphasis on facilitating student development of mathematical explanations rather than continuing to provide the majority of explanations herself. To achieve this, the teacher trialled the use of prompts such as: "I want you to think because I'm sitting here and I'm dead confused, how you could explain it to us. So I'm not just interested in your answer, I'm interested in you explaining it."

The introduction of the mathematical practice of using representations was an important aspect in the second phase of the study. This included facilitating students' use of representations as a key way for them to support their own reasoning and to access the structure of tasks and develop understanding. The teacher also promoted the use of different representations (e.g., verbal, concrete materials and written) as a way of developing the clarity of explanations and to link tasks and representational forms. In the final phase, the teacher continued to encourage use of multiple representations. But more than just using a selected representation, she now developed an expectation that the students would translate between different representations. This included asking students to draw on multiple representations in relation to a task and to listen to explanations by their peers and then to use an alternative representation for the explanation.

In the second and third phase of the study, the teacher introduced her students to the mathematical practices of generalisation, justification, and proof. She began by purposefully planning an investigation of identity elements similar to the approach advocated by Carpenter et al., (2003). This familiarised students with the processes of making conjectures and finding examples to illustrate these. The teacher initiated a growing expectation that generalisations would be expressed and treated as conjectures. In doing this, she facilitated a 'conjecturing atmosphere' such as described by Bastable & Schifter, (2008) and Mason (2008) where students readily expressed conjectures. This meant that the teacher was able to draw on the conjectures and then use these to engage students in the mathematical practices of generalisation, justification and proof. Also in the third phase, representations were introduced as a powerful form of concrete justification. With further classroom experiences focused on justification, students more readily drew on material to prove reasoning.

In summary, the teacher actions are illustrated in Table 4.

Table 4
Teacher Actions to Develop Mathematical Practices that Support the Development of Algebraic Reasoning

Phase One	Require students to explain their reasoning
Phase Two	Require students to develop mathematical explanations that refer to the task and context. Facilitate students to use representations to develop understanding of algebraic concepts. Ask students to develop connections between tasks and representations. Provide opportunities for students to formulate conjectures and generalisations in natural language. Lead students in examining and refining conjectures and generalisations. Listen for conjectures during discussions. Facilitates examination of these. Require use of different representations to develop the clarity of explanations. Model and support the use of questions that lead to generalisations; Does it always work? Can you see any patterns? Would that work with all numbers?
Phase Three	Listen for implicit use of number or operational properties. Uses these as a platform for students to make conjectures and generalise. Facilitate students to represent conjectures and generalisations in number sentences using symbols. Ask students to consider if the rule or solution strategy they have used will work for other numbers or for a general case. Promote use of concrete forms of justification. Require students to translate between different representations.

Conclusions and Implications

This study sought to illustrate the pathway that a teacher took in shifting her practice to integrate algebra into her everyday mathematics lessons. Similar to the findings of other researchers (Bastable & Schifter, 2008; Blanton & Kaput, 2005), it was evident that it is the teacher who makes the integration of algebraic reasoning into the learning community possible. The findings highlight the important role that the teacher takes in implementing and leading change within the classroom. In the first phase of the study, although the teacher began to consciously plan to integrate algebra into lessons, some of the existing classroom practices limited opportunities for engagement with algebra. Through the second and third phase, the teacher continued to extend her planning for algebraic reasoning and also began to notice and respond to spontaneous opportunities during lessons. Increasingly, the classroom practices and mathematical practices supported the students to engage with algebraic reasoning. These changes meant that the students became engaged in the key mathematical practices linked with algebra.

Overall, this study illustrates that the integration of early algebraic reasoning requires more than the introduction of algebraic concepts. It was necessary for the teacher to reflect on both the planning and implementation of tasks. Also of importance was attending to the development of the classroom community and facilitating the growth of classroom practices and mathematical practices that supported collective student participation and engagement with algebraic reasoning.

Practical Implications

A challenge for teachers in recent years has been to develop classroom contexts that integrate arithmetic and algebra and facilitate learners to shift from arithmetical to algebraic reasoning. The results of this study provide some important practical implications for thinking about ways in which early algebraic reasoning can be integrated into primary mathematics classrooms. A clear contribution is seen in the broad perspective of algebra that is taken to include both areas of content and classroom and mathematical practices that support student engagement in algebraic reasoning.

The *Framework of Teacher Actions to Facilitate Algebraic Reasoning* that is outlined in the paper is offered as a contribution to the field. Importantly this framework integrates four separate, interlinked components that the study identifies as key to the development of early algebraic reasoning. These include:

- Teacher awareness of and a purposeful focus on algebraic concepts
- Teacher actions to develop and modify tasks and enact them in ways that facilitate algebraic reasoning
- Teacher actions to develop classroom practices that provide opportunities for engagement in algebraic reasoning
- Teacher actions to develop mathematical practices that support the development of algebraic reasoning.

Each of the four key aspects integrated within the framework has been linked with specific supportive teacher actions. Based on evidence of 'what works' in terms of teacher practice, this is an important contribution to enhance professional learning and development opportunities to build capacity to enact reforms in early algebra teaching and learning. This framework can be used both by teachers to investigate and develop their own practice and as a productive model for researchers and designers of professional development to use while working with teachers.

This study illustrates the complexity and challenges of teacher change and enactment of changes within the classroom. The integration of algebraic reasoning into classroom mathematical activity was a gradual process. It required a focus on developing teacher understanding of algebraic concepts and involved changes to task implementation and design, shifts in pedagogical actions and the facilitation of new classroom and mathematical practices. It is important that teachers view algebra as encompassing classroom culture. This means that both pedagogical content knowledge of algebra and a focus on classroom and mathematical practices that facilitate algebraic reasoning opportunities needs to be incorporated into professional learning and development.

Of importance is the need for teachers to develop understanding of algebra beyond their schooling experiences. Initially the teacher in this study held understandings of algebra that were grounded in her own schooling experiences. This involved more traditional approaches where computational arithmetic was taught in primary school followed by the introduction of abstract algebra in secondary school. In her own words, she described her previous view of algebra as: *the missing number and shoving in an X here*. An important factor in the shift in the teacher's understanding and practice was the re-conceptualisation of her understanding of algebra.

Planning for algebraic opportunities was a key element in the teacher's development. However, an important implication for both teachers and teacher educators is that simply planning and developing algebraic tasks is insufficient to ensure that early algebra is integrated into mathematics lessons and learners shift from arithmetical to algebraic reasoning. Attention also needs to be focused on how tasks are implemented and enacted in the classroom. Enacting a task successfully requires teachers to identify the focus of the task, the purpose of any adaptation, and anticipate the possibilities that may happen in the task enactment. The framework provides some key teacher actions that relate to task implementation and enactment. It highlights the importance of implementing tasks in ways that focus on structural and relational aspects as well as drawing on spontaneous opportunities arising from both task enactment and student responses to engage all students in algebraic investigation.

Also evident from the findings of this study is that there are a number of key pedagogical strategies and classroom and mathematical practices that support student engagement in algebraic reasoning. Understanding of the classroom and mathematical practices that link to the development of algebraic reasoning are a further key aspect of teachers developing classrooms that integrate algebra into everyday mathematics lessons. The teacher in this study progressively introduced new classroom practices. There was an increased expectation on students to talk and work collaboratively. This collaborative work included developing shared understanding of a jointly constructed solution strategy. Another key emphasis was on student development of mathematical explanations. Also illuminated in this study is the importance of teacher understanding of mathematical practices such as generalising and justifying. An initial lack of understanding of these mathematical practices resulted in the teacher shifting student focus from general cases to specific examples. Developing understanding in this area enabled the teacher to draw on student generated conjectures and use these to engage students in justifying and generalising.

In summary, the important implication of this study for both teachers and teacher educators is that if we want to develop classroom contexts in which early algebra is a focus and students engage in algebraic reasoning, we must take a multi-faceted approach that addresses not only algebraic concepts but also task design and implementation as well as classroom and mathematical practices.

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