Authority and Agency in Young Children's Early Number Work: A Functional Linguistic Perspective.

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This paper presents a preliminary study of three six year-old children's use of functional language when engaging collaboratively on a mathematics task. The analysis is presented as an illustration of young children's authority and agency in mathematics as evidenced in their discourse. Modality, as a function of language, was seen to indicate reasoning as a semantic process that expressed a state of knowledge as the children explored number comparison relationships. It is proposed that the children's use of modality indicated an element of internal authority in arbiting mathematical correctness and that related to the nature of the task.

Introduction

It would seem a worthwhile aim to encourage learners to participate as creative agents, who think and reason for themselves in mathematics, rather than being passive recipients of knowledge (Boaler & Greeno, 2000). Bandura (1997) considered how self-efficacy related human agency to the capacity to coordinate learning and motivations. Hence self-efficacy and agency would seem important in supporting learners to be creative agents.

This paper presents a preliminary study of a task that intended to support collaboration and self-efficacy in young learners. Three six-year old children worked together on a puzzle designed to encourage the children to think and reason for themselves. The episode was analysed in relation to the children's use of language as evidence of authority. Whilst just one episode, the analysis provides a rich interpretation of the children's engagement with the task, from which more can be learnt about children's agency and self-efficacy in mathematics, and the nature of supporting tasks.

Exploring Agency and the Use of Language in Mathematics

Lange (2009) defined human agency as the "faculty to act deliberately according to one's own will and thus to make free choices" (p. 2588). This interpretation of agency is extended further by Pickering's (1995) and Cobb, Gresalfi, and Hodge's (2009) distinctions between human agency (choice and discretion of a learner), conceptual agency (developing meanings and relations between concepts and principles), and disciplinary agency (the established procedures of a discipline). Pickering referred to the free and forced moves between these distinctions as a 'dance of agency.' As such, the choices made in learning mathematics are forced or tempered by the intrinsic authority of the discipline of mathematics, and agency in mathematics is further defined as the opportunity to exercise discretion in making choices by drawing on mathematics ideas to solve problems (Grootenboer & Jorgensen, 2009).

Traditionally the teacher has authority in influencing, or controlling, the flow of ideas in a mathematics classroom (Amit & Fried, 2005). That is, the teacher controls the 'dance of agency' and shapes the authority of knowledge for the students. However, if students control the 'dance of agency', they learn to rely on the disciplinary agency of mathematics, and not the authority of the teacher. Such students would have the confidence to become "arbiters of mathematical correctness" (Schoenfeld, 1992, p. 62). This ability would seem intrinsic to the

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relationship between self-efficacy and agency, and so lead to students as creative agents who think and reason for themselves.

Studies regarding the distribution of authority in teaching mathematics have shown how secondary students rely on their teacher's authority (Wagner & Herbel-Eisenmann, 2009). So it would seem desirable to establish agency with students from an early age. Recent studies have considered play and agency with kindergarten children (e.g. Erfjord, Carlsen, & Hundeland, 2015), but little research has been carried out with primary school children. Whereas there is the potential for free choice in the play activities of pre-school children, primary school children are introduced, more formally, to key mathematical ideas that are often modelled by the teacher. Hence primary school children are required to engage with conceptual and disciplinary agencies of mathematics in a more structured way. That is, they are being led by the teacher to engage in the dance of agency. So, how might we develop tasks where the children are managing this dance of agency, rather than the teacher?

Furthermore, consideration is needed on how to investigate agency and authority in the mathematics classroom. One way is to analyse discourse as patterns of interaction between teachers and students (Wagner, 2007). Other methods of discourse analysis focus on the functional use of language. In relation to Halliday and Matthiessen's (2004) theory of systemic functional linguistics (SFL), functional linguistics provides a way of examining meaning making in a given environment. In examining agency in relation to language use, a key distinction is made between the primary tense, that expresses what is present at the time of speaking, for example 'it is' or 'it isn't,' and modality that expresses certainty or possibility, for example 'it has to be' or 'it can be.' Modality is further divided into deontic and epistemic. Deontic modality indicates the necessity or possibility of acts, that is, socially regulated behaviour, and these are more commonly known to young children (for example, 'you have to sit still' or 'you can't go out to play'). Epistemic modality indicates the speaker's beliefs based on the available evidence (for example, 'that has to be …').

Much mathematical language relies on the use of modality, both deontic and epistemic. The use of deontic modality suggests authority through the control of behaviour in how to carry out a procedure, and epistemic modality suggests the certainties or possibilities regarding mathematics, and hence is part of the dance of agency in relating to the discipline of mathematics. De Freitas and Zolkower (2010) have focused on modality in studying authority and agency of the teacher in mathematics classrooms, but modality has not been used as a focus to study young children's interactions.

Developing the Task

I had been working with a class teacher over a school year to develop tasks to encourage collaboration with six year-old children. The intention was to move away from direct instruction, and to shift authority away from the teacher. As such, the nature of the task was important in providing access to conceptual and disciplinary agency, where the students were put in charge of making decisions. The task presented here was developed as a puzzle with intrinsic logic (see figure 1). In solving a puzzle, as in playing a game (van Oers, 2014), there are rules: rules of the puzzle (how to act based on the rules of the puzzle) and conceptual rules (how to act based on specific concepts). As such the task resembled a play activity, where students make choices, but where the correctness of a choice is based on the rules of the puzzle.

A further aim in developing the task was to support the children's learning in number. In particular we focused on the comparison relationship 'more or less than'. Finding a number with say two more or two less, relies on the comparison of two cardinal units and involves more than counting. The relational nature of numbers is abstract. Relations between numbers do not refer directly to concrete objects; they can only be represented by concrete or symbolic

objects (Steinbring, 2005). This abstract notion was seen as important in supporting the children's learning in number, as they often relied on counting processes.

Structural dot patterns, based on ten frames (see Figure 1), were used to represent cardinal units for comparison, as a way to encourage part-whole thinking rather than counting. Developed from earlier research on young children's counting models and subitising abilities (Steffe & Cobb, 1988; Steffe, von Glasersfeld, Richards, & Cobb, 1983), the importance of pattern and structure in early mathematics learning has now become fully recognised. Studies on children's use of representations and structure, such as egg boxes as six and ten frames, have been shown to support part-whole thinking (Young-Loveridge, 2002). Further studies have examined young children's awareness, recognition, and visualisation of pattern and structure. (Mulligan & Mitchelmore, 2009), and the examination presented here provided an opportunity to explore the use of pattern in comparing numbers.

The task required the students to complete a rectangle by placing ten frame cards, which were more or less than the previous one, according to a given condition recorded on the arrows around the rectangle (Figure 1). For example, the ten frame following the start 10 ten frame could be either two more or two less than ten. Eight ten frame cards with values from three to ten were provided to complete the task. The rectangle was to be closed, meaning that the last ten frame had to meet both the previous and the final conditions. As can be seen in Figure 2, the 9 ten frame has been placed, so that it is three more than the 6 ten frame and one less than the 10 ten frame.

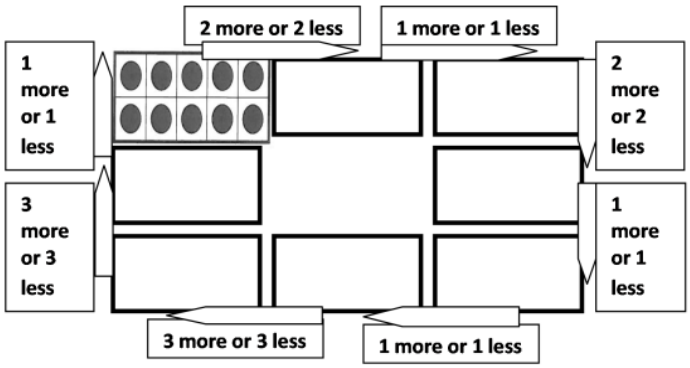

Figure 1. The More or Less task

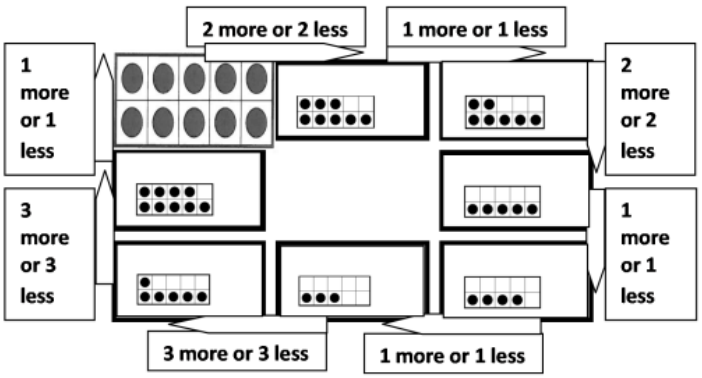

Figure 2. A completed version of the More or Less task

Research Methods

In the episode examined in this paper, the focus is on how the pattern structures, within the nature of the task, might mediate the students' arbitration in determining the correctness of their choices. The task was presented by me, as the researcher, to three six year-old children, Kim, Emma, and Helen. The children had worked together, and with me, on previous collaborative tasks, but the More or Less task was a new introduction. The episode was video recorded and transcribed. The research method followed the principles of the clinical interview (Ginsburg, 1997). As the researcher, I observed, probed, and prompted the children as they worked on a task. The intention was to enter the children's minds, but through discourse analysis that focused on functional linguistics, and in particular the children's spontaneous uses of primary tense and modality. The use of the primary tense indicated what was present and known to the children, and modality indicated the children's reflections on possibilities or certainties. In introducing the task to the three children, Kim, Emma, and Helen, I emphasised that they needed to close the rectangle, so that, for the last space, the ten frame would have to be one more or one less that the first card (the 10 ten frame).

Examples of Critical Incidents in Use of Language

As this was just one episode, analysis was carried out through viewing the video material alongside the transcript. The students' use of present tense and modal terms in critical incidents were identified. Transcripts of the critical incidents are presented below. Actions are presented in italics in parenthesis for clarification. References are made to figures 3 to 6, showing images taken from the video recording.

1. Helen: What shall we do, two more or two less? Two less, two less. *(Helen placed the 8 ten frame next to the 10 ten frame.)*
2. Kim: One less. (*Kim read from the next arrow.)*
3. Helen: It's eight. *(Helen counted the dots on the 8 ten frame and Kim placed the 7 ten frame next.)*
4. Kim: Two more or two less. *(Kim read from the next arrow.)*
5. Helen: Two more. *(Emma handed Helen the 9 ten frame. Helen pointed to two dots on the 9 ten-frame.)*
6. Helen: See there's two more. *(Helen placed the 9 ten-frame.)* (See Figure 3.)
7. Kim: We need to decide which one goes where. Do you want to do one more or one less?
8. Helen: One less, I mean one more *(The children looked at the ten frames they had left (3, 4, 5, and 6) and Helen pointed to the 9 ten-frame and the next space on the rectangle.)*

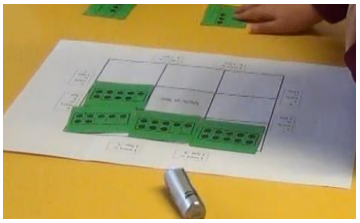

Figure 3. The children's first attempt at completing the task

9. Helen: So that will be one less, it'll have to be one less.

10. Kim: It will have to be eight. No. We have to do one more, because we don't have…. *(Kim looked to the 10 ten frame)* Oh no we have to start again. (*The children removed all the ten frames, apart from the 10 ten-frame, and started again.)*

11. Helen: What about we do this – more, less, more, less, more, less *(Helen pointed to the spaces around the rectangle.)*

12. Kim: But there's no bigger number, that's the biggest number. (*Kim pointed to the 10 ten-frame.*)

13. Kim: Why don't we do this one less, this one less, this one more, this one more, this one less, this one less? What do you want that to be? *(Kim pointed to the spaces around the rectangle and then stopped at the last closing space.)* (See Figure 4)

14. Emma: More.

15. Helen: No less, less.

16. Kim: That has to be nine. *(Kim pointed to the space before the 10 ten-frame and Helen placed the 9 ten frame in the space.)* (See Figure 4)

17. Kim: Two more? *(Kim indicated the space next to the 10 ten-frame.)*

18. Helen: Two less, seven. *(Helen pointed to the space next to the 10 ten frame, and Kim placed the 8 ten frame.)* (See Figure 4.)

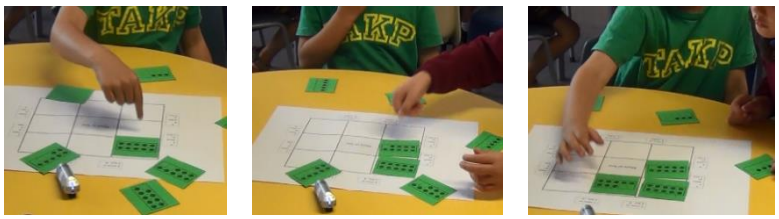

Figure 4. The children start again on the task

At this point the children realised the positioning of the 9 ten frame before the 10 ten frame and, even though Helen had stated seven as two less than ten, Kim placed the 8 ten frame after the 10 ten frame (Figure 4). They then became confused over the positions of the 8 and 7 ten frames.

19. Kim: One more or one less. Eight where's eight? *(Kim suggested eight as one more than seven.)*

20. Helen: On no, this is an eight. *(Helen picked up the 8 ten frame and counted the dots.)* It's eight. That's seven. *(Helen moved the 8 ten frame and placed the 7 ten frame between the 8 ten frame and the 10 ten frame.)*

The children then chose which ten frames to place next, with Kim asking "Which do you want, one more or one less?" but they did not place the ten frames according to the conditions given in the arrows (Figure 5).

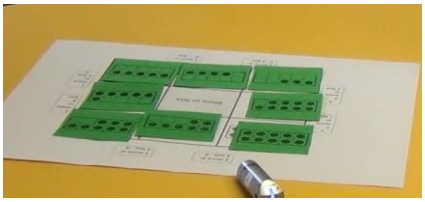

Figure 5. An incorrect solution to the task

All three children turned to the researcher to say "Done!" The researcher returned to the group and asked the children to explain their decisions for placing the ten frames.

21. Researcher: So you decided that one had to be a nine. *(Researcher pointed to the 9 ten-frame.)* Why did you decide that?
22. Kim: Because ten's the biggest number. We can't do one more, that would be eleven and we don't have eleven.
23. Researcher: So why did you decide to put this one there? *(Researcher pointed to the 7 ten frame next to the 10 ten frame.)*
24. Kim: Cos it's, what's this? Because it's…
25. Helen: No it's three less.
26. Kim: It's supposed to be two less. *(Kim swapped the 7 with the 8 ten frame.)*
27. Researcher: *(Researcher pointed to the new position of the 7 ten frame.)* Why wouldn't you put the 9 ten frame there? You could have made that one more?
28. Kim: Cos maybe you couldn't put anything here. *(Kim pointed to the 9 ten frame before the 10 ten frame.)*
29. Researcher: So where do you go after that? You need two more or two less. Can you use nine? *(Kim shook her head.)* So what you are going to have to use?
30. Kim: Five? *(Kim moved the five next to the 10 ten frame but then moved it away and replaced it with the 4 ten frame.)*
31. Helen: No you need the five, you need the five, you need the five there. *(Helen moved the 4 ten frame away and replaces it with the 5 ten frame.)*

The students needed reassurance from the researcher in placing the last three ten frames but they did complete the task with the correct solution (Figure 6).

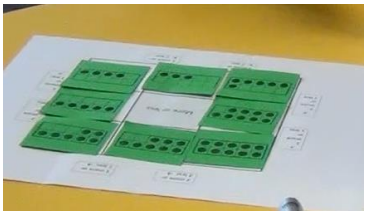

Figure 6. The children complete the task with a correct solution

Analysis and Discussion

In transcript lines 1 to 7 the children used the primary tense. Helen asked, "What shall we do?" suggesting a free choice. The children also used the primary tense in stating "It's eight" or "There's two more." The numbers and the number relationships were present and known to the children. Even though Kim used a deontic modal term "We need to decide," she then asked what they wanted to do, not what they had to do, again suggesting free choice. In transcript lines 8 to 9, it seemed the children realised they did not have a ten frame more than nine, so they decided they needed a ten frame less than nine, and, for the first time, they used epistemic modality in the phrases, "It'll have to be one less" (transcript line 9) and "It will have to be eight" (transcript line 10). As Kim noticed the 8 ten frame had been used, she then used deontic modality in the phrase, "We have to do one more" (transcript line 10). The children were beginning to reason what they had to do, and what numbers were needed to meet the rules of the puzzle, rather than referring to free choice. Hence, the children were beginning to relate to conceptual rules and the rules of the puzzle.

As the children started a second attempt (transcript lines 11 and 13), and plotted possible positions around the rectangle, they seemed to experiment with systems. This systemic approach, whilst still tempered by free choice in choosing a system, suggested the children

were attempting to relate to authority within the rules of the task. Their experimentation with the systems also led them to look at the final closing position. Kim used the primary tense in noting that there was no bigger number than ten (transcript line 12), and then used epistemic modality in the phrase, "That has to be nine," in determining the value of the ten frame in the closing position (transcript line 16 and Figure 4). Ten, as the biggest number, was present and known to the students, but Kim then used epistemic modality to realise a necessary number for the closing ten frame.

The children made an error in stating seven as two less than ten, but the 8 ten-frame was placed next. The confusion with the 8 ten-frame and the 7 ten-frame (transcript lines 19 and 20) resulted in the children checking by counting the dots. The children referred to the numbers on the ten frames and used the primary tense "…this is an eight….It's eight. That's seven," in confirming knowledge that was presented to them. The children then further confused the completion of the task by swapping the 8 and 7 ten frames, but later, after questioning from me, Kim and Helen changed the placing of the 8 ten frame as two less than ten (transcript lines 24 and 25). Kim's phrase, "It's supposed to be…" (transcript line 26) showed use of epistemic modality that linked knowledge of the number on the ten frame with the number relationship and the rules in the task.

Children's use of both the primary tense and modality were further evidenced as I questioned the positioning of the ten frames. For example, Kim's statements "We can't do one more... we don't have eleven" (transcript line 22) and "you couldn't put anything here" (transcript line 28) further suggested Kim was linking the knowledge of the number on the ten frame with the number relationship and the rules of the task. Later, Helen was clear in her use of necessity as deontic modality in the repeated phrase "you need the five" (transcript line 31). Whilst deontic, this use of modality must also have related to epistemic modality in noting that the next ten frame had to be 5.

Analysis of the children's dialogue indicated that, as they realised the limiting conditions in completing the task ("But there's no bigger number...[than ten]"), they began to use modal terms both deontic ("So we have to..." and "You need…") and epistemic ("It'll have to be one less" and "That one has to be nine..."). This suggested they were linking knowledge that was present to them in the numbers as quantities, the number comparison relationships, and the intrinsic logic or rules of the task. As they realised the limitations of their choices, the children moved from free choice (human agency) to conceptual agency. Whilst the children received some prompting from me, as the researcher, the rules of the puzzle in the task had the potential to mediate the children's arbitration in determining the correctness of their choices. Hence authority was determined within the task.

Concluding Remarks

From the analysis of the use of modality in this episode, it is proposed that young children are capable of conceiving of possibilities and certainties and reflecting on these. Modality was seen to indicate reasoning as a semantic process, where it depended on understanding the meaning of the premises and expressed a state of knowledge. It is further proposed that tasks presented as puzzles, with an intrinsic logic, have the potential to support students in determining the correctness of their choices, and in realising the dance between human, conceptual and disciplinary agency. The task was new to these children, and it remains to be seen if further use of such tasks would enable young children to work independently in completing the task. Whilst this task was based on the comparison relationships, more than and less than, the closed rectangle could be further investigated with other number relationships, including multiplicative thinking, and other mathematical functions.

Further studies are needed to investigate how extended use of such tasks might effect a shift of authority in learning from the teacher, and so support learner agency with young children. However, this preliminary study of one episode has shown the potential of investigating agency through discourse analysis focusing on modality as a function of language. More extensive studies, and finer analysis of the use of language using software such as NVivo, would be important in further understanding how tasks can be developed to support young children's self-efficacy and agency in relation to learning mathematics.

References

Amit, M., & Fried, M. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. Educational Studies in Mathematics, 58(2), 145-168.

Bandura, A. (1997). Self-efficacy: The exercise of control. New York, NY.: W. H. Freeman and Company.

Boaler, J., & Greeno, J. G. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.), Multiple perspectives on mathematics teaching and learning (pp. 171-200). Westport, CT: Greenwood Press.

Cobb, P., Gresalfi, M., & Hodge, L. L. (2009). An interpretive scheme for analyzing the identities that students develop in mathematics classroom. Journal for Research in Mathematics Education, 40(1), 40-68.

de Freitas, E., & Zolkower, B. (2010). Discursive authority in the mathematics classroom: Developing teacher capacity to analyze interactions in terms of modality and modulation. Paper presented at the Mathematics Education and Society 6 Proceedings, Berlin, Germany.

Erfjord, I., Carlsen, M., & Hundeland, P. S. (2015, 4-8 Feb 2015). Distributed authority and opportunities for children's agency in mathematical activities in kindergarten. Paper presented at the CERME 9.: 9th Congress of European Research in Mathematics Education, Prague, Czech Republic.

Ginsburg, H. P. (1997). Entering the child's mind: The clinical interview in psychological research and practice. NY: Cambridge Press.

Grootenboer, P., & Jorgensen, R. (2009). Towards a theory of identity and agency in coming to learn mathematics. Eurasia Journal of Mathematics, Science and Technology Education, 5(3), 255-266.

Halliday, M. A. K., & Matthiessen, C. (2004). An introduction to functional grammar. Third edition. London: Arnold.

Lange, T. (2009). "Tell them that we like to decide for ourselves" - children's agency in mathematics education. Paper presented at the Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education. January 28th - February 1st 2009, Lyon

Mulligan, J., & Mitchelmore, M. (2009). Awareness of Pattern and Structure in Early Mathematical Development. Mathematics Education Research Journal, 21(2), 33-49.

Pickering, A. (1995). The Mangle of Practice: Time, Agency, and Science. Chicago: University of Chicago Press.

Schoenfeld, A. (1992). Reflections on doing and teaching mathematics. In A. Schoenfeld (Ed.), Mathematical thinking and problem solving (pp. 53-70). Hillsdale, NJ: Erlbaum.

Steffe, L. P., & Cobb, P. (1988). Construction of arithmetic meanings and strategies. New York, NY.: Springer-Verla.

Steffe, L. P., von Glasersfeld, E., Richards, J., & Cobb, P. (1983). Children's counting types: Philosophy, theory, and application. New York, NY.: Praege.

Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: an epistemological perspective (Vol. 38). Berlin, NY.: Springer.

van Oers, B. (2014). The roots of mathematising in young children's play. New York, NY.: Springer.

Wagner, D. (2007). Students' critical awareness of voice and agency in mathematics classroom discourse. Mathematical Thinking and Learning, 9(1), 31-50.

Wagner, D., & Herbel-Eisenmann, B. (2009). Re-mythologizing mathematics through attention to classroom positioning. Educational Studies in Mathematics, 72(1), 1-15.

Young-Loveridge, J. (2002). Early childhood and numeracy: Building an understanding of part-whole relationships. Australian Journal of Early Childhood, 27(4), 36-42.