History-infused Lessons in Introductory Calculus at the Secondary level: Students' Learning and Perceptions

Wei Beng Poh
Ministry of Education Singapore
poh_wei_beng@moe.edu.sg

Jaguthsing Dindyal
National Institute of Education Singapore
jaguthsing.dindyal@nie.edu.sg

A history-infused lesson package developed by a team of teachers in a professional learning community was used to teach introductory calculus in a secondary school. First, we report a quasi-experimental design that showed that students in the experimental group performed significantly better than students in the control group. Second, we report on the qualitative data collected from a larger group of students who were generally positive about the use of the history-infused lessons.

When and how calculus is introduced to students learning mathematics is an issue that mathematics educators have to grapple with. For example, Szydlik (2000) has claimed that, "[c]urrently educators debate whether calculus should be taught in an informal manner that allows students to discover the important ideas or presented to students as rigorous and structured" (p. 274). In Singapore the former approach is used, that is, calculus is introduced in an intuitive or informal way to students in Secondary 3 or 4 (grades 9 or 10) in Additional Mathematics, a more advanced course in mathematics for O-level students. The emphasis is on finding simple derivatives and integrals of some very simple algebraic, trigonometric and exponential functions and some related applications. A majority of the students solve problems in calculus using a set of prescribed procedural rules taught by their teachers, a point highlighted in a study by Ahuja, Lim-Teo and Lee (1998) at the Junior College level. In this study with 178 calculus teachers, these researchers revealed that there was a call by teachers for more opportunities for students to appreciate the applications and ideas of calculus in real life. One of such suggestions by the respondents was the call for a short history of calculus in problems relating to calculus.

Calculus has a rich history on which teachers can leverage to help students to learn about calculus. The use of the history of mathematics as a pedagogical tool has been advocated by many mathematics educators (Katz, 1993; Liu, 2003; Wilson & Chauvot, 2000). Despite the rich historical development of mathematics, the use of the history of mathematics appears to be absent in Singapore mathematics classrooms. Any infusion of the history of mathematics in the teaching of mathematics has to deal with the comfort level of the teachers themselves in this domain and as well with the interest and motivations of the students. Several reasons are put forward by the proponents of using the history of mathematics in the teaching of the subject. For example, Liu (2003) has suggested five reasons why teachers should teach the history of mathematics. First, the history of mathematics helps to increase students' motivation towards mathematics. Second, it improves students' attitude towards the learning of mathematics. Third, by looking at the historical development of mathematics, it is a useful tool to help understand why students have difficulties learning mathematics. Four, learning from mathematics problems in the past helps promote students' mathematical thinking. Lastly, history of mathematics helps to make mathematics a livelier subject and provide mathematics teachers with a structure in guiding their teaching of mathematics in the classroom. Several other researchers have proposed similar reasons for including the history of mathematics in

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the teaching of the subject (see Bidwell, 1993; Byers, 1982; Radford, Furinghetti, & Katz, 2007). While there may be strong reasons for using the history of mathematics as a pedagogical tool, teachers need practical information about how to implement history-infused lessons. Tzanakis and Arcavi (2000) listed 13 ways in which history of mathematics can be incorporated into the classroom. One of the suggestions was the use of a "historical package", an approach used in this study.

In this research, we were interested in the teaching of one particular area of mathematics, namely calculus, using its rich history as a pedagogical tool in a secondary school context. To assess students learning after the intervention, which included a history-infused lesson package on calculus, the following hypothesis was tested: students in the experimental group taught using the history lesson package will perform better than students in the control group.

Several studies have reported using some elements of history in the teaching of mathematics and these have mostly looked at the implementation from the teachers' perspectives. In this study, one additional point of interest was to look at students' views on using the history of mathematics in the teaching of calculus. Accordingly, the following questions were investigated: (1) how do students view the use of the history of mathematics in the teaching of calculus? and, (2) to what extent do students report benefiting from the use of the history of mathematics in teaching calculus?

Method

The study took place in an autonomous secondary school in Singapore. Four mathematics teachers (Ms. A, Mr. B, Ms. C and Mr. D) worked together as a team in a professional learning community (PLC), a community of practice (see Wenger, 2008). Stoll et al. (2006) have advocated that in a PLC, educators are motivated by a shared learning vision to explore effective teaching, curriculum and instruction aimed at impacting students' learning. The details of the how the PLC worked and developed the lesson package is not reported here. Further details on PLCs are available in Hord (2004) and DuFour and Eaker (1998).

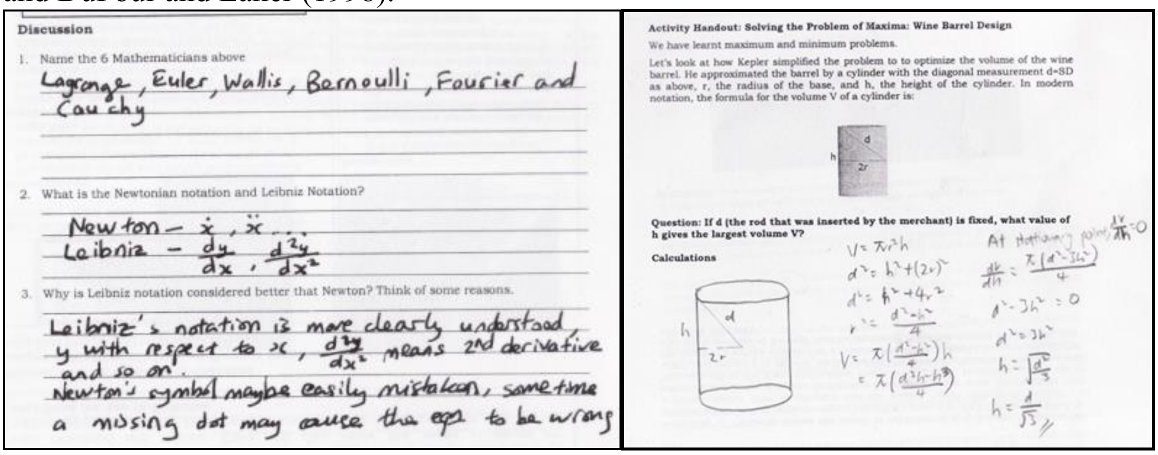

Figure 1. Some sample activities for students after watching the video clips

The PLC team worked on developing a historical package, which Bruckheimer and Arcavi (2000) have described as a collection of activities designed for a particular topic. The PLC team developed a historical lesson package containing six sets of activities. Each activity contained either historical facts or copies of original sources and was meant for two to three class lessons. The lesson package containing the activities was self-contained and included notes, videos, samples responses from students and guidelines for implementations by the teachers in the classrooms (see Figure 1 for a sample).

The target students were from secondary four (grade 10) and from three different classes. The 20 students from 4A were relatively low performing, the 21 students from 4B were average and the 15 students from 4C were high performing students, in fact they were scholars. 4A and 4B were really half-classes as the regular class was split into two for the teaching of Additional Mathematics, with a different teacher for each half-class. To assess the learning of the students through the intervention, a quasi- experimental design was used with students in 4B taught by Mr. D acting as the experimental group and the other half-class taught by a different teacher, not using the historical package, acting as the control group. Class 4B was chosen for this quasi-experimental part of the study as it was quite homogeneous in terms of students' performance. The following four subtopics from the school's scheme of work were examined in the post-test (historical aspects were not tested): (1) techniques of differentiation, (2) rates of changes, (3) maxima and minima, and (4) derivatives of trigonometric and exponential functions. There were no major threats to internal validity in this quasi-experimental part of the study.

To look at the students' views, we used pre- and post-study questionnaires with a larger group of students having different performance levels (4A, 4B and 4C). The reason for doing so was that, the 4B group was fairly homogeneous as mentioned above and also because we had the data from the implementation of the lesson package in the other classes. All participating students had to complete a pre-study questionnaire and four students from each class (12 in all) were interviewed. The items in the pre-study questionnaire that we report here are focused on two main domains: (a) the students' perceptions on the use of history in the learning of mathematics, and (b) the students' preferences on how history of mathematics can be infused into the teaching of mathematics. After the students had been taught using the lesson package, they completed a post-study questionnaire and 10 students of different performance levels were interviewed. The post-study questionnaire also contained an open-ended section asking students to write their thoughts about the history of mathematics in the learning of the subject. A five-point Likert scale was used in relevant sections of the two surveys.

Results

For the quasi-experimental part of the study, the data showed that experimental group (M=69.3, SD=20.3) and control group (M=72.9, SD=14.9) were similar as there was no significant difference in the pre-test means. The data for the post-test for the experimental group (M=85.4, SD=13.5) and the control group (M=76.1, SD=17.6) were used to test the hypothesis that students in the experimental group taught using the historical package will perform better than students in the control group. The t-value of 1.9368 (41df) is significant, $p<0.05$. The effect size is 0.59 using Cohen's d.

For the two research questions, the data collected from the students through the surveys and interviews helped to gauge their views on the use of the history of mathematics in the teaching of calculus and the extent to which they benefitted from the use of the history of mathematics in the teaching of calculus. We first report some findings about the students' preferences about the use of the history of mathematics from the pre-survey, then the rest of the pre-survey about the history of mathematics together with and the pre-implementation interviews and finally, some findings from the post-survey, interviews and the open-ended part of the post survey questionnaire. As it is not possible to comment on each and every item in the survey, we present some selected ideas.

In part (b) of the pre-study questionnaire, the students had to select preferred ways of infusing the history of mathematics in learning the subject from seven options. The students' top three choices were, (1) the learning of historical problems in mathematics, (2) learning about the controversies and arguments of mathematicians, and (3) the development of mathematical concepts and ideas. These suggestions about the students' preferences were then considered by the PLC team in the designing of the activities in the lesson packages.

In part (a) of the questionnaire, three domains were considered: (1) awareness of the historical development of mathematics, (2) beliefs about the usefulness of history of mathematics in the learning of mathematics, and (3) knowledge of the history of mathematics.

Pre-survey and Interviews

Students' awareness of the history of mathematics

Students from all three classes showed improvements in their awareness about the history of mathematics. For example, in the post-survey 90% of the low performing students (4A) agreed with the statement that mathematics has played an important role in all civilisations (up from 80%). The corresponding result was 100% for the average group (up from 80%) as well as the high performing group (4C).

Beliefs about the usefulness of the history of mathematics

In the post-survey, 35% of the 4A students agreed that "Everyone should learn about the history of Mathematics when learning mathematics" (up from 20%). Also, 40% of the students in this band agreed that "Knowing the history of mathematics will help me to understand both mathematics and other subjects" (up from 10%). There similar positive findings in the other two classes and again students from 4B, the average group, showed more gains. Some students' comments on the usefulness of learning the history of mathematics:

> Yes, because I think we learn mathematics as a subject, for me, I am interested to find out about the mathematics itself. I want to know how it developed and who developed it.
>
> Yes, because it is interesting. Then can know how they came about, all the equations and all laws come about. So that students can understand better and remember it better, for example, history of it then you will know it more.
>
> Well, because if you know the origin, you will understand the formula rather than memorising it.

Knowledge of the history of mathematics

Students demonstrated very limited knowledge about the history of mathematics. When asked about what they understood about by the term "history of mathematics" or "name some famous mathematicians", the students responses were very vague. Some responses follow:

> Who founded certain formulae and how it came about?
>
> How mathematics originated, who invented them. How it has help to better the society.
>
> Pythagoras, actually other than him, I also don't know already. It was always said very briefly, they just say "oh, this guy found it and that is it"

Post-survey and Interviews

After the implementation of the lesson package, infused with the history of mathematics, the data from the interviews and the open-ended section of the post-survey yielded some interesting comments from the students. There were four main areas: appreciation of mathematics, interest in mathematics, learning of calculus concepts, and reflecting on issues of mathematics.

Appreciation of mathematics

Many students quoted getting a deeper appreciation of the subject:

> The history of calculus has given me a background understanding of calculus and led me to appreciate the subject more.
>
> At least I know what is the history of calculus instead of just studying and learning it. I know how calculus can be used in our daily lives. I have learnt not to take mathematics for granted, as many people had dedicated their lives in finding out about Calculus.

Interest in mathematics

The lesson package on the history of calculus helped students gain a greater interest in the topic of calculus. Students found it enjoyable and interesting and felt that it was a good break from the normal mathematics lessons. They felt more motivated to learn more about the topic.

> By adding history of calculus to the learning of calculus, it spices up the lesson so that it will not be so boring and not merely about practicing calculus questions. Hence, it makes a maths lessons a more interesting experience.
>
> Learning the history of calculus makes my learning of the topic of calculus more interesting and engaging. As I have heard from my seniors in China that calculus was a difficult and tiring topic. After learning it in Singapore, I felt it was fun and I was motivated to learn more while listening to the stories of famous Mathematicians.

There were very few negative comments, which came mainly from students in the low performing group.

> The history of calculus did not have much impact on the learning of the topic of calculus as the concepts and the way in which the mathematicians worked out the rules of calculus. However, it does allow me to understand more about the history behind calculus and made me more interested in the topic of calculus.

Learning of calculus concepts

Students claimed that the pedagogical approach on infusing the history of calculus did help them to understand the topic of calculus. They felt excited learning and solving past historical problems of the calculus and viewing the videos on the mathematicians who had contributed towards the development of calculus.

> It enabled me to understand how calculus came about and this made my learning of calculus more interesting. It also helps to improve my understanding on calculus, especially on questions relating to maxima and minima. All in all, I have a deeper understanding of this topic and this made learning calculus enjoyable as I understand the meaning of learning calculus.
>
> I find calculus more interesting now, seeing how it has evolved from the past. I also find this topic easier to comprehend compared to the other topics in maths. When it is easier to comprehend, I can do better in this topic. Also, it makes me more interested to find out the history of other topics in mathematics.

However, there were a few students who felt that history of calculus was only suitable for students who are already good in mathematics, and they would rather concentrate on spending time practicing more questions than spending time learning about the history. At best, it was only a kind of break away from the normal mathematics lessons.

> Telling the truth, the lessons didn't help much in improving scores of calculus. It is only useful to those who already scored well and was interested in math and wanted to know about math. So, it is not suitable for all the students. I know how calculus was invented in the lesson, thus I won't be worry about forgetting the formula of differentiation.

Reflecting on issues about mathematics

The history infused lesson gave the students the opportunities to develop rich understanding of mathematics and to deepen their insights towards mathematics. It helped them to develop their thinking skills and to open up their minds.

> It makes the topic more interesting and linked to reality. It makes me curious to know of more mathematics and how they derived what they did. It makes me think of Newton and Leibniz occasionally when I am doing questions on the topic. It gave me a better overview on the purpose of Calculus and mathematics.

> I think it (mathematics) has come a long way; it is deeper than I thought. It requires a lot more study to obtain the things that we study now. It is not as easy as I thought.

Discussion and Conclusion

The idea of combining the history of mathematics with the teaching and learning of mathematics led to the founding of the International Study Group on the relations between History and Pedagogy of Mathematics (HPM) in 1976, a group affiliated to the International Commission on Mathematical Instruction (ICMI). In the ICMI study on *History in Mathematics Education*, Tzanakis and Arcavi (2000) listed down five main areas where the history of mathematics can play a role in mathematics education. The five areas are in (a) the learning of mathematics, (b) the development of views on the nature of mathematics and mathematical activity, (c) the didactical background of teachers and their pedagogical repertoire, (d) the affective predisposition towards mathematics, and (e) the appreciation of mathematics as a cultural-human endeavor (see pp. 203-207). These areas were amply visible in the study we report here. Although, we do not report the teachers' professional development in the PLCs here, we have no doubt that the experience was beneficial to all of them in the PLC team.

The significant difference between the post-test results of the experimental and control groups demonstrates that the intervention was successful. So, there is value in using the historical development of calculus in the teaching of the subject in school. The results of this study concur with results of earlier research conducted by Lim (2011) at the Junior College level and the results from Ng (2003) (about ancient Chinese mathematics at the secondary level) that the history of mathematics seemed to help the students to achieve better achievement scores in their tests. We may argue whether a significant post-test result for the experimental group is a good proxy for the students' learning. Although, the Additional Mathematics syllabus focuses on a restricted set of competencies in the use of an intuitive approach to calculus, we nevertheless feel that the students in the experimental group showed better performance in solving some calculus problems.

In the other areas highlighted by Tzanakis and Arcavi (2000), the students certainly improved. The students were generally receptive in the new approach of learning calculus. Many of the students felt that learning the history of mathematics had allowed them to appreciate mathematics better and they were more interested in the topic of calculus, and were motivated to read up more about the topic themselves. Students like to hear stories and hearing stories of the lives of mathematicians and their contributions towards mathematics made them interested in the topic that they are learning. Greater motivation and increased interest through the history infused lessons could be possible reasons why students involved in the history of calculus scored better than the other groups of students not involved in the study. These results strongly support that learning can be more effective through the learning of the history of mathematics. The comments by the students who participated in the study attest to the fact that they had enhanced motivations and interests, as also highlighted by Liu (2003).

However, on the beliefs of the usefulness of history of mathematics, results vary among the three different ability groups. Although there was an increase in the percentage of students from all the three bands agreeing to the usefulness of infusing history of mathematics, students from the lower performing band seemed the least convinced in the actual practicality in helping them to learn mathematics. This can be gathered from the post-questionnaire findings that only 15% of this group of students agreed to the statement "Learning the history of mathematics will allow me to have a better understanding of concepts and the techniques in mathematics ", compared to 74% and 92.9% for average and high ability students. Likewise, only 40% of the low performing students agreed that "knowing the history of mathematics will help me to understand both mathematics and other subjects." compared to 87% and 100% for average and high performing students. Insights gathered from the open-ended responses and the interviews also confirmed that low performing students felt that they would rather spend time on mastering the content than spending time on the learning the history of mathematics, although the history of mathematics itself was not the focus of the lessons. Thus, care must be taken into consideration when implementing any new pedagogical tool on this group of students who may already be struggling with mathematics.

Implementation of lessons infused with the history of mathematics is quite hard for most teachers. If such lessons are not properly thought about then instead of helping students the lessons may have quite the opposite effect in distancing students from mathematics. While many researchers have argued for the inclusion of the history of mathematics, there are a few others who cautioned about its use in the classrooms. Fried (2001) warned that mathematics history and mathematics education cannot co-exist unless mathematics educators deal with the issue of a "whiggish" approach to the history of mathematics. It must be emphasised here that in this study the approach was carefully planned by the group of teachers in the PLC and was trialled and improved through a series of lesson study sessions. The students' preferences were taken into consideration in the planning of the activities. The materials developed for the teaching of the subject had the right amount of historical connections to gain the students attention.

We acknowledge that the study was focused on a relatively small group of students from one secondary school and as such the findings may not apply to all of the secondary schools students in Singapore. Nevertheless, the study demonstrates that history-infused lessons can be beneficial to students. We suggest that similar approaches could be used in teaching the same topic or different topics.

Calculus is an important strand of the Additional Mathematics syllabus, and has uses in many other fields of study. It is therefore important that students acquire a broader knowledge about calculus and the role it plays in mathematics and other disciplines. One possible way as seen from this study is through the infusion of the historical development of calculus into the teaching of calculus.

The study stemmed from a desire to encourage teachers to learn about the use of the history of mathematics in the teaching of the subject. The study has also enabled us to understand students' perceptions of mathematics and the use of the history of mathematics in their learning of mathematics. The findings of this study will certainly contribute to the existing knowledge about incorporating the use of history of mathematics in the classroom and act as an impetus for future work both for researchers and curriculum planners. We strongly urge teachers in the mathematics fraternity to incorporate the history of mathematics in their teaching of mathematics and hope this study would help in convincing curriculum leaders to see the value of the history of mathematics and infuse it in future mathematics curriculum.

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