Supporting Children with Special Needs in Learning Basic Computation Skills: The Case of Mia

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This paper introduces a revised model for the development of basic computation skills. The model draws on four key phases, which have proven to be important for the development of calculation strategies and stresses the use of gestures and the verbalisation of concrete and mental images. This seems to be of crucial importance for children with special needs as the case of Mia illustrates. Context is a university based intervention program that seeks to support children who struggle with the learning of basic arithmetic concepts and skills.

Introduction

Children with severe mathematical learning difficulties frequently struggle from early on with the development of basic computational ideas. In many occasions they leave their teachers and parents helpless and continue to struggle with their mathematics learning during their entire school years. In order to find ways to help children with the learning of basic arithmetic in 1980 Jens Holger Lorenz founded a *counselling centre* at Bielefeld University. It was the first centre of this kind in Germany following the establishment of so called "math clinics" at North American universities. From its foundation the centre aimed at three strands: (1) to provide an intervention program for children experiencing severe difficulties with the development of computational understanding and skills, (2) to prepare future teachers of primary schools mathematics to support these children in both intervention and preventive teaching, and (3) to use the video data from the one-on-one interventions for research. The analysis of this data and its interpretation on the basis of relevant research on the development of basic arithmetic ideas and skills indicates that the children participating in the intervention program predominantly struggle in three areas. They have not yet managed to develop

- a deep understanding of place value, e.g., the majority of students up to Grade 4 cannot tell the difference between the numbers 34 und 43 and frequently claim that they are "the same", because they involve the same digits (Rittle-Johnson & Siegler, 1998),
- operational insight and basic ideas, in German "Grundvorstellungen" (vom Hofe, 1998), that enable them to understand the concept of addition and subtraction (e.g., addition as taking together quantities and subtraction as taking away a quantity from another) including changes between different modes of representation,
- derived-fact strategies for addition and subtraction (Gaidoschik, 2012), i.e., they solve respective problems with varying counting strategies, such as *count all*, *count on* and *count down* (Fuson, 1992; Geary & Hoard, 2005).

Research suggests that characteristic indications of severe learning difficulties in whole number arithmetic are primarily the exclusive and rigidified dependence on counting strategies for solving basic addition and subtraction problems (Gaidoschik, 2012; Geary & Hoard, 2005) and an insufficient understanding of place value (Rittle-Johnson & Siegler, 1998; Resnick, 1983). Low achieving (primary) students in early arithmetic typically use

2016. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia), pp. 568–575. Adelaide: MERGA.

counting strategies especially for problems with numbers up to 20 (Gray & Tall, 1994) and *split number strategies* such as *separate-tens* and *mixed method* (Fuson et al., 1992) for two-digit numbers, which are less successful than *complete number strategies* such as *sequence-tens* and *adding-on* (Foxman & Beishuizen, 2002). In addition, students who experience difficulties with the acquisition of computational skills often use materials and manipulatives exclusively for counting activities without utilising any structure of the material such as units of 5 and 10 (Geary & Hoard, 2005).

Instructional approaches focus on physical actions with manipulatives and on assisting the gradual shift from material to mental images. Especially for students with learning difficulties a meta-analysis of instructional components in several interventions (Gersten et al., 2009) demonstrates a consistent significant effect for *using visual representations while solving problems* on the mathematics proficiency of students as “the second most strongly influencing factor after *explicit instruction*, which is characterized by the demonstration of a problem specific step-by-step plan or strategy” (p. 1228). Wartha and Schulz (2011) have developed a so called *four-phases model* to assist teachers and learners with the gradual transition from the active use of manipulatives to mental strategies which has been informed by and guided the intervention program at Bielefeld University in the last decade.

In this paper we argue that while this *four-phases-model* identifies four crucial steps in the learning process, it is too broad to effectively assist all children struggling with basic computation tasks as described above. Hence, we introduce a revised model that includes sub-steps that foster the use of gestures and the (further) verbalisation of concrete and mental actions while solving computation tasks. The case of Mia, a 7-year-old girl attending a special school for children struggling with the acquisition of language and communication, is used to illustrate how the sub-steps help her with the development of mental calculation skills.

Theoretical Framework: The Role and Choice of Manipulatives for the Development of Basic Computational Understanding

An important purpose for using manipulatives is to foster the development of mathematical concepts and (calculation) strategies. In this sense, manipulatives are concrete representations of abstract mathematical concepts (Chao, Stigler, & Woodward, 2002) and “allow children to establish connections between their everyday experiences and their nascent knowledge of mathematical concepts and symbols” (Uttal, Scudder, & De Loachie, 1997, 38). Nevertheless, the role of physical material as a *representation* is not necessarily obvious for children. In this context Uttal, Scudder, and DeLoachie (1997) point out a *dual-representation hypothesis*, i.e., any concrete manipulative can be thought of as a representation that stands for something else or as an object on its own. The latter view might be a reason for learning difficulties:

> Concrete objects can help children gain access to concepts and processes that might otherwise remain inaccessible. However, there is another side to the use of concrete objects: children may easily fail to appreciate that the manipulative is intended to represent something else – that it is a symbol. If so, the manipulative will be counterproductive. (p. 52)

Hence, in order to use a manipulative in a constructive way it is necessary for a child to become familiar with its structure. Therefore, it is important for school mathematics instruction to first identify a suitable manipulative and then to portray and explore it as a learning aid. For a number of children the teacher’s explicit instruction is required to support the development of mathematical concepts and strategies based on direct modelling activities (Uttal et al., 1997; Rottmann & Schipper, 2002).

The intervention that provided the context of this paper is based on manipulatives that model the strategies to be ultimately developed on a cognitive level. However, it has to be acknowledged that all manipulatives and visualizations, that are used to illustrate mathematical concepts, need to be learned and understood before they can be drawn on in developing mathematical understanding. Hence, only two specific materials were chosen that are widely used in international classrooms as they best lead to the respective mathematical concepts and internal models. In order to enhance children's understanding of place value "Multibase Arithmetic Blocks" (MAB), see Figure 1, also called "Dienes Blocks" (e.g., see Rittle-Johnson & Siegler, 1998), which stress the cardinal understanding of number were chosen and used. In addition, the twenty-frame (Figure 2) and the arithmetic rack (Figure 3) was selected because it fosters the replacement of counting-based calculation strategies with derived-fact strategies. The use of the arithmetic rack helps learners to visualize any number up to 100 quasi-simultaneously and the derived fact strategy 23+7+2=32 can be modeled without having to count all single objects.

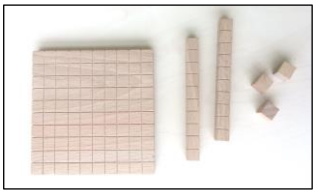

Figure 1
Multibase Arithmetic Blocks

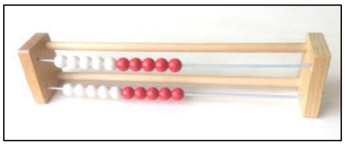

Figure 2
Twenty-frame

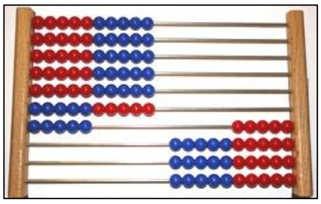

Figure 3
Arithmetic rack

The role of manipulatives for learning processes in mathematics has been extensively discussed since the 1970s. Concrete actions on manipulatives are commonly seen as a necessary foundation for mathematical concepts (Piaget & Inhelder, 1971; Bruner, 1973). However, Gray, Pinto, Pitta, and Tall (1999) point out that there are substantial differences between low achievers and high achievers in the way they perceive and remember the physical actions. Whereas high achievers focus upon the semantic mathematical aspects, low achievers mainly focus upon the physical and procedural aspects of the activity and remember incidental, often irrelevant detail of the material.

> On the one hand, we see a cognitive style strongly associated with invoking the use of procedures, on the other a style more in tune with the flexible notion of procept. Those using the latter have a cognitive advantage; they derive considerable mathematical flexibility from the cognitive links relating process and concept. (p. 120)

Gray and his colleagues (1999) stress that high achievers distinguish themselves with the ability to connect numerical symbols with an action schema to perform any required computation. According to these findings concrete actions on manipulatives do not necessarily "automatically" lead to the development of mathematical concepts and mental strategies. A successful learning process depends on the ability to focus upon the relevant aspects of the actions and thereby to link the symbolic representation and the enactive representation (in terms of concrete actions on manipulatives). Complementary to these two representational systems (enactive and symbolic) the iconic representation is frequently described as a third system. Bruner (1973), who attributes pictures and mental images to the iconic representation, strongly links learning processes to translations of one representational system into another. Extending Bruner's theory the Swiss psychologist Aebli (1976) additionally describes gradual internalization processes from enactive to mental actions, which focus on the transition from one representation to another.

To support the development of mathematical concepts and mental strategies especially for students that experience severe learning difficulties, assisting such internalization processes is seen as essential (Wartha & Schulz, 2011; Rottmann, 2004). Mental images and representations should gradually replace concrete actions on manipulatives. Children, who are extremely vulnerable with the learning of basic computation skills, tend to use the material as a sole counting aid and/or do not manage to develop strategies beyond the use of the concrete objects (Rottmann & Schipper, 2002). Frequently, they cannot describe their actions, which can be interpreted in a way that they do not understand what they do and how this relates to a computation strategy other than counting.

A Revised Conceptual Model for Intervention and Classroom Instruction

In order to support the gradual shift from the use of manipulatives to the development of mental images, Wartha and Schulz (2011) introduced a *four-phases-model*. This model involves the three types of representational systems – enactive, iconic and symbolic – as described above. It deliberately acknowledges the need for verbal descriptions when using manipulatives as well as a gradual progression from concrete actions to mental operations that activate a mental concept which allows the child to imagine the actions required in order to solve an addition or subtraction problem such as 16 + 27 or 41 – 28.

Table 1
Four-phases model to support the development of computational ideas and skills by Wartha and Schulz (2011)

Phase 1	Concrete use of manipulatives and respective verbalisation of operations
Phase 2	Verbal description of the use of the manipulative in sight
Phase 3	Verbal descriptions of the imaginative use of the covered manipulative
Phase 4	Verbal description of the mental operation

In Phase 1 teacher[1] and child actively use the material and verbalise their actions while solving a(n) addition/subtraction task. When the child is confident in using the structure of the material (i.e., strategies other than counting), they take over and verbalise their actions. In Phases 2 and 3 the concrete use of manipulatives is replaced by its verbal description: The child describes the material based actions (without touching it herself) and the teacher performs the described operations. While in Phase 2 the child can see the manipulative in use, in Phase 3 it is covered and therefore the teacher's operations are invisible for the child. Finally, in Phase 4 the child verbally describes her mental operations without the manipulative being present in any form other than the child's imagination (ibid; also see Rottmann & Peter-Koop, 2015).

While this *four-phases-model* provided a suitable guiding structure of the 12-week intervention program for some of the children (for example see the case of Ole in Peter-Koop & Rottmann, 2015), it needed adaptions for others. Especially the transitions from Phase 1 to 2 and from Phase 2 to 3 were difficult to master for a number of children. The revised

[1] In the university-based intervention program pre-service primary school teachers conduct the interventions in pairs under the close supervision and with the support of experienced staff.

model (see Table 2) therefore introduces two sub-steps in each phase to ease the transition from one phase to another involving an increasing degree of difficulty.

Table 2
A revised conceptual model for intervention and classroom instruction

Phase 1	**Concrete use of the manipulative**
1a	The child uses the manipulative for a strategy based on counting in ones.
1b	The child uses the specific structure of the manipulative for a non-counting strategy.
Phase 2	**Verbalisation of the operational action with the manipulative (partially) in sight**
2a	With the manipulative in sight, the child describes the material based actions to the teacher/a peer, who performs the actions according to the child's descriptions.
2b	The child describes the display of the first number, which is enacted by the teacher/a peer, before the material is covered and the child describes the operational actions.
Phase 3	**Verbalisation of the imaginative use of the covered manipulative**
3a	With the manipulative covered by a screen the child describes the respective actions to be executed by the teacher/a peer. When needed, the screen is lifted temporarily.
3b	The manipulative remains covered during the entire description of the imaginative use of the manipulative.
Phase 4	**Mental calculation**
4a	The child verbalises the calculation process based on the imaginative use of the manipulative.
4b	The child solves the task mentally without any verbal references to imaginative actions with the manipulative.

We argue that this extended model not only helps to further structure individualised intervention programs for children who severely struggle in learning basic arithmetic and to better cater for their individual needs, but also informs classroom instruction with respect to the teaching and learning of computational understanding and skills.

The Case of Mia

Mia, a second-grader attending a special needs school with a focus on language acquisition and communication, attended the university-based 12-week intervention program between October 2015 and February 2016. When entering the intervention program Mia's sole strategy for solving simple addition and subtraction problems with numbers up to 20 was counting with the help of her fingers or the twenty frame/arithmetic rack. She did not demonstrate any use of derived fact strategies. After an initial assessment in August 2015 that led to her acceptance in the program, the first intervention started in October 2015. Apart from focussing on the extension of her counting skills and the development of insight into part-part-whole schema (Resnick, 1983), the aim of the first sessions for Mia was to learn to use the manipulative, in this case the twenty-frame (see Figure 2) and to develop an understanding of its structure with respect to subitizing and displaying numbers by using bigger subunits.

Data Collection and Analysis

With the consent of her parents, all weekly intervention sessions have been video-taped and partially transcribed; written documents of the detailed planning and reflection of each 60 min session completed the data collection.

In order to identify the obstacles that children face in the different phases and with respect to transition processes from one phase to another, the data analysis was based on the comparison of relative quantities. Hence, based on the video data of the twelve sessions, we analysed the solution processes of the selected task type across all the sessions in order to identify the different phases the child was working in, and to quantify the success rate for each of the phases.

In addition we documented (transcribed) and classified the gestures Mia applied in the different phases in addition to her verbal explanations, which were also transcribed. In this way four types of gestures were identified with respect to the twenty-frame/arithmetic rack: (i) touching *and* indication of moving one or more beads, (ii) touching of one or more beads, (iii) pointing at individual beads from a short distance, and (iv) pointing at the manipulative from a distance indicating how to move the beads. Table 3 demonstrates the data analysis for the respective section of the video transcript in session 5.

Table 3
Example of the data analysis

Session	Date/Time	Task	Phase	Result	Gesture
5	18.11.15 11:20 min	7+5	1b	correct	(material based action)
		9+5	1b	correct	(material based action)
		7+5	2a	incorrect correct	1st attempt: (5 (gesture iv) + 2) + 3 + 3 2nd attempt: - 1 (gesture iii) = 12
		6+5	2a	correct	(5 (gesture iii) + 1 (gesture iii)) + 4 (gesture iii) + 1 = 11

Summary of key results

The overview of the different phases of Mia's solution processes during the entire 12-week intervention (see Table 4) focuses exclusively on addition and subtraction tasks of the type *1-digit-number/2-digit number plus/minus 1-digit number*[2]. The intervention seeks to enable children to successfully apply the strategy *bridging tens* (Foxman & Beishuizen, 2002), e.g., 8 + 7 = 8 + 2 + 5 or 34 – 7 = 34 – 4 – 3). In Phase 1 Mia predominantly demonstrated counting strategies when adding the second addend or subtracting the subtrahend. This represents Phase 1a. In this phase she does not verbalise her solution apart from giving the result. In the transition from Phase 1 to Phase 2 in session 4 however, gestures become important when Mia demonstrates her growing understanding of the part-part-whole concept as the following excerpt from the video-transcript indicates:

Session 4, Phase 2a (use of gestures), Twenty-frame Time (18:08)

Pt*	*nine plus five*
Mia	*first the nine*
Pt	*how do I make nine?*
Mia	*first the five* [points at the fifth bead in the top row, pt moves fives beads to the left] *and then the four* [points to the ninth bead in the top row; pt moves four more beads to the left]
Pt	*plus five ... how do I do that?*
Mia	*this* [touches the tenth bead in the top row]
Pt	*one* [moves the bead to the left] *and how many are still missing?*
Mia	*four* [points at the beads in the bottom row]
Pt	[moves four beads from the bottom row to the left] *what is the result?*
Mia	*… fourteen*

* Pre-service teacher

[2] Other contents that were covered in the different intervention sessions were counting activities, place value understanding including the translation of number symbols into number words and part-part-whole-schema.

While working in Phase 2, Mia uses distinctive gestures in order to accompany the verbal description of the solution process. In contrast to Phase 1, her verbalisations clearly increase in Phase 2 due to their growing relevance for the interaction with the pre-service teacher. Furthermore, as the analysis of the video data from session 8 onward indicates that with Mia's increasing competence in the verbalisation of her solution strategy, her use of gesture decreases.

With respect to the overall intervention across the twelve weeks (Table 4) two more aspects arise. The transition from Phase 2 to Phase 3 seems to be crucial for Mia. The demand to conduct the use of the manipulative entirely on the mental level correlates with a substantially decreased success rate.

Table 4
Overview of Mia's solution processes on tasks of the type 1-digit number/2-digit number plus/minus 1-digit number during the 12-week intervention program

	Week of intervention												% correct
	1	2	3	4	5	6	7	8	9	10	11	12	
Phase 1a			6/1*	7/0	2/0	0/3	1/0	1/0		1/0		0/1	78
Phase 1b			1/0		4/0		3/0			1/0			100
Phase 2a				1/2	6/4	5/0	9/1	5/0	6/3	6/2	1/1		75
Phase 2b								3/0					100
Phase 3a										3/7	0/4	3/2	32
Phase 3b								0/3			1/2	1/3	20
Phase 4a													
Phase 4b								4/0	3/0			4/0	100
% correct			88	80	75	63	93	81	75	55	29	57	

* Number of tasks being correct/incorrect

Surprising is the significantly decreasing success rate in weeks 10 to 12. During weeks 1 to 8 the number space was limited to 20. In week 9 the pre-service teachers responsible for the intervention extended it to numbers up to 40. However, in this number space Mia still struggled with the formation of number words, which explains the increased rate of error when solving tasks involving numbers larger than 20.

Discussion and Implications

The case of Mia highlights the difficulties children may experience when shifting from material based-actions to mental images. While the revised conceptual model introduced in this paper demonstrates an attempt to ease the transition from Phase 2 to Phase 3, which turned out to be a crucial developmental step for Mia, further analyses of other case studies are needed to better understand the obstacles and challenges children face when learning computational skills. For example, while Phases 2b and 3a appear to be rather similar, they surprisingly and unexpectedly differ with respect to the success rate Mia demonstrates. This aspect needs further investigation.

Mia's case is one example of our attempt to individually adapt the intervention in a way that supports the transition from Phase 2 to 3. Further case study analyses of children experiencing a variety of individual difficulties are needed including detailed descriptions of respective arrangements of the sub-steps introduced in the revised model. For example, the pre-service teachers working with Mia demonstrated two different approaches to deal with Phase 3a. While the intention of this phase initially was to lift the screen only if the child showed difficulties, Mia's teachers fostered her solution process by lifting the screen when the first addend (or minuend) had been arranged on the hidden twenty-frame to assist her in finding out how to split the second addend (or subtrahend) appropriately.

Further stages of the research project will also include variations of the intervention setting, for example small groups or pairs instead of individual children as in the *Extending Mathematics Understanding* (EMU) intervention (Gervasoni, 2015).

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