Teaching Computation in Primary School without Traditional Written Algorithms

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Concerns regarding the dominance of the traditional written algorithms in schools have been raised by many mathematics educators, yet the teaching of these procedures remains a dominant focus in in primary schools. This paper reports on a project in one school where the staff agreed to put the teaching of the traditional written algorithm aside, replaced with computational strategies. The results reinforce a belief that I have held for many years that the traditional algorithms should be removed from the primary mathematics curriculum.

Background

Computation involving the four operations (addition, subtraction, multiplication, and division) is a major content area in primary school mathematics. Curriculum documents advise teachers to take an approach focusing more on strategies and less on traditional written algorithms. For example, the current Australian Curriculum: Mathematics Version 7.3 (ACARA, 2015) states that students "apply a range of strategies for computation and understand the connections between operations" (p. 5). Despite mathematics education research stating concerns about overdependence on procedural thinking (e.g., Hiebert & Lefevre, 1986), and those stating the benefits of computational strategies as leading to deeper understanding of the structure and properties of numbers (e.g., Plunkett, 1979; Reys, 1984; Thompson, 1999), the development of number sense (e.g., Sowder, 1988), the development of problem solving and thinking skills (Callingham, 2005; Plunkett, 1979), and better alignment of school mathematics to the mathematics used beyond the classroom (e.g., Australian Education Council, 1991; Callingham & Watson, 2008; Hedren, 1999; Northcote & McIntosh, 1999), the teaching of computation in primary classrooms is still dominated by the traditional written algorithms.

The dominance of the traditional written algorithms in schools can be traced back to times before calculation machines had been invented and schools needed to prepare students for jobs where they would need to manually add long columns of figures with accuracy. To enable others to check the calculations a standard method was preferred. Today we have several electronic calculation methods with calculators, spreadsheets, and other applications readily available in all classrooms, as well as in the world beyond the classroom. Back in 1999, Northcote and McIntosh conducted a study into how adults completed computations. In a twenty-four hour period only 11.1% of the calculations involved any written component and 6.8% used a calculator. Today, I would predict that the percentage of adults who used a calculator would be much higher given the availability of these devices, especially on mobile phones. These researchers also found that in 60% of the computations situations only required an estimate for the calculation task. The need for traditional written algorithms in the world beyond school today is limited if not non-existent. However, the need to think and reason mathematically is high. The place of traditional written algorithms as a dominant aspect of primary school programs deserves to be seriously questioned. However, teachers and parents maintain a belief that the place of

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algorithms in primary school is deserved and that to teach mathematics *properly* its inclusion is important.

The Project

I was asked to begin a teacher professional development project with the staff of one primary school north of Brisbane in January 2012. My brief was to work with a team of teachers to develop their mathematics pedagogy The school had conceived and been using a professional development system that involved the use of *experts* working with teams of teachers across year levels using classroom demonstrations and reflection, followed by the teachers supporting their year level peers toward whole school implementation. I was employed to be the mathematics *expert*. The project's overall aim was to improve student learning outcomes. Student data was consulted which included their current NAPLAN data. The school was performing below cohort, below state, and below national average scale scores (see Figure 1). While the data was not excessively below average, the school and system wanted to see it improve. The school staff also articulated a desire for the students to be more confident in their approach to mathematics and to be able to reason mathematically and problem solve, which are proficiency strands in the current Australian Curriculum (ACARA, 2015).

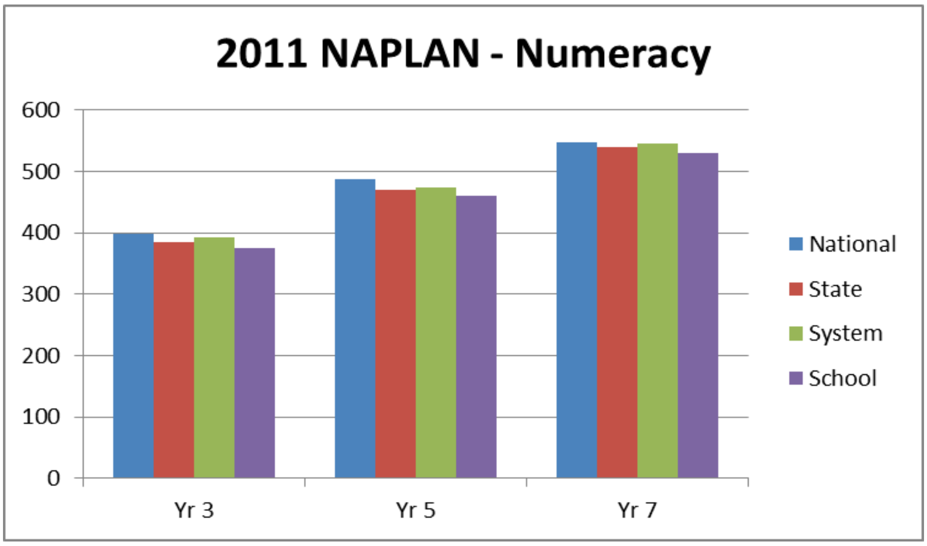
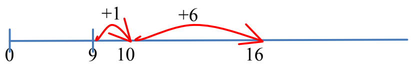


Figure 1. 2011 NAPLAN data for the project school

After discussion with the school administration we decided to focus on encouraging the teachers to challenge the students to think and reason mathematically. We discussed research including work done on student thinking related to the use of computation strategies rather than traditional written algorithms (Hartnett, 2008). The school administration was interested in challenging the teachers to approach the teaching of mathematics in a more investigative way. Given the dominant place of the traditional written algorithm and its procedural focus it was agreed to work with the *Maths team* (at least one teacher from each year level) to develop their own understandings of strategies that could be used instead of algorithms to challenge traditional views of the teaching of mathematics, while offering a professional development pathway that would support the teachers to work differently. The Maths team teachers would then mentor their peer teachers, working with same year level, to change the focus of computation instruction to strategies. The staff, supported by the school administration, agreed to stop teaching the traditional algorithms completely and instead to encourage students use strategies for computation. The school adopted a computation strategy categorisation framework as the organiser of the content to be learnt (based on Hartnett, 2007; see Table 1) and the project began in January 2012.

Table 1.
Categorisation of Computation Strategies used in the Project (based on Hartnett 2007)

Strategy Categories	Examples (addition 27+19)
Break Up 1 Number	27+10=37; 37+3=40; 40+6=46
Break Up 2 Numbers	20+10=30; 7+9=16; 30+16=46
Change 1 Number and Fix	27+20=47; 47–1=46
Change 2 Numbers and Fix	30+20=50; 50–3–1=46
Change 2 Numbers	26+20=46 (27–1 + 19+1)
Count on to Subtract (e.g. 16–9)	

This categorisation framework was chosen for consistency of strategy names across the four operations. The category names described the action of the strategies in language students could understand. In a previous study where this framework was used, students started to use the strategy category labels even though they had initially been designed to assist teachers with their planning for the development of the strategies (Hartnett, 2008). The plan was for the students to make a simple choice between whether they would *break up* numbers, or whether they would *change* one or both of the numbers and decide on the *fix*, if needed. The thinking and number sense required to use the strategies was an identified deficiency with the students at this school.

The project began with Year 3 to Year 7 teachers focussing on introducing the strategies for addition to their students. Teachers in Prep to Year 2 focussed on developing number sense and operation concepts as well as working on basic fact development. The Maths team worked to develop a whole school plan for developing the strategies during the first year of the project. A program of professional development and mentoring was actioned and teachers began to work with their students to develop the strategies and related number sense. Support was provided to the Maths team, as needed, as they worked with their year level peers to introduce the strategies to their classes. Because all of the strategies were new to the staff and the students, most of the first year was spent focussing on strategies for basic addition and multiplication facts and the development of the strategies for addition. The Maths team teachers worked ahead of their peers trying strategies with other operations, as appropriate, and developing lessons and activities to support student understandings and sharing these with their peers.

Initially, students and parents reactions (reflected on through students sharing perspectives from home as well as teachers interacting with parents formally and informally) indicated that the algorithms were viewed as having higher importance than the strategies. It was this perception that influenced the decision in this project to not teach the algorithms at all so as to raise the status of the strategies. Students were not banned from using the algorithms but were encouraged to use strategies and to show their thinking in the way they recorded their responses. This was especially important the older students who were quite familiar with algorithms, but teachers gently shifted the focus from the procedural algorithms to conceptual understanding of numbers and operations.

Observations

The project is ongoing and data presented is observational. The data is anecdotal and qualitative in nature. It is presented as a commentary of the process so far: outlining factors influencing the project, problems encountered, and reflections by the education advisor *expert* supporting the school (the author), teachers, and parents. In the first year of the project, qualitative data was collected where all students in Years 1 to 7 completed a range of computations showing their thinking or working out. Each response was coded for accuracy and strategies used. This data collection has not been repeated as yet. It is planned to conduct this data collection at the end of this year to capture change in the cohort that was in Year 3 at the start of the project. This cohort has not had the traditional written algorithms taught to them at all, unless they have come from a different school. The data would not be able to be used for student comparison but for overall change in the range of strategies used.

At the beginning of the project, students in the upper grades were reluctant to *let go* of the traditional written algorithms they had learned to use already. This was understandable but interesting in terms of their reasoning. When questioned students had difficulty articulating why they preferred the algorithms or why they were not keen on learning other ways to approach the operations. One possible reason was that they were successful with the algorithms and predicted they would not be as successful with something that was new and different. This seemed to be the case with students identified by their teachers as *good at maths*. These students may have decided that it was better to not try than to try and be unsuccessful.

The project included parent information sessions to share the school's direction with the wider community. During these sessions I found there was a need to make a distinction between the use of strategies for computation as an end user beyond school, and as part of a learning program in school. While many parents recognised and acknowledged that some of the computation strategies presented were ones they used in their everyday lives, there were other strategies that they would not choose to use. At school the students were being exposed to a wide range of possible strategies as a learning activity to encourage them to develop their number sense, reasoning, and operation sense as well as the strategies. As the students developed their understandings it was predicted that they too would choose they found personally effective and which made sense to them from the strategies studied. The use of calculation technologies was also discussed as a practical means to finding answers as an end user and that during the learning process the focus was on development of number sense and operation sense that could inform the choice of computational method. This distinction was discussed with teachers in the Maths team during professional development sessions as well.

NAPLAN Data

One set of quantitative data that has been analysed to identify the impact of the project has been the school NAPLAN data. While it is recognised that only a small proportion of the questions on the Yr 3/5/7 tests each year can be directly linked to computation or potential use of computation strategies, the overall aim of the project was to assist teachers to use pedagogy that would improve the students' understanding about maths and their ability to think mathematically so the data could be used to reflect progress on this overall aim. The pre-project NAPLAN data from 2011, the year before the project, is summarised in Figure 1. After three years working with the staff and developing a relationship with them and the students at the school we are starting to see changes. The NAPLAN data below shows the Year 5 and Year 7 school data above system, state and national average scale scores for the first time. The Year 3 data has improved but not passed the other scores (Figure 2). Figure 3 shows the Year 5 data and Figure 4 shows the Year 7 data.

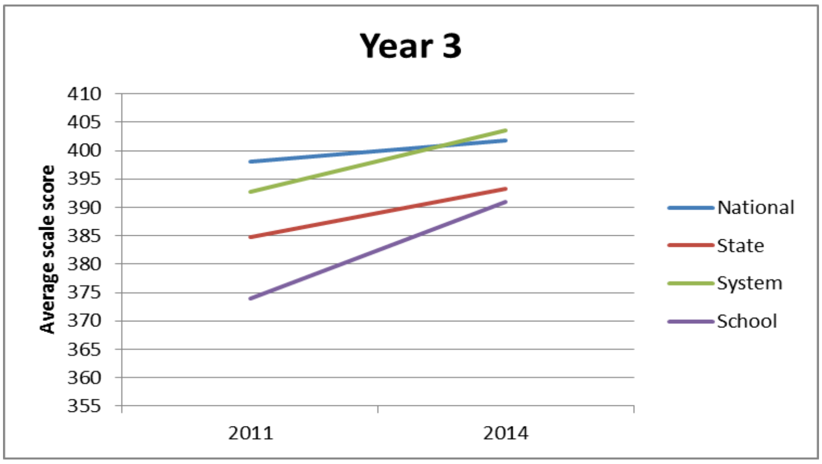


Figure 2. NAPLAN data Year 3 2011 (before the project) and 2014 (current data)

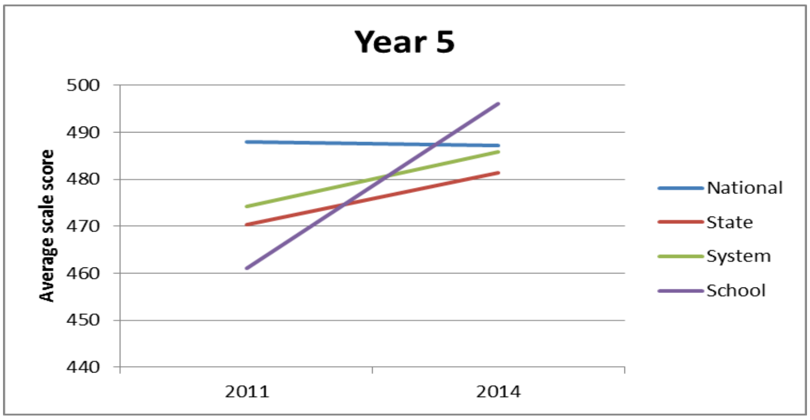


Figure 3. NAPLAN data Year 5 2011 (before the project) and 2014 (current data)

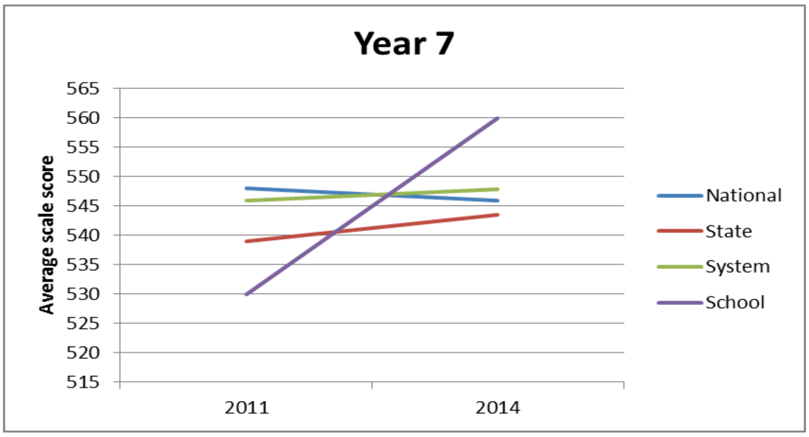


Figure 4. NAPLAN data Year 7 2011 (before the project) and 2014 (current data)

Teacher Reflections

The reflections below provide some further anecdotal data as the project progresses.

After a whole day professional development session with the Maths team where we discussed the computation strategies for each operation, one teacher commented that she could see the reasoning behind the use of strategies instead of the algorithms, but that she still believed there was a place for the traditional methods in primary classrooms. I returned to the school the following week to be greeted by the same teacher who asked me to disregard her previous comments and that she "was now convinced". She had started working with her Year 5 class on a *Break U*p strategy for multiplication where the multiplication was represented using an area model. She reported how the students "loved the strategy" and how it "made so much sense to them" and to her. Her other observation was how confident the students were as they approached multiplication problems; something she had not experienced teaching traditional multiplication algorithms to students this age. (Education Advisor leading the project)

When I began teaching here at [school name] I wasn't sure about how not using an *algorithm* would work. At the end of my first twelve months I was delighted with what I had learned and the progress my class had made in *thinking* about what they were learning and doing in Maths. As I became more confident with the teaching strategies, I was able to clearly see how beneficial it was to teach the children a range of skills, which not only made sense, but also enabled them to solve problems using a variety of strategies, which enhanced their understanding of what they were actually doing. **(**A teacher who came to the school in 2012**)**

I came to [school name] with no concept of these strategies and at first I found it hard to comprehend and tended to stick with the algorithm concept. After teaching these strategies, I found that the students and I really began to improve our mathematical thinking. I was never the strongest in Maths but now I have learnt many new strategies to work with numbers and no longer need to write down algorithms. The improvements I have made using these strategies has given me the confidence and enthusiasm to teach the children and never use the old methods again. (Yr 5 early career teacher who came to the school in 2012)

Parent Reflections

My daughter seemed to have lost confidence in her ability with maths as she moved from Year 3 into Year 4 in 2013. Her Year 5 teacher last year used the strategies and as the year went on her number knowledge grew. At home we noticed she was engaging in conversation involving maths and she was using strategies in everyday situations, like with her pocket money. We noticed that her confidence grew and are now quite confident she be more comfortable in high school next year with a better attitude to maths. (Parent of a current Yr 6 student)

My son is in Yr 5 this year. At the start of this project, my husband was very resistant to the strategies focus. As Jack has become more proficient he has been able to explain to his Dad how the strategies work. His Dad is now seeing Jack learning rather than just doing it quickly and getting answers. When Jack makes mistakes he can look back and understand what he did. He has confidence and considers himself good at maths. Anyone who can convince my husband he was wrong must be doing something right. (Staff member and parent)

Conclusions

The project is ongoing. Having the opportunity to work in one school on a long-term project has been a factor in the success so far. Being able to build rapport with the staff and students as an *expert* builds their trust in me to lead them through the process. This school entrusted me to lead them to make the decision about this project. It is to their credit that the results are showing improvement in what they set out to achieve–improvement in the students' ability to think mathematically and to be confident users of mathematics and to improve the teachers' pedagogy in relation to mathematics. By changing a very traditional aspect of the school program, we sent a message to the staff and school community that we wanted to do things differently. I had held a belief for many years that changing students perceptions of mathematics as a subject, as well as changing their ability to think and reason mathematically, could be achieved by starting with a change to the focus for computation. This project has allowed me to test this theory.

We have shown that students can be successful in mathematics without the traditional written algorithm as part of the school mathematics program. The traditional algorithms are procedures that can assist students to get answers to computations but by using strategies and number sense instead students gain more than just answers.

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