# Developing a Theoretical Framework for Classifying Levels of Context Use for Mathematical Problems

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This paper aims to revisit and clarify the term problem context and to develop a theoretical classification of the construct of levels of context use (LCU) to analyse how the context of a problem is used to formulate a problem in mathematical terms and to interpret the answer in relation to the context of a given problem. Two criteria and six indicators form the basis of the construct. While this construct connects to a previous classification of uses of contexts, several theoretical considerations are considered and clarified. Quantitative analysis suggests that the construct is effective in distinguishing between LCU.

## Introduction

Over the last two decades the existing research in mathematics education has recommended measuring how well students are able to use their knowledge and mathematical skills and how well they use them to solve mathematical problems embedded in meaningful contexts for students (Blum, Galbraith, & Niss, 2007). Thus, the incorporation of context in problems has been highly recommended by current reform documents and mathematics curricula around the globe (see for example, OECD, 2013) which started to develop new forms of connectedness of the instructional mathematical content by focussing on problem solving, and modelling. The latter was not only because of the potential for "motivating students and for the meaningful development of new mathematics concepts and skills" (Depaepe, De Corte, & Verschaffel, 2010, p.138), but also to develop in students the capability to apply and communicate efficiently the mathematics they know in different real-world and everyday contexts (OECD, 2013).

While the importance of using mathematics problems in context seems to be well acknowledged, this field lacks a firm body of convincing empirical evidence for the effects of context on the students' performance (Stacey, 2015). In addition, clear definitions and classifications of context that guide and synthesise research results in this area are difficult to find. Thus, for the available classifications of contexts it is required to develop more comprehensible and reliable classifications that can be used as an analytical tool in further research regarding the effects of context on the students' performance.

This work is the beginning of a sequence of attempts to revisit and scrutinise a possible effect of the problem context on students' performance by employing a revised distinction and classification of uses of context in mathematical problems. For this paper, the purpose is therefore: (1) to clarify the term (problem) context and (2) to report a theoretical development and a later validation of the construct of levels of context use thorugh a small empirical study.

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## The Definitions of Context and Levels of Context Use

### *(Problem) Context*

Several names and meanings for problem context can be found in the literature. Terms such as cover history, thematic content, content effects, situation, and figurative context are used as alternatives names for the term problem context in the literature. Busse and Kaiser (2003) further refine the notion of figurative context by distinguishing between objective figurative context and subjective figurative context. According to these authors, the objective figurative context refers to "the description of the scenario given in the task [problem]" (p. 4) contrasting with the subjective figurative context associated to the "individual interpretation of the objective figurative context" (p. 4). According to these authors, the objective figurative context is "often implicitly meant by researchers when referring to the context" (Busse & Kaiser, 2003, p. 4). However, it is considered that the latter definition, close to what researchers may intuitively call 'context', is fractional. The objective figurative context, in the way it is stated, seems to draw attention only on what is described in a problem's statement rather than on extra information that is packed within the context in which a mathematical problem is situated, which students may also need to decode sensibly when mathematising a problem. To make sense of the above contention, consider the following. A problem can be related to the estimation of the number of fans attending to a sold out rock concert taking place at a given rectangular field (see Table 1).

Table 1

*Problem context example*

| Rock concert problem* | | | | |
|---|---|---|---|---|
| For a rock concert, a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert? | | | | |
| A) 2 000 | B) 5 000 | C) 20 000 | D) 50 000 | E) 100 000 |

* Source OECD (2006, p. 94)

The rock concert context of the problem is required to find the estimation of the number of people that can be accommodated per square metre. In the problem, context provides a chance to identify assumptions and constraints to use a model and validate the answer in relation the context. The latter is a very precise example that highlights that what is described in a scenario of a given problem cannot always be regarded exclusively as problem context; there is information that surrounds an objective figurative context, which may be also used in the problem. Therefore, and for the purpose of this paper, an operational definition bounding what problem context means is required. Thus, (problem) *context* is defined as follow:

> Context is the information that is contained and, at the same time, surrounds the statement of a mathematical problem that needs to be mathematised. The containing and surrounding information might be necessary or unnecessary for the mathematisation of the problem, but is independent from the problem's syntax and stimulus.

In the above definition, problem's syntax refers to the problem's grammar structure whereas stimulus refers to the actual material about the problem that is presented to the student. While syntax encompasses words, stimulus can involve pictures, graphs, diagrams and formulas, or even to its physical and visual layout, and multimedia material.

To clarify and exemplify the intended definition of context, consider the example provided in Table 1. In the example below, the context is related to a rock concert to be held in a rectangular field of size 100 m by 50 m with all the fans standing. Context involves aspects such as dimensions of the rectangular field, facilities for the crowd (e.g., inside or outside the rectangular field, emergency exits, etc.), and more general aspects of the concert including the purchasing of the tickets, and venue details (e.g., in a stadium). However, not all of them are necessary to mathematise this problem. In fact, only the lengths of the field and the density of the crowd in a rock concert are needed. The estimation of a static crowd, in theory, is straightforward (i.e., area of the field multiplied by density of the crowd). The problem requires students to make and relate their own estimation of the amount of area that a person would take up in such a type of concert in order to solve this problem. The clues 'field was full', 'completely sold out' and 'fans standing' are there to guide students in their estimation. The fact that this is a multiple-choice question further helps them. In the above example, the words and the grammatical structure give the syntax, whereas the physical and visual layout (i.e., the set of words that is presented to the students) provide the stimulus.

## *Levels of Context Use (LCU)*

Several classifications of context can be found in the literature. These describe, on one hand, how context is used to embed mathematics in a scenario without supporting the students' learning and, on the other hand, to provide a model to think with (Vappula & Clausen-May, 2006). Space limitations preclude a proper review of literature here on different classifications of contexts, but see Greatorex (2014). Among categorisations of contexts the LCU introduced and revisited by de Lange in 1979 and 1997 respectively, poses an interest for the author of this paper. This is because de Lange's classification focusses on the mathematical relevance of contexts to solve problems. In this classification, solving a problem requires students to engage, in different degrees, with the problem context. The requirement described above is not well understood yet, hence part of the attention. The LCU were further refined by the Mathematics Expert Group (MEG) for PISA 2003, which de Lange chaired. The three-levels distinguished by de Lange are now described briefly. The *zero order use of context* is when a problem generally involves just mathematical terms, shapes, data, and the translation of textually packaged mathematical problems. Alternatively, some real-world terms might be included just to camouflage or add a little interest to the mathematical operations required. *First order use of context* takes place when the context is "needed for solving the problem and judging the answer" (OECD, 2009, p. 31). Lastly, the *second order use of context* takes place when "students need to move backwards and forwards between the mathematical problem and its context in order to solve the problem or to reflect on the answer within the context to judge the correctness of the answer" (OECD, 2009, p. 31). Information drawn from knowledge of the context (not just the problem statement or known mathematical facts) is required to solve the problem. For example, the problem presented in Table 1 requires for students to make and relate their own estimation of the amount of area that a person would take up in such a type of concert in order to solve the problem. The need for that estimation is what must be realised in students' interaction with the context. These definitions of LCU, however, need clarification; first and second order uses of context blur because in the examination of their descriptions essential differences between them are difficult to be distinguished. Therefore, this paper aims to provide, to the best extent possible, a reliable system to classify mathematical problems in terms of the opportunities that context offers for students to engage with it to solve mathematical problems. The following section sets out the rationale for the theoretical development of the construct of LCU.

## Theoretical Development of a Classification of the Levels of Context Use

This section sets out the rationale for constructing a theroretical framework for the revised classification of levels of context use. The classification to be introduced is a synthesis of previous theory by de Lange (1979, 1997) and work from the OECD (2009) to create a framework that allows classifying this construct in a reliable manner. The theoretical classification of LCU, although somewhat simplistic, comprises a set of two criteria and six indicators -three per criterion-, which are theoretically hypothesised to be important to consider in classifying levels of context use. The rationale behind this construct is based on the mathematical relevance of contexts; thus, *context use* is understood as:

> The degree to which solving a problem requires engagement with the context to either formulate or devise a problem in mathematical terms, solving it mathematically, and to interpret and validate the answer in relation to the context of the given problem.

### *Relationship between LCU and Transition Stages of a Modelling Cycle to Develop the Theoretical Classification*

The criteria introduced relate to the mathematical processes of *Formulation* and *Interpretation* encountered in the transition stages of the modelling cycle of model of mathematical literacy in practice (see OECD 2013, p.7). In general, the use of these particular processes responds to the interplay between the solver and the problem context to formulate a problem in mathematical terms, and to interpret and evaluate the mathematical solution against the context of the problem. Specifically, criteria that underpin this categorisation include the following:

(1) The nature and degree of the opportunities to use mathematics: this includes the provision for the use of mathematics as well as the availability of information, variables and mathematical elements required for formulating the problem mathematically, if any, which might arise from the problem context, and

(2) The degree of interpretation and evaluation of the mathematical results: this includes a judgement of the correctness of the mathematical results obtained from the mathematical model in relation to the context as well as a global reconciliation of the solution process and the problem in context, when needed.

### *The Two Theoretical Criteria Considered to Develop the Classification*

Formulation. The criterion of formulation of the mathematical problem refers to the different degrees that the problem context offers to put into practice mathematics. In general, context could offer room to put in practice mathematics straightforwardly, or to apply or devise a mathematical model accordingly. The latter situations pose a chance for the solver to interact with the context; the interaction with the context may happen at different levels when trying to formulate the mathematical representation of the problem, which pose a different use of the context while formulating the problem.

*Interpretation*. After completing the mathematical formulation of a problem in context, relevant mathematics concepts and techniques are employed to solve it mathematically. Subsequently, context may play a role on the interpretation process. This can be explained by the fact that in this process, the mathematical results, when required, need to be interpreted or validated in terms of the problem context. Hence, this criterion relates to the interpretation of the mathematical results in terms of the context of a problem. It is expected that the combination of these criteria reflects conceptually levels of broadly increasing context use in both the formulation and interpretation processes.

*The Revised Classification of LCU*

The effort of the author of this paper for classifying levels of context use has resulted on a set of two criteria the *Formulation of the Mathematical Problem* (denoted by the letter A) and *Interpretation of Mathematical Results* (denoted by the letter B), and three indicators per criterion. These are presented in Table 2.

Table 2

*Criteria and indicators for classifying levels of context use*

| Criteria | Indicators associated to criteria |
|---|---|
| Formulation of the mathematical problem (A) | (A0) Context is not needed for formulating the problem in mathematical terms. From instructions given, direct actions can be taken. |
| | (A1) Formulating mathematically the problem requires more than just straightforward actions to be taken from the given instructions. One has to identify relevant information, variables, relationships, or make assumptions. Context provides explicit cues for this. |
| | (A2) Given cues are not sufficient and further consideration of the context is needed to retrieve relevant information, variables, relationships, or make assumptions, no longer explicitly hinted. |
| Interpretation of mathematical results (B) | (B0) Context is not needed to interpret, decide or evaluate mathematical results or arguments. |
| | (B1) Context is used to decide whether the mathematical results or a mathematical argument satisfies the requirements of the problem. |
| | (B2) Context is used to judge the adequacy of the mathematical results and arguments in terms of the assumptions made for the mathematical formulation of a problem. That is to say, context and mathematical results or arguments need to be reconciled globally for a valid solution that satisfies the requirements of the problem. |

## *Indicators Associated to the Formulation Process*

Three indicators (A0, A1, and A2) are related to this criterion. These indicators intend to reflect an increasing use of context from the lowest level (i.e., A0) to the highest level of context use (i.e., A2) in the formulation process. If the context of the problem provides room for using mathematics straightforwardly, the indicator $A_0$ is chosen; this is because context is not required to formulate a mathematical problem, context instead acts as a camouflage to use mathematics. In a different manner, the code $A_1$ is assigned if context provides room to identify and represent information mathematically in a mathematical model or argument. Lastly, the code $A_2$ is given if context offers room to retrieve extra information needed to devise a mathematical model or argument that satisfies the requirement of the problem, that is to say, context is importantly required for the mathematical formulation of the problem. Heuristic values associated to $A_0$, $A_1$, and $A_2$ are respectively 0, 1, and 2.

### *Indicators Associated to the Interpretation Process*

There may be a difference in context use when linking the results to the problem context; there have been theoretically identified three different levels of relations in the interpretation process. There may be the case when mathematical results are not needed to be interpreted in again to the context, because the context is not needed or it is disregarded for the mathematical formulation. The other case is when the mathematical results of a problem are needed to be interpreted within the context; that is to say, mathematical results need to satisfy the requirements of the problem in terms of the context of the problem. The final case is when mathematical results need to be justified within the context to the problem; here, the student needs to defend adequate reasons for the mathematical results obtained. For instance, in the PISA Rock Concert problem, the justification of the realistic knowledge related to the number of people that can be accommodated per square metre is essential to solve and support the mathematical result of this problem (e.g., 'I assumed that 4 people can be accommodated per square meter and that number of people seems to be a reasonably estimation for a rock concert'). In these cases, indicators $B_0$, $B_1$, and $B_2$ are assigned correspondingly and their heuristic values are respectively 0, 1, and 2. As mentioned above, these indicators intend to reflect an increasing use of context from the lowest level (i.e., B0) to the highest level of context use (i.e., B2) in the interpretation process.

### *Revised Levels of Context Use: Defining and Profiling the Construct*

To summarise the criteria, indicators and codes from Table 2, a three-level scale is introduced. The scale is presented in Table 3. The overall scale is used to describe and classify the three-levels of context use (i.e., zero, first and second order use of context) introduced previously by de Lange (1979, 1997). This scale is not statistically created, it rather uses a conceptual definition approach to scale ordered outcome punctuation related to the criteria and indicators introduced previously in Table 2. As stated previously, LCU to be introduced consider linkages concerning how the context is used in the formulation and interpretation processes of the model of mathematical literacy in practice. Problems belonging to zero and first orders use of context use should involve the recognition mathematically well formulated problems in context, which include the reproduction of known mathematical operations or procedures. At these levels, an interaction with the context of the problem either is barely or required within limits. In contrast, the second order use of context use should combine the amalgamation of context of a problem and information found in the problem statement to either apply or formulate the problem mathematically. The postulated score range for every postulated category of context use is conceptually drawn from the punctuation of the indicators of every criterion. The indicators associated to the zero level add up to one, hence the possible range in this level is 0. At the next level, the first level, the indicators associated add up to 3, thus the possible range at this level is 1-3. At the final level (second order), scores associated to this level yield a maximum of 4, then, a total score of 4 indicates the second level of context use.

Early versions of Table 2 and Table 3 were subjected to expert advice to provide qualitative feedback on the proposed theoretical classification of LCU. Feedback consisted of participants (six people among mathematics educators and mathematicians)_providing detailed comments on wording of the tables, the clarity and progression of indicators as well as on the proposed three-level classification introduced. After the qualitative process, earlier versions were further refined, resulting in the tables presented above.

Table 3
*The three redefined levels of context use, their score range, and their features*

| Postulated levels of context use | Postulated score range of the level | Level features |
|---|---|---|
| Zero order | 0 | At this level, provides the opportunity to take direct actions or make direct inferences from the instructions given in a mathematics problem. Therefore, context of a problem is not used to interpret mathematical results or arguments. |
| First order | 1-3 | At this level, context is used to either identify or select relevant information, variables, or relationships for the mathematical formulation of a problem. Also, context is used to determine the adequateness of the mathematical results. |
| Second order | 4 | At this level, context is the source to either define or retrieve relevant variables, relationships, or assumptions for the mathematical formulation of a problem. Also, context is used to judge the adequacy of the mathematical results/arguments in terms of the original problem. |

The following section reports a validation of the classification thorugh empirical study.

## Validation Study of the Proposed Classification of the LCU

To assess the extent to which the proposed classification of LCU is reliable among coders, a small validation study was conducted in order to determine how functional and reliable the classification introduced above is for classifying mathematical problems in terms of LCU. Four volunteer coders (coders were students undertaking a Master of Teaching -secondary- at one faculty of education in Melbourne) classified individually two sets of three different mathematical problems (not presented here) at each LCU (problems categorised by the author of this paper using the classification above). The problems in the two sets were characterised by the same common core (i.e., same mathematical content, competency and model), but they differ in the context in which the problem was embedded. Participants used criteria, indicators and the proposed classification from Table 2 and 3 respectively. The coding process took place within a seminar class. A Cohen's κ-test for multiple coders was performed to determine if there was agreement between the coders' judgements.

### *Results of the Validation of LCU*

The results indicated that there was a substantial agreement (Landis & Koch, 1977), $\kappa = 0.611$, $p = 0.005$, that is statistical significant among the coders' judgements in the set of problems. As there was a substantial agreement, there is a possibility, but by no means a guarantee, that the set of criteria and indicators proposed do in fact reflect coders' agreement on the proposed classification LCU. However, this result must be interpreted with caution. This is because despite the agreement among coders for the proposed classification of LCU, the coding process revealed discrepancies between the anticipated coding and the participant's coding. For example, there was complete agreement between the anticipated coding and coders at the zero order use of contexts' problems, whereas inconsistencies occurred when coding second order use of contexts problems (when context is needed to formulate and evaluate the results of a mathematical problem). Some coders classified second orders problem as first orders problems and vice versa.

## Discussion and Conclusion

This paper has revisited the definition of context in mathematics problems and offered a theoretical classification and small validation study of the construct of LCU. The construct is built on a previous classification of levels of context use. However, two criteria and three indicators per criteria are considered to classify functional levels of context use of a mathematical problem. The rationale behind the classification of LCU is based on the mathematical relevance of contexts, which was interpreted here as the opportunities that context offers for students to engage with it to solve mathematical problems. Results of the validation of the theoretical classification may indicate that criteria and indicators in which this classification was built on suggest that the criteria and indicators are reliable, but by no means a guarantee, in distinguishing different levels of context. With a small sample of participants in the validation process, the theoretical development of the classification of the LCU presented here needs more careful testing with deeper analysis and experimental control (e.g., using a broad range of problems) in order to improve the construct's validity and reliability. To conclude, the development of the classification reported on this paper has offered an analytical tool to guide and synthesise further research results on the possible effect of context on the students' performance.

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