Learning from Lessons: Teachers' Insights and Intended Actions Arising from their Learning about Student Thinking

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A central premise of this project is that teachers learn from the act of teaching a lesson and that this learning is evident in the planning and teaching of a subsequent lesson. We are studying the knowledge construction of mathematics teachers utilising multi-camera research techniques during lesson planning, classroom interactions and reflection. This paper reports on the learning of two Year 7 teachers, one in Melbourne and one in Chicago, teaching the same initial lesson focusing on division, remainders and context. Both teachers claimed to have learned about their students' mathematical thinking after teaching the initial lesson, but found planning a second lesson to accommodate this learning challenging.

Literature and Conceptual Framework

Our overarching research question is:

When reflecting on a recently taught lesson, what do teachers claim to learn about their students' thinking and how does this influence subsequent lesson planning?

In recent research, teacher knowledge has been viewed as a key determinant of good teaching practice (e.g., Ball, Thames, & Phelps, 2008). Our project, *Learning from Lessons,* inverts that relationship by identifying productive teacher learning as a consequence and, possibly, a constitutive component of good teaching practice.

Research is clear that the teacher is the key to improved student learning (e.g. Ball et al., 2008; Hattie, 2003; OECD, 2014). Both teacher knowledge and teaching quality are the focus of current research internationally (MET, 2013; OECD, 2011). Teacher knowledge construction *in situ* is less well researched. Margolinas, Coulange and Bessot (2005) and Leikin and Zazkis (2010) are useful exceptions. A model of teacher professional growth (Clarke & Hollingsworth, 2002, see Fig. 1), provides the orienting framework for our research.

An essential aspect of this model is the mediating role played by *Salient Outcomes* (those outcomes of classroom practice to which the teacher attaches significance), which provide both the basis for change in beliefs and knowledge and, once changed, the motivation to engage in classroom experimentation in recognition of changes in those outcomes considered salient by the teachers.

The research design recognises the lesson's function as a catalyst for teacher knowledge construction. Purposefully-designed and trialled lesson plans are a key design feature in catalysing teacher learning. Data include the teacher's adaptation of the pre-designed lesson, the teacher's actions during the lesson, the teacher's reflective thoughts about the lesson, and the consequences for the planning and teaching of a second lesson. It has already been demonstrated (Clarke, Clarke, Roche, & Chan, 2015) that through the teaching of a mathematics lesson and the planning and delivery of the next lesson teacher

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learning can be investigated in a structured fashion. Creating optimised controlled conditions for these teacher actions can therefore provide an effective tool for researching teacher learning and for its promotion.

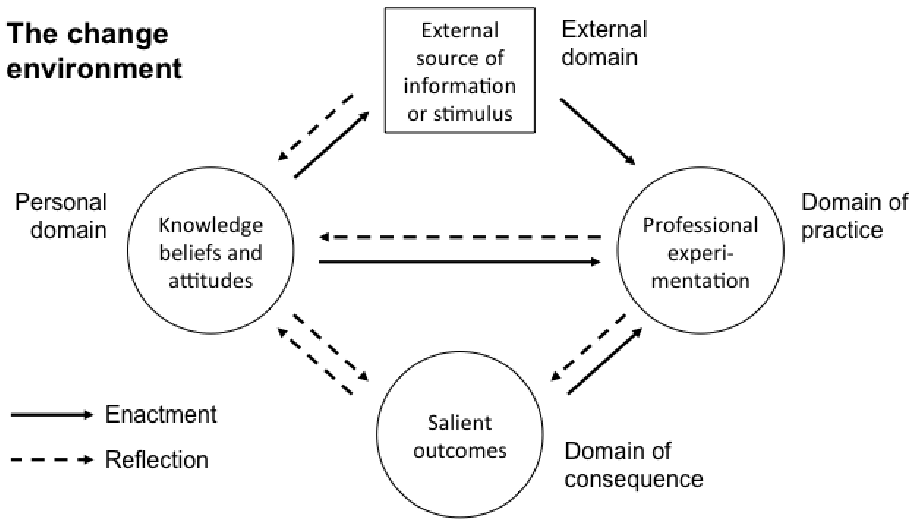

Figure 1. The Interconnected Model of Teacher Professional Growth (Clarke & Hollingsworth, 2002).

The “pre-designed lesson” discussed in this paper involved division, remainders and the role of context. Given the focus of this paper on student thinking, the literature related to this mathematical content is now discussed. Carpenter, Lindquist, Matthews and Silver (1983) reported on the results from the third National Assessment of Educational Progress in Mathematics in the USA. They explained that when solving the following problem: “An army bus holds 36 soldiers. If 1128 soldiers are being bussed to their training site, how many buses are needed?” 47% of 13 year olds tended to ignore the reasonableness of the answer in relation to the problem situation, reporting a non-integral number of buses as the answer, and that 30% performed the incorrect calculation. Problems of this kind have been the focus of much research in the past (e.g. Nunokawa, 1995; Silver, Mukhopadhyay, & Gabriele, 1992; Silver, Shapiro, & Deutsch, 1994).

Silver, Shapiro, and Deutsch (1993) found that students had difficulty in interpreting the result of their calculation consistent with the context of the problem. They investigated the written solutions of students in Grades 6, 7 and 8 to three versions of a division word problem where the solution differed only in the size of the remainder. The context of the problem involved determining the appropriate number of buses to transport a large group of people. The students were asked to show their work, record the solution, and explain their reasoning. The study found that overall students could complete the calculation correctly, but only 45% provided a whole number solution that was consistent with the meaning of the problem. Only one-third of the students provided an appropriate interpretation of their solution to the calculation, and approximately one-half explained their reasoning inadequately, by simply providing a description of the algorithm or procedure they had used.

Silver, Mukhopadhyay, and Gabriele (1992) demonstrated that students’ appropriate interpretations of these problems improved with exposure to a series of such problems. The present study provided an opportunity to describe teacher learning about student thinking in the context of the use of these kinds of problems.

Research Design

In a previous paper, we outlined in detail the process for data generation (Clarke et al., 2015). In brief, teachers were given purposefully-designed experimental mathematics lesson plans, which provided the initial context for this study of teacher selective attention, reflection and learning. During a preparatory (pre-active) interview, the teacher was asked to complete the same mathematics tasks as those employed in the lesson about to be taught. The teacher then annotated the pre-designed lesson plan to meet the needs of her students. A pre-lesson interview just before the lesson focused on the teacher's thinking regarding the lesson to be taught. An open-ended interview protocol offered teachers the opportunity to discuss such things as: key mathematical or pedagogical points, likely student difficulties, anticipated important moments in the lesson, intended student learning outcomes, and so on.

The initial lesson chosen for the part of the study reported here was called *Division with Remainders,* developed by Peter Sullivan (see Clarke & Roche, 2014, for a complete lesson outline). The word problems for the students to complete all involved the calculation 1144 ÷ 32 (see Figure 2). However, the solutions were not all 35.75, as this depended upon the context of the problem. The students were allowed to use a calculator, and were required to explain their reasoning. The lesson was chosen because the authors had found previously that it provided considerable opportunity for teacher learning.

1. We need to book buses to take all the students in the school to a concert. There are 1144 students and each bus can take 32 students. How many buses do we need to order?
2. We are making up packets of chocolates. Each packet must have exactly 32 chocolates. If I have 1144 chocolates, how many complete packets can we make?
3. Our basketball club won a prize of $1144. The 32 members decided to share the prize exactly between them. How much money will each of them get?
4. The train from Melbourne to Sydney travels at an average speed of 32 km/hr. How long would it take to travel the 1144 km to Sydney if the train does not stop?
5. Our year level of 32 students together won a prize of 1144 pizzas. If we share the prize equally, how much pizza do we each get?
6. There are 1144 people who need to cross a crocodile infested river. The ferry can carry 32 people each trip. If everyone is in a hurry to cross the river, how many people will be left for the last trip?

Figure 2. Sample problems from Division with Remainders.

The teacher taught the lesson to their usual class, and the teacher was then asked to identify salient events in the lesson, sometimes referring to the lesson video. Teachers were then asked to develop a written plan for "a follow-up lesson" (Lesson 2) using a structured template provided by the researchers, offering the opportunity to build on the first lesson (Lesson 1), in relation to content, student understanding, student engagement, instructional flow, classroom organisation, or any other aspect of the previous lesson that the teacher might consider salient. The interview process for Lesson 2 (L2) was similar to that of Lesson 1 (L1).

One week after the filming was completed, teachers were given a written assessment of content knowledge and pedagogical content knowledge and a beliefs survey adapted from the test instrument developed for the 17-country TEDS-M study (Tatto et al., 2012).

Data Analysis

The analysis reported in this paper focused primarily on interview data for two experienced Year 7 teachers ("Amy" in Chicago, "Jennifer" in Melbourne), supplemented by results from the TEDS-M instruments (Tatto et al., 2012). All interviews were fully transcribed.

For this paper we focus on what the teachers learned with respect to students' thinking. That is, student thinking about the mathematics, student thinking when making sense of the problem and the solution, and articulating this thinking in written and spoken words. We anticipated that this particular lesson lent itself to selective attention to student thinking.

In reading through the transcripts of all interviews with both teachers, we recorded all instances where the teacher claimed to have learned something about student thinking when asked (i.e. knowledge claim), or if they reported "surprise" when they noticed that something had occurred, or indicated that they had not anticipated that this might occur (i.e. indicating lack of knowledge). If a form of student thinking was anticipated prior to the lesson (i.e. incorrect or correct), and then noticed during the lesson, this was not considered to be teacher learning. We also took learning to have occurred if their practice was adapted in light of what was observed or heard (i.e. adaptive practice) leading to the inclusion of novel practices (for that teacher) in the plan for L2.

Results and Discussion

We found it helpful to first consider what the teachers claimed to know (or anticipated) prior to teaching L1, as this helped us separate those things they noticed that were "new" or "learned" from those they already knew or anticipated. Prior to teaching L1, both teachers suggested possible misunderstandings or errors that they anticipated their students having, as well as those problems they considered would not be a challenge for their students.

Jennifer anticipated that her students might incorrectly answer 35.75 for the buses and chocolates problems (see Fig. 2), where the context requires a whole number solution, and that some would incorrectly report 35.75 hours as 35 hours and 75 minutes for the train problem. We were not surprised that Jennifer anticipated these errors in her students because she made the error of not providing whole number solutions to the bus and chocolate problems when she worked through the problems herself in the pre-active interview, and the incorrect interpretation of the decimal as minutes was provided as a sample student error in the lesson plan. She also anticipated that some students would have difficulty explaining their reasoning. She believed the sharing money and sharing pizza problems would not be difficult for her students.

Amy did not anticipate that the first two problems would be difficult for her students. She believed that her students would realise that "I can't cut a bus in half so I need to round up to the next bus or whatever" (Pre-L1). She also believed that the sharing money problem would not be difficult. However, she did anticipate that the train question involving rates and time would be difficult: "So I think the hardest part for them is going to be when they have to interpret the decimal as part of another type of measurement. So 0.75 of a..." (Pre-L1). She also anticipated that her students might struggle with explaining their reasoning.

Both teachers commented that the "most important part of the lesson" was that their students might learn to interpret the result of the calculation or make sense of the solution. Given that this appeared to be a salient outcome for both, it is likely that this belief influenced those lesson elements or events to which they attended. Jennifer and Amy spent around 60% and 53% respectively of lesson time "roaming" the room listening to or communicating with individuals or pairs of students about their thinking and solutions. Jennifer chose not to provide support or scaffolding around incorrect thinking but used this time solely to learn about her students' thinking rather than to direct or guide it. However, Amy affirmed correct answers and probed faulty thinking.

For each teacher we describe what they appeared to have learned from L1 (with regard to student thinking) and the consequences of this for the planning of L2.

What Jennifer Claimed to Learn from Lesson One

Jennifer commented on three aspects of her students' thinking that she had not anticipated or which had surprised her in the first lesson. While Jennifer had anticipated that some students might struggle with explaining their reasoning, she was surprised about "how many kids couldn't reason" (Post-L1). She noticed that many of them left the reasoning aspect of a question blank or just explained the division process and not the logical sequence that led to an answer consistent with the context of the problem.

She also noticed one student wrote "35 r 24" as the solution for a number of the questions and this surprised her. Both of these errors we would argue are a result of a lack of sense making of the solution, consistent with the problem situation.

After Jennifer examined student worksheets between L1 and L2, she noted a pair of students who solved two of the later problems by dividing 1144 by 32, and then the solution by 32 again. Only one other student appeared to have a similar difficulty of not determining the appropriate initial calculation.

What Amy Claimed to Learn from Lesson One

Amy commented on three types of faulty thinking that she had noticed but had not anticipated. The first came in the lesson introduction from a discussion of the meaning of each number in the equation $7 \div 3 = 2$ R 1. The associated scenario was also provided: "There are seven pieces of candy shared between three kids". She noticed that some students did not interpret the "R" as a remainder but instead considered it a variable.

Amy also noticed that some students were rounding up to 36 when the calculator showed 35.75. She questioned some students and realised their reason to "round up" was not because an extra bus was required for the remaining 24 students, but that they had mechanically rounded 35.75 to 36, and that if the calculator had shown 35.2 they would have rounded down. Thirdly, she noticed that for the money question, some students were writing the solution as 35 remainder 75 rather than $35.75.

The Planning for Lesson Two

Prior to L2 (the lesson to be created by her, building upon what was learned from L1) Jennifer identified four issues for her students that would be the focus of L2. They were incorrectly concluding 35.75 hours represented 35 hours and 75 minutes, and incorrectly choosing not to "round up or ignore the remainder", and recording 35 r 24 for all solutions. She also hoped her students might improve on how they explained their reasoning. Jennifer had anticipated three of the four issues prior to L1. When asked what she hoped her students would learn, she suggested calculating parts of an hour, which she referred to as conversions or ratios (e.g., "half an hour that's equivalent to 30 minutes"*)*.

For L2, Amy chose to focus on similar questions to the first two from L1 in which the students needed to determine if the context required a whole number solution. Amy had not anticipated this issue prior to L1. She also wanted to focus on the issue of making sense of 35.75 with regard to hours, which she had anticipated. For L2, Amy hoped her students might also learn to connect the mathematics with real life experiences, which we consider synonymous with "making sense" of the problem. Her focus for L2 appeared to be related to those aspects of students' thinking that she had learned from L1.

Both teachers chose to create more word problems involving division with remainders in which the students had to make sense of the solution. This appears to be evidence of adaptive practice as it seemed that both teachers had not used these problem types before. We would argue that some were not well-constructed, but this reflects inexperience in the use of these types of problems by the teachers, rather than lack of learning.

Jennifer created seven questions (three were multiple choice) and the students were again expected to explain their reasoning. However, the lesson concluded after only three questions were attempted and discussed, with the whole class moving through them in lockstep manner. Jennifer admitted that she struggled to plan the lesson by herself and this may have been connected to her unfamiliarity with the content. She also said "we keep saying that these misconceptions tend to happen throughout the years but we've never formally done anything about it" (Post-L2).

Amy created three questions for the warm up and then a series of cards that contained division word problems that the students shared in pairs. She also showed students the model of a double number line with which the students attempted to match some decimal portion of an hour with its equivalent in minutes. After L2, Amy reflected that this part of the lesson may have been promoting a procedural understanding rather than conceptual.

> I was worried that I was getting to a point that they were following a procedure maybe, but not really making sense of it. So they were doing it and they might even have gotten the right answer, but I'm not sure if they were realising why it was working to get them the result (Post-L2).

Amy explained that students may be already predisposed to expect certain solutions if their classroom experiences have promoted that belief.

> But we've, for so long, given students so many experiences where the answer is a whole number answer. Throughout much of their education, they know that they've got a right answer probably if it's a whole number answer. If they've got a decimal answer, they probably made a mistake somewhere. They've come to that realisation not because of their problem, but because of the way that we've taught them. (Pre-L2)

The way in which the teachers' own understanding of the relevant mathematics was structured seemed to make understanding students' thinking and acting upon it challenging for them. The TEDS-M data placed Jennifer at the 58^{th} and 52^{nd} percentile approximately when compared to graduating primary preservice teachers in the 17-country study, for mathematical content knowledge (MCK), and pedagogical content knowledge (PCK), respectively. For Amy, the figures were the 68^{th} percentile and 99^{th} percentile, respectively. The TEDS-M results were therefore quite different for the two teachers, but both teachers commented that this (division with remainders) was not content they had taught in the past, and this may explain the similar confusion around interpreting the students' thinking (Jacobs, Lamb, & Philipp, 2010). In this study, it appears that the MCK and PCK specific to the particular mathematical focus of the lesson was key to appropriate follow up in the second lesson.

Conclusions and Implications

Our findings show that teachers do learn from the teaching and planning of mathematics lessons. Jennifer and Amy gave great importance to their students' understanding and listened carefully to determine their students' thinking. In relation to our underpinning model (see Figure 1), their "professional experimentation" (i.e. the teaching of the provided lesson) on reflection led to learning about their students' thinking, which they clearly regarded as salient. They reflected upon the common misunderstandings that they had noticed, and planned a follow-up lesson to attempt to address these. However, they were not always clear about the precise nature of the misconception (e.g. how 0.75 and the remainder of 24 were connected). Given that this content was new to these teachers and their students, it is understandable that the appropriate next steps would not necessarily be clear for them. Nonetheless, the uncertainty arising from the novel content served to illustrate even more graphically the content-specific nature of both MCK and PCK and to re-emphasise the incremental and iterative nature of teacher in-class learning.

Jacobs, Lamb, and Philipp (2010) showed that experienced teachers were more able to recall students' strategies than novice teachers and were more likely to interpret them appropriately. They suggested one reason for this may be that "they had learned to chunk strategy details in meaningful ways" (p. 184). Teaching is incredibly complex and teachers cannot be expected to attend to and respond to everything that arises in a lesson, particularly one as potentially rich as this one. However, Jennifer and Amy might benefit from knowing categories of faulty thinking in these kinds of problems; such as distinguishing an incorrect procedure (or choice of operation) from ignoring the context.

There is an emerging literature in mathematics education around teacher noticing (Sherin, Jacobs, & Philipp, 2011; Thomas et al., 2014/2015; van Es & Sherin, 2008), referred to in our study as teacher selective attention. In that literature, the process of noticing commonly included three phases: attending, interpreting, and deciding how to respond. In this study, the teachers anticipated and noticed salient aspects of student thinking (attending), but sometimes had difficulty interpreting students' faulty thinking, and deciding on how to respond in the next lesson.

The data from this study suggest that whatever a teacher's general mathematical or pedagogical knowledge might be, it is the MCK or PCK specific to the task in hand that determines both the teacher's instructional effectiveness and their capacity to engage in further learning with respect to the teaching of the relevant content (see also, Fennema & Franke, 1992).

If, as we surmise, major opportunities for teacher learning arise through the daily experience of teaching lessons, then a better understanding of this learning process should provide a key to enhanced teacher learning (and therefore teacher performance) on a much larger scale than could ever be accomplished through professional development courses. However, as the experience of these two teachers indicates, the challenge of building on this learning in subsequent lessons remains a challenge, particularly when operating in unfamiliar mathematical territory.

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